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ABSTRACT

This curriculum for mathematics is designed as a teachable curriculum directed toward workplace needs. It is intended to be workplace specific and to move the learner toward a full 12th-grade level attainment. The two modules in this curriculum are further divided into modules dealing with specific topics. The first module, Basic Mathematics, consists of these seven parts: whole numbers, decimals, fractions, percents, measurement, ratio, and graphs and charts. The second module, Algebra 1, Geometry 1, & Statistics, has these seven parts: introduction to algebra, introduction to geometry, further development of algebra, ratio and proportion, further development of geometry, statistics, and the mathematics of linear equations. Each module is broken down into concepts, which are explained with examples and applications. Sources of drill and practice exercises are provided for each concept.
(YLB)

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WORKPLACE MATHEMATICS MODULES 1 & 2

A WORKING CURRICULUM OF THE FINGER LAKES REGIONAL EDUCATION CENTER

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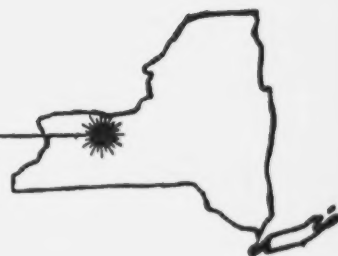
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1988



WORKPLACE MATHEMATICS MODULE 1 & 2

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Rochester City School District



**FINGER LAKES REGIONAL EDUCATION CENTER
FOR ECONOMIC DEVELOPMENT**
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FOREWORD

Tom Peters indicated in Thriving on Chaos that Workforce training needed to become a national obsession. We are now in a time when it must become the obsession described by Peters. As we begin to realize the need for Workplace Literacy skills it has become clear that the skills of the workplace of the past are not those of the present or (indeed) of the year 2000. We have found that sometimes as many as six of every ten of us who work in industry, work in service sector business, labor behind a CRT, or function in management fail in at least one of the five areas of adult workplace literacy (Reading, Writing, Speaking, Listening, Math) at the full high school level. Some sectors of business and industry have discovered this fact earlier than others. Some have identified the need for basic skills at this "12th grade" level as they have changed technology.

Please note that we are not necessarily talking about the typical ESL, ABE, GED need alone! We are also speaking of the need to problem solve, work in groups, know basic computer functions, or give an informal speech in support of an idea or innovation. The truth is, however, that over 20% of us do not have a high school diploma, still others speak English as a second language, and others have basic education skills which (for lack of a better term) have atrophied over time. There is nothing magical about the '12th grade' level except that it is the level of attainment that we generally identify as attaining basic skills. I tried to address the overall issue of Workplace Literacy at the 12th grade level in the paper "Workplace Literacy: Raising the Floor of Education in the Workplace" This paper defined Manufacturing Literacy as "the ability to read and write at the twelfth grade level, to speak and listen with clarity and understanding in a team or work group, to have mathematics skills at a twelfth grade level sufficient to handle SPC (Statistic Process Control) problems, to have problem solving skills, and to have basic computer literacy skills." What is clear is that Workplace Literacy is different from general adult literacy in that Workplace literacy seeks to meet the goals of the individual and the employer at the same time. Since we began the development workplace literacy programs in our region we have moved from providing very specific curricula for each project to the more generic approach of this curriculum. That is not to say that the specific needs of the workplace are not important, indeed, they are. Tailored use of this curriculum is absolutely desired! What is present here is a shell that can be moved from setting to setting, workplace to workplace without the necessity of "reinventing" the wheel for each industry or business.

The Finger Lakes Regional Education Center has identified several areas of CORE SUBJECT MATTER which make up Workplace Literacy Skills for our region . At the end of this curriculum document you will find of full listing and timetable for the development of these material. The curriculum that follows has been developed by the center as a "teachable" curriculum directed toward workplace needs. It is not designed to be "easy" or necessarily the "end-all and be-all" of workplace curricula. It is not a "quick fix" for a very significant problem either. Instead, it is designed to be workplace specific and to move the learner toward a full 12th grade level attainment. Finally, I would encourage companies that use this material in their setting to make at least some of this training "on the clock." It is only in this way that a uniform level of attainment can be reached which will allow more technical firm specific training to take place.

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David M. Mathes
Executive Director
Finger Lakes Regional Education Center

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I. INTRODUCTION

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III. Bibliography

MODULE 1

BASIC MATHEMATICS

A. WHOLE NUMBERS

1. CONCEPT: UNDERSTANDING OUR NUMBER SYSTEM

Our number system is called the decimal system. Decimal means that the system is based upon ten symbols (1, 2, 3, 4, 5, 6, 7, 8, 9 and 0). All numbers can be represented by these symbols in conjunction with some other special symbols. These ten symbols are called digits. The position of the symbol is important. This is called place value.

5	means	5 ones
50	means	5 tens
500	means	5 hundreds
5000	means	5 thousands
50000	means	5 ten thousands
500000	means	5 hundred-thousands
5000000	means	5 million

0 means "zero" and is often used as a place holder; that is, it holds a place when that position is not occupied by another symbol. (Example: 103)

Example: 936 means 9 hundreds, 3 tens, 6 ones

Example: Read 2436. Two thousand four hundred thirty-six

(Do not use the word "and".)

Example: 4,056,906. Four million, fifty-six thousand nine hundred six.

Numbers such as 9, 34, 312, 1654 are called whole numbers or integers. Zero is also considered an integer.

Application 1: The departmental budget is \$56,072.
Read this number.

Ans. Fifty six thousand seventy-two dollars

Application 2: A counter on a production machine shows 6497.

Read this number.

Ans. Six thousand, four hundred ninety-seven.

Drill and Practice:

Shaw, Kane, and Merseth, Fundamentals of Mathematics,

Houghton-Mifflin, 1986, pp. 2-3

Howett, Jerry, GEO, The Mathematics Test,

Contemporary Books, Chicago, 1984, pp. 23, 24

Dressler, Preliminary Mathematics, Amsco, New York, 1962, p. 7, 8

2. CONCEPT: THE DECIMAL SYSTEM AS A SYSTEM OF BASE 10

If we write 3×2 , we mean 3 times 2 or 3 multiplied by 2. However, if we write 3^2 where the 2 is a smaller size than the 3, we mean 3×3 . Therefore $3^2 = 3 \times 3 = 9$. If $3^2 = 9$, the 3 is called the base. The 2 is called an exponent.

Example: $7^2 = 7 \times 7 = 49$
 $8^2 = 64$
 $4^3 = 4 \times 4 \times 4 = 64$
 $10^4 = 10 \times 10 \times 10 \times 10 = 1000$

We read these: 7^2 as 7 to the 2nd power or 7 squared
 4^3 as 4 to the 3rd power or 4 cubed
 10^4 as 10 to the 4th power
 9^5 as 9 to the 5th power

The bases are 7, 4, 10 and 9.

The exponents are 2, 3, 4 and 5.

Example: Write 64 with a base of 8:
 $64 = 8 \times 8 = 8^2$
 Write 32 with a base of 2:
 $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$
 Write 1000 with a base of 10:
 $1000 = 10 \times 10 \times 10 = 10^3$

The Powers of 10

$$\begin{aligned} 10 &= 10^1 \\ 100 &= 10 \times 10 = 10^2 \\ 1000 &= 10 \times 10 \times 10 = 10^3 \\ 10000 &= 10 \times 10 \times 10 \times 10 = 10^4 \\ 100000 &= 10 \times 10 \times 10 \times 10 \times 10 = 10^5 \\ 1000000 &= 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6 \end{aligned}$$

Example: Write 5000 as the product of a one-digit number and a power of 10.

$$5000 = 5 \times 1000 = 5 \times 10^3 \text{ [5 times ten cubed]}$$

Example: Write 8327 as a sum of terms which (except for the unit's digit) are represented by product of one digit and a power of 10.

$$8327 = 8000 + 300 + 20 + 7 = 8 \times 10^3 + 3 \times 10^2 + 2 \times 10^1 + 7$$

Application 1. The construction budget of a large corporation was \$8,000,000. Write this number as the product of a one-digit number and a power of 10.
 $8,000,000 = 8 \times 1,000,000 = 8 \times 10^6$
or "eight times ten to the sixth power".

Application 2. A space probe to the sun traveled 90,000,000 miles. Write this number as the product of a one-digit number and a power of 10.
 $90,000,000 = 9 \times 100,000,000 = 9 \times 10^7$

Application 3. An odometer shows 63471. Rewrite this number as the sum of terms which are a product of a digit and a power of 10 (except for the unit's place).
 $63471 = 60000 + 3000 + 400 + 70 + 1$
 $= 6 \times 10^4 + 3 \times 10^3 + 4 \times 10^2 + 7 \times 10^1 + 1$

Drill and Practice:
 Dressler, p. 12, 13

Application 1: The following figures are production data showing the number of small machine parts turned out during a year. Read or write in words.

<u>Month</u>	<u>No. of Parts</u>	<u>Write (Words)</u>
Jan.	1,607	One thousand, six hundred seven
Feb.	209	---
Mar.	2,414	---
Apr.	10,532	Ten thousand, five hundred thirty-two
May	22,603	---
June	9,816	---
July	12,004	Twelve thousand, four
Aug.	18,760	---
Sept.	24,091	---
Oct.	56,221	---
Nov.	63,409	Sixty three thousand, four hundred nine
Dec.	<u>66,519</u>	---
Annual Total	288,185	<u>Two</u> hundred eighty-eight thousand, one hundred eighty-five

If this production were maintained for five years, the total number of machine parts would be:

5x288,185 or 1,440,925 -- which is read:
 "one million, four hundred forty thousand nine hundred twenty-five".

Application 2. At a term meeting the supervisor gave several production cost figures without showing them on an overhead or chalkboard.

He said "for 1985: one million three hundred sixty five thousand, seven hundred thirty-two.

for 1986: seventeen million, four hundred five thousand, sixteen..

What figures should you have written in your notes

for 1985: 1,365,732

for 1986: 17,405,016

Drill and Practice:

Howett, p. 24

Dressler, pp.14-16

4. CONCEPT: ROUNDING OFF WHOLE NUMBERS (INTEGERS)
(Rule)

1. Underline or box in the digit to which you intend to round off.
2. Look at the digit to the right of the underlined digit.
3. If the digit to the right of the underlined digit is equal to a number less than 5 keep the underlined digit.
4. If the digit to the right of the underlined digit is equal to 5 or more than 5, add 1 to the underlined digit.
5. Replace with zeros all digits to the right of the underlined digit.

Example: Round off 63,172 to the nearest thousand.

- 1) 63,172
 - 2) Since the digit to the right of 3 is 1 keep the 3
 - 3) Replace 1, 7, 2 with zeros.
- Therefore 63,172 becomes 63,000 correct to the nearest thousand.

Application 1: An inside diameter of a tube is 127 mm. Round this number off to nearest ten millimeters.
1 [2] 7 becomes 130 mm. correct to the nearest ten millimeters.

Application 2: 3276 carburetors are stored in an automated warehouse. To the nearest hundred, how many carburetors are stored? 3 [2] 76 carburetors become 3300 carburetors

Drill and Practice:

Howett, pp. 25-26

Dressler, pp. 17-18

Shaw, p. 4

5. CONCEPT: ADDITION OF WHOLE NUMBERS (INTEGERS)
(Rule)

To add (find the sum) of several whole numbers:

- 1) Arrange the unit's digits under each other, the ten's digit under each other and so on.
- 2) Add each column starting from the right.
- 3) If the sum of that column is less than 10 write the sum at the foot of the column.
- 4) If the sum of that column is 10 or more write the unit's (one's) digit at the foot of the column and "carry" the ten's digit to the top of the next column.
- 5) Ten, add the next column, including the "carried" ten's digit from the previous column.
- 6) Continue until all columns have been added.

Example: Add (find the sum) of 632, 77 and 1420.

632	632	632	632	632
77	77	77	77	77
<u>1420</u>	<u>1420</u>	<u>1420</u>	<u>1420</u>	<u>1420</u>
	9	29	129	2129

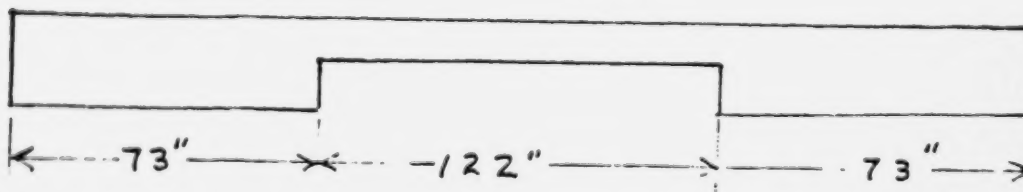
Note: To check an addition problem, repeat the addition in the reverse order:

Problem: 971	Check: <u>5308</u>
16	971
<u>4321</u>	16
5308	4321

Application 1. In an inventory, the records show that there are 63 meters of wire on one spool, 132 meters on a second spool and 1472 meters on a third. Find the total number of meters of wire available.

63	63
132	132
<u>1472</u>	<u>1472</u>
	1669 meters of wire

Application 2.



Find the total length of this metal plate as shown in the drawing.

73
122
73
268" total length

Drill and Practice:

Howett, pp. 31-32

Shaw, pp. 12-13

Sullivan, R.L., Practical Problems in Mathematics for
Electronics Technicians, Delmar, Albany, 1982,
pp. 1-6

6. CONCEPT: SUBTRACTION OF WHOLE NUMBERS

(Rule)

To subtract two whole numbers (or to find the difference):

- 1) Writing the larger number on top, arrange the unit's digits under each other, the ten's digits under each other and so on. The number written on top is called the minuend. The number underneath is called the subtrahend, and the result is called the remainder (or difference).
- 2) Begin by subtracting in the unit's column, then subtract in the ten's column and so on.

Example: Subtract 632 from 4857:

Minuend	4857	4857	4857	4857	4857
Subtrahend	<u>632</u>	<u>632</u>	<u>632</u>	<u>632</u>	<u>632</u>
Remainder or Difference		5	25	225	4225

- 3) If, in any column, the digit in the minuend is smaller than the corresponding digit in the subtrahend, then we must borrow. That is, take 1 from the next highest place value and add 10 to the digit whose value needs to be raised. Then subtract.

Example: From 532 subtract 428:

532	500+30+2	500+20+12	500+20+12
<u>428</u>	<u>400+20+8</u>	<u>400+20+ 8</u>	<u>400+20+ 8</u>
			100+ 0+ 4

Ans.: 104

or Simply:

532	² 532
<u>428</u>	<u>428</u>
	104

Example: Subtract 3425 from 5217:

5217	5217	¹ 5217	⁴ 5217	⁴ 5217
<u>3425</u>	<u>3425</u>	<u>3425</u>	<u>3425</u>	<u>3425</u>
	2	92	792	1792

or Simply:

5217	5217
<u>3425</u>	<u>3425</u>
	1792

Application 1. A regional parts distributor ordered 2647 exhaust pipes from the manufacturer. 1732 were delivered. How many exhaust pipes were back-ordered?

$$\begin{array}{r}
 1 \\
 2647 \\
 \underline{1732} \\
 915
 \end{array}
 \begin{array}{l}
 \text{exhaust pipes on} \\
 \text{back-order}
 \end{array}$$

Application 2. Of 2502 reams of paper stored in a lithography shop, 337 reams were used during one month's production. How many reams were available for next month's production?

$$\begin{array}{r}
 4 \qquad 4 \qquad 49 \\
 2502 \quad 2502 \quad 2502 \\
 \underline{337} \quad \underline{337} \quad \underline{337} \\
 2165
 \end{array}$$

Since there is no borrowing from 0, move to the hundreds column.

or Simply:

$$\begin{array}{r}
 2502 \\
 \underline{337} \\
 2165 \text{ Reams available}
 \end{array}$$

Note: To check a subtraction problem:
Add the subtrahend to the remainder
(difference). The result should equal
the minuend.

Example:

Minuend	6732	Subtrahend	1491
Subtrahend	<u>1491</u>	Remainder	<u>5241</u>
Remainder	5241	Minuend	6732
Answer		Check	

Drill and Practice:

Smith, Robert D. Mathematics for Machine Technology,
Second Edition, Delmar Albany, 1984, pp. 7-11

Howett, pp. 31-32

Dressler, pp. 29-30

Shaw, pp. 14-15

7. CONCEPT: MULTIPLICATION OF A WHOLE NUMBER BY
10, 100, 1000

{Rule}

To multiply a whole number by 10, 100, 1000:
Write the whole number and follow it with one, two, or three
zeros as indicated in 10, 100, 1000.

Example: $63 \times 10 = 630$
 $524 \times 100 = 52400$
 $47 \times 1000 = 47000$

To multiply a whole number by 700:
Multiply the whole number by 7, then follow that
product with two zeros.

Example: $6 \times 700 = \underline{4200} = 4200$
or $9 \times 50 = \underline{450} = 450$

Application 1. Production in a VCR chassis assembly
department during the first day of
operation was 69 units. By the end of
the month the daily production had
increased to 10 times the first day's
production. Find the daily production
at the end of the month.
 $69 \times 10 = 690$ units per day

Application 2. In a production department 8 spot welds
were needed to assemble one metal
filing cabinet. How many spot welds
would be necessary to assemble 2000
cabinets?:
 $8 \times 2000 = \underline{16000} = 16000$ spot welds.

Drill and Practice:

Dressler, p. 38 (19-22)
 Shaw, pp. 36-37

8. CONCEPT: MULTIPLICATION OF WHOLE NUMBERS

{Rule}

To multiply a number by a one-digit number, place the number with the greatest number of digits on top. Then multiply each digit in the top number by the number on the bottom.

Example: Multiply 43 by 2.

43	43	Multiplicand
<u>x 2</u>	<u>x 2</u>	Multiplier
	86	Product

Sometimes carrying is necessary.

Example: Multiply 79 by 3

If the product of the multiplier digit and a multiplicand digit is 10 or greater write down the unit's digit and carry the ten's digit mentally. Now multiply the next higher placed digit in the multiplicand by the multiplier. To this product add the number carried. Repeat the process as needed.

79	79	2
<u>x 3</u>	<u>x 3</u>	<u>x 3</u>
	7 (3x9=27)	237

Multiplication of a number by a two or three digit number can be done as follows: (3x7 + 2)

Example: Find the product 63x24
Remember 63 means 60 + 3
24 means 20 + 4

Therefore:

63	Multiply each
<u>x 24</u>	number in the
	top row by four

63	Now multiply each
<u>x 24</u>	number in the
252	top row by 20

63	
<u>x 24</u>	
252	} ← Partial products
1260	

$$\begin{array}{r} 63 \\ \times 24 \\ \hline 252 \\ 1260 \\ \hline 1512 \end{array}$$

Add the partial products

1512 is the product (63x24)

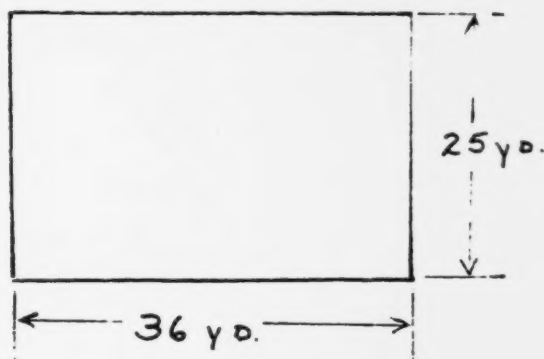
Example: Multiply 832 x 716
$$\begin{array}{r} 832 \\ \times 716 \\ \hline 4992 \quad (832 \times 6) \\ 8320 \quad (832 \times 10) \\ \hline 582400 \quad (832 \times 700) \\ 595712 \quad \text{Product} \end{array}$$

$$\begin{array}{r} 832 \\ \times 716 \\ \hline 4992 \\ 832 \\ \hline 5824 \\ 595712 \end{array}$$

zeros can be dropped

Example: Zeros can cause problems:
Find 603 x 420
$$\begin{array}{r} 603 \\ \times 420 \\ \hline \end{array}$$

$$\begin{array}{r} 603 \\ \times 420 \\ \hline 000 \quad (0 \times 603) \\ 12060 \quad (20 \times 603) \quad \text{or} \\ \hline 241200 \quad (400 \times 603) \\ 253260 \end{array}$$

$$\begin{array}{r} 603 \\ \times 420 \\ \hline 000 \\ 1206 \\ \hline 2412 \\ 253260 \end{array}$$
Application 1.

Concrete is to be poured to cover an area 25 yd. wide and 36 yds. long. If the area of a rectangle is given by the formula:

Area = Length x Width
Find the area to be covered.
Area = 36x25
$$\begin{array}{r} 36 \\ \times 25 \\ \hline 180 \\ 72 \\ \hline 900 \end{array} \text{ sq. yds. (Area)}$$
Application 2. A wholesale automotive parts department is charging \$18 for a directional systems switch assembly. Find the cost of 24 assemblies.
$$\begin{array}{r} 18 \times 24 = 24 \\ \times 18 \\ \hline 192 \\ 24 \\ \hline \$432 \end{array}$$

Cost of 24 assemblies = \$432

Drill and Practice:

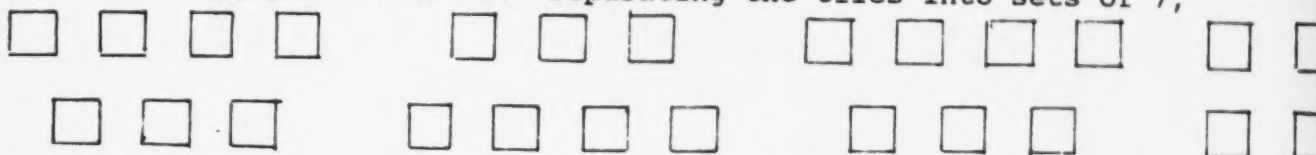
Howett, pp. 31-32

Dressler, pp. 37-38

Shaw, pp. 44-47

9. CONCEPT: THE DIVISION OF A WHOLE NUMBER BY A ONE-DIGIT WHOLE NUMBER

Sarah, a mason, is planning to install a design of seven decorative tiles in each of 3 bathrooms in a motel. She has a supply of 25 tiles. Separating the tiles into sets of 7,



produces 3 sets of 7 with 4 tiles left over. This operation of separating a number into equal sets is called division. The division sign is $\div$. We say 25 divided by 7 equals 3 with a remainder of 4.

$$\begin{array}{l} \text{or} \quad 25 \div 7 = 3R4 \\ \text{or} \quad 7 \overline{)25} = 3R4 \end{array}$$

Where
7 is the divisor
25 is the dividend
3 is the quotient
4 is the remainder.

Sarah now knows that she has enough tiles for 3 bathrooms and 4 extra tiles.

Example: Divide 63 by 9

$$63 \div 9 = 9 \overline{)63} \quad 9 \overline{)63} \begin{array}{r} 7 \\ 63 \\ \hline 63 \end{array} \quad \text{Ans. 7}$$

Example: Divide 44 by 6

$$44 \div 6 = 6 \overline{)44} \quad 6 \overline{)44} \begin{array}{r} 7 \\ 42 \\ \hline 2 \end{array} \quad \text{Ans. 7 R2}$$

Example: Divide 832 by 9

$$9 \overline{)832}$$

How many times does 9 divide 8?
None!

How many times does 9 divide 83?
9 times; since $9 \times 9 = 81$

Write 9 above 3 and 81 below 83

$$9 \overline{)832} \begin{array}{r} 9 \\ 81 \\ \hline 2 \end{array}$$

$$9 \overline{)832} \begin{array}{r} 9 \\ 81 \\ \hline 22 \end{array}$$

Subtract $83 - 81 = 2$
Write 2 under 1 and "bring down"
the 2 in 832

$$\begin{array}{r} 92 \\ 9 \overline{)832} \\ \underline{81} \\ 22 \\ \underline{18} \end{array}$$

How many times does 9 divide 22?
2 times; since $9 \times 2 = 18$
Write 2 above the 2 in 832
and 18 under 22.

$$\begin{array}{r} 92 \\ 9 \overline{)832} \\ \underline{81} \\ 22 \\ \underline{18} \\ 4 \end{array}$$

Subtract 22-18; write 4 under 18.
Therefore $832 \div 9 = 92 \text{ R}4$

Application 1.

432 cu. centimeters of organic solvent are to be poured into each of 9 glass containers in equal amounts. How many cu. centimeters will each container have?

$$432 \div 9 = 9 \overline{)432} \quad \begin{array}{r} 4 \\ 9 \overline{)432} \\ \underline{36} \\ 72 \end{array} \quad \begin{array}{r} 4 \\ 9 \overline{)432} \\ \underline{36} \\ 72 \end{array}$$

$$\begin{array}{r} 48 \\ 9 \overline{)432} \\ \underline{36} \\ 72 \\ \underline{72} \end{array}$$

Ans. 48 cu. centimeters

Application 2.

Electric wall switches are to be shipped to retailers in lots of 8 to a package. If 1240 wall switches are to be packaged, how many packages will be ready for shipment?

$$1240 \div 8 = 8 \overline{)1240} \quad \begin{array}{r} 155 \\ 8 \overline{)1240} \\ \underline{8} \\ 44 \\ \underline{40} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

Ans. 155 packages

Drill and Practice:

Howett, pp. 31

Shaw, pp. 66-67

Dressler, p. 47

Smith, pp. 17-20

10. CONCEPT: DIVISION OF WHOLE NUMBERS BY TWO DIGIT AND THREE DIGIT WHOLE NUMBERS

Discussion

To divide a whole number by a two-digit or three digit whole number requires much practice, carefulness, and skill. A certain amount of "trial and error" technique needs to be used.

Example: Divide 6432 by 74

$$74 \overline{) 6432}$$

Will 74 divide 6? No.

Will 74 divide 64? No.

Will 74 divide 643? Yes.

To determine the number of times, divide 7 into 64 mentally.

Since $7 \times 9 = 63$, try 9.

Write the 9 above the 3

in the number 6432, since we are really dividing 643 by 74.

Now $9 \times 74 = 666$.

Write 666 under 643.

9 is too large, since 666 is greater than 643.

Try 8

$$74 \overline{) 6432} \begin{array}{r} 8 \\ 592 \\ 51 \end{array}$$

$8 \times 74 = 592$

Write 8 above the 3 in 6432

Subtract $643 - 592 = 51$

$$74 \overline{) 6432} \begin{array}{r} 8 \\ 592 \\ 512 \end{array}$$

Write 51 under the bar.

"Bring down" the 2 from 6432

Divide 74 into 512

To estimate, divide 7 into 51.

Try 7. $7 \times 74 = 518$, too large.

$$74 \overline{) 6432} \begin{array}{r} 8 \\ 592 \\ 512 \\ 518 \end{array}$$

Too large.

$$74 \overline{) 6432} \begin{array}{r} 86 \\ 592 \\ 512 \\ 444 \end{array}$$

Try 6. Write 444 under 512, subtract.

68 remainder

Note: On each subtraction make sure that the initial remainder is less than 74.

Ans. 86 R68

B. DECIMALS

Example: Divide 93462 by 471

$$\begin{array}{r} 2 \\ 471 \overline{) 94362} \\ \underline{942} \end{array}$$

Will 471 divide 943?
(Think 400 into 900). Try 2.
Since 942 is less than 943,
2 is correct.

$$\begin{array}{r} 20 \\ 471 \overline{) 94362} \\ \underline{942} \\ 16 \end{array}$$

Subtract, "bring down" 6.
Will 471 divide 16? No.
Place a zero above the
6 in 94362.

$$\begin{array}{r} 20 \\ 471 \overline{) 94362} \\ \underline{942} \\ 162 \end{array}$$

Bring down 2.
Will 471 divide 162? No.
Place another zero
above the 2 in 94362.

$$\begin{array}{r} 200 \\ 471 \overline{) 94362} \\ \underline{942} \\ 162 \end{array}$$

This means that the
answer is 200 R162

Application 1. A printing department uses 8136 reams of paper per year. Find its monthly rate of paper use.

$$8132 \div 12 = 12 \overline{) 8132}$$

$$\begin{array}{r} 72 \\ 93 \\ \underline{84} \\ 96 \\ \underline{96} \\ 0 \end{array}$$

678 reams/month

Application 2. During the year by working five days per week plus several weekends, the printing department was in operation for 339 days.

If 8136 reams of paper were used, how many reams were used per day?

$$8136 \div 339 = 24 \overline{) 8136}$$

$$\begin{array}{r} 678 \\ 1356 \\ \underline{1356} \\ 0 \end{array}$$

24 reams/day

Drill and Practice:

Shaw, pp. 68-69

Howett, pp. 31-32

Dressler, pp. 17-20

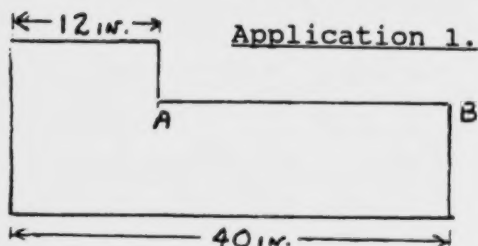
Smith, pp. 17-20

11. CONCEPT: CHOOSING AN OPERATION TO SOLVE A NUMBER PROBLEM

Many problems can be solved by choosing the correct operation from among the four we have studied, addition, subtraction, multiplication and division.

Solving Word Problems

1. Read the problem slowly and carefully.
2. Reread the problem if necessary.
3. Try to understand the essential information.
4. Decide which operation (or operations) will help you to solve the problem.
5. Check your arithmetic.
6. Determine whether the result you obtain makes sense.

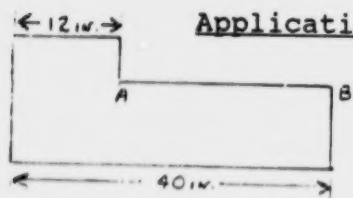


Application 1.

Determine the perimeter of the floor plan shown here.

Perimeter means the sum of the lengths of the boundary lines. Therefore, addition is the main operation here.

$$\text{Perimeter} = 21 + 18 + 30 + 8 + 9 + 8 = 94\text{m.}$$



Application 2.

In this layout of a piece of sheetmetal, find the length of A to B. The length of AB indicates finding the difference between 40m and 12m.

Therefore subtraction is the operation to be used here.

$$\begin{array}{r} 40 - 12 = 40 \\ -12 \\ \hline 28\text{m is the length of AB} \end{array}$$

Application 3.

Twelve bricklayers laid 275 bricks each. Find the total number of bricks laid. We could find the solution by adding twelve sets of 275 bricks each, but this would be very tiresome. Multiplication can be used instead of addition when all the numbers added are equal. Therefore, multiplication can be used in this case.

$$\begin{array}{r} 275 \\ 12 \\ \hline 550 \\ 275 \\ \hline 3300 \text{ bricks laid} \end{array} \quad 20$$

Application 4.

The overhead cost of a small manufacturer for one month was \$15,050. During that time 175 electronic assemblies were turned out. Find the overhead cost of one assembly.

Since this involves separating the total overcost \$15,050 into 175 equal parts, then division is implied.

$$15050 \div 175 = 86$$

15050	86	
1400		
1050		
<u>1050</u>		\$86 per assembly

Drill and Practice:

Howett, pp. 41-46

Dressler, pp. 51-54

Shaw, pp. 95, 99

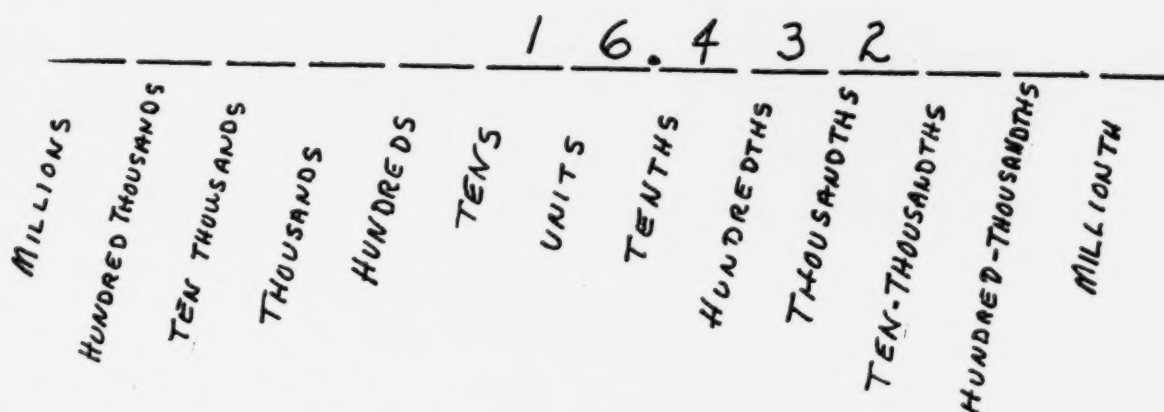
Smith, pp. 21-24

1. CONCEPT: THE MEANING OF A DECIMAL

The cost of a custom-made cam was determined to be \$523.84. This is read, "Five hundred twenty-three dollars and eighty-four cents". Notice that the "and" is reserved for the decimal point.

Remembering the idea of place value, 523 dollars (ones or units.) are familiar to us from our discussion of whole numbers. The .84 represents a number called a decimal. The 8 represents $\frac{8}{10}$ of one dollar and the 4 represents $\frac{4}{100}$ of a dollar. .84 represents $\frac{84}{100}$ of a dollar or 84 cents.

Extending the chart used to explain place value for whole numbers we can now read such numbers as 16.432cm which is the length of a piece of steel rod in a fabrication process.



We read, "sixteen and four hundred thirty-two thousandths."
(read 432 as a whole number and follow it with the name of the place occupied by the last digit read, 2.)

Example:

63.17
1497.0023

Sixty three and seventeen hundredths.
One thousand four hundred ninety-seven and twenty-three ten thousandths.

Note: Read 1497 as a whole number. use "and" for the decimal point.
Read 23 as a whole number.
Follow with the label of the last place in the number (ten thousandths)

63,468.932

Sixty three thousand, four hundred sixty eight and nine hundred thirty two thousandths.

Application 1.

The distance between two points on a metal bracket is 4.765 inches. Read this number. Four and seven hundred sixty five thousandths.

Application 2.

A voltmeter indicated a voltage of 13.65 v. Read this voltage. Thirteen and sixty-five hundredths volts.

Drill and Practice:

Shaw, pp. 2, 3

Dressler, pp. 123-124

Howett, p. 57

2. CONCEPT: ROUNDING OFF DECIMALS

A machine tool is set up to drill a hole correct to the nearest one-thousandth of an inch. A technician reads 1.274327 on her calculator. Round this number off to the nearest one thousandth. We get 1.274 in.

Method:

Write 1.274327
Underline thousandth place

1.274327
Since the digit to the right of the
4 is 3 we write 1.274

(Rule)

1. Underline or "box in" the digit to which you intend to round off.
2. Look at the digit to the right of the underlined digit.
3. If the digit to the right of the underlined digit is equal to a number less than 5, keep the underlined digit.
4. If the digit to the right of the underlined digit is equal to 5 or more than 5, add 1 to the underlined digit.
5. Drop all digits to the right of the underlined digit.

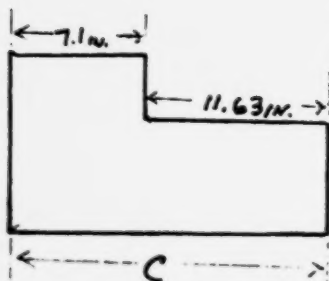
Applications

1. A micrometer reading indicated a width of 3.371 inches. Write this measurement correct to the
 - a) nearest tenth: 3.371 = 3.4 inches
 - b) nearest hundredth: 3.371 = 3.37 inches
 - c) nearest whole number: 3.371 = 3 inches
2. The print out on a technician's calculator reads 17.43271. Write this number correct to the nearest ten thousandth.
17.43271 = 17.4327

Drill and Practice:

Shaw, p. 5
Dressler, pp. 126, 127
Howett, p. 71
Smith, pp. 29-32

3. CONCEPT: ADDITION OF DECIMALS



The length of a metal bracket can be obtained by adding the lengths: 7.1" and 11.63"

Thus we write

$$\begin{array}{r} 7.1 \\ + 11.63 \\ \hline \end{array}$$

Align the decimal points under each other.
Add the columns for each place.

$$\begin{array}{r} 7.1 \\ + 11.63 \\ \hline 18.73 \end{array}$$

18.73" = the length of C.

Examples:

1. Add $6.43 + 119.04 + 0.064$

$$\begin{array}{r} 6.43 \\ 119.04 \\ + 0.064 \\ \hline 125.534 \end{array}$$

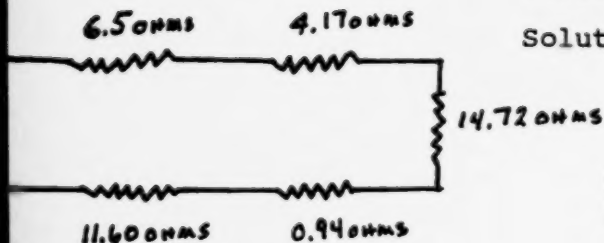
$$\begin{array}{r} 6.430 \\ 119.040 \\ + 0.064 \\ \hline 125.534 \end{array}$$

Zeros may
be written
in empty
decimal places

2. Add $19.16 + 75 + 147.043$

$$\begin{array}{r} 19.160 \\ 75.000 \\ + 147.043 \\ \hline 241.203 \end{array}$$

Application 1: The total of resistances in an electrical circuit connected in series is obtained by addition of the values of the individual resistors.



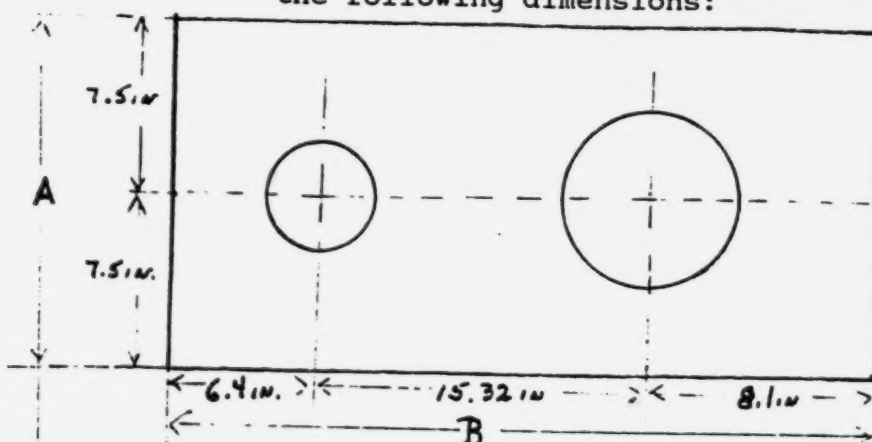
Solution:

$$\begin{array}{r} 6.5 \\ 4.17 \\ 14.72 \\ 11.6 \\ + .94 \\ \hline \end{array}$$

Total
Resistance

37.93 OHMS

Application 2. In a sheet metal layout the location of two circular cutouts is given by the following dimensions:



Find distance A

7.5
<u>7.5</u>
15.0 inches

Find distance B

6.4
15.32
<u>8.1</u>
29.82 inches

Drill and Practice:

Shaw, p. 22

Dressler, pp. 129-131

Howett, p. 61

Smith, pp. 33-36

Sullivan, pp. 52-56

4. CONCEPT: SUBTRACTION OF DECIMALS

A piece of wire is used in an electrical assembly job. If the original wire is 12.75m. long and a piece 7.8m. is cut from it, find the length of the remaining wire.

If we subtract 7.8m. from 12.75m. the length of the remaining wire is obtained.

$$\begin{array}{r} 12.75 \\ -7.80 \\ \hline \end{array}$$

Note that the decimal points are aligned. Zeros may be used to fill empty decimal places.

$$\begin{array}{r} 12.75 \\ -7.80 \\ \hline 4.95 \end{array}$$

Subtract as with whole numbers.
4.95m. remain

Examples: 1. From 113.647 subtract 97.604

$$\begin{array}{r} 113.647 \\ -97.604 \\ \hline 16.043 \text{ Ans.} \end{array}$$

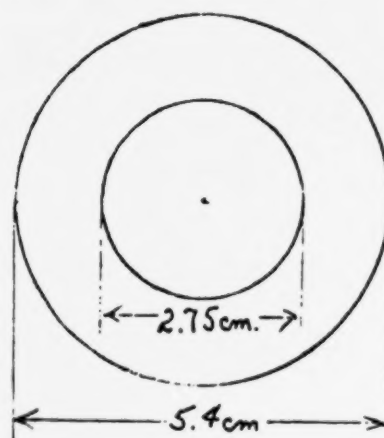
2. From 9 subtract 4.73

$$\begin{array}{r} 9.00 \\ -4.73 \\ \hline 4.27 \text{ Ans.} \end{array}$$

Application 1.

The inside diameter of a metal washer is 2.75 centimeters. The outside diameter is 5.4 centimeters. Find D, the width of the washer.

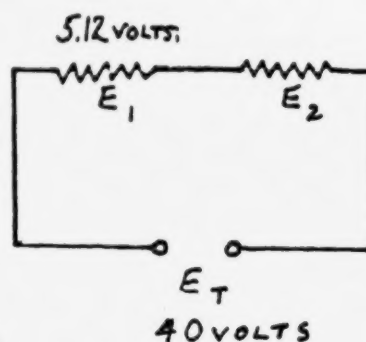
$$\begin{array}{r} 5.4 - 2.75 = 5.40 \\ \quad \quad \quad \underline{2.75} \\ 2.65 \text{ cm.} = D \end{array}$$



Application 2.

The voltage across a circuit is 40 volts. The voltage across one resistor in series with another registers a voltage of 5.12 ohms. Find the voltage across the other resistor

$$\begin{array}{l} \text{Note: } E_2 = E_T - E_1 \\ \text{Therefore } E_2 = 40 - 5.12 \\ E_2 = 34.88 \text{ ohms} \end{array}$$



$$\begin{array}{r} 40.00 \\ -5.12 \\ \hline 34.88 \text{ ohms} \end{array}$$

Drill and Practice:

Shaw, p. 23

Dressler, pp. 133, 134

Howett, p. 62, 63

Smith, pp. 33, 36

Sullivan, pp. 57-61

5. CONCEPT: MULTIPLICATION WITH DECIMALS

Voltage is determined by the formula:

$E = I \times R$ where E = voltage (volts)

I = current (amperes)

R = resistance (ohms)

Calculate the voltage if the current is 9.4 amperes and the resistance is 14.46 ohms therefore, $E = 9.4 \times 14.46$

14.46	2 decimal places
<u>9.4</u>	1 decimal place
5784	
<u>13014</u>	[1+2 = 3]
135.924	mark off 3 decimal places.

135.924 volts

Rule: Multiply two decimals by finding the product in the same manner used to multiply two whole numbers. The number of decimal places in the product equals the sum of the decimal places in the two numbers multiplied.

Example:

7.3 x 16.4	16.4	1 decimal place
	<u>x7.3</u>	1 decimal place
	492	
	<u>1148</u>	
	119.72	Answer (2 decimal places)

32 x 1.753	1.753	3 decimal places
	<u>x32</u>	0 decimal places
	5656	
	<u>5259</u>	
	58.246	3 decimal places

Application 1.

The cost of electrical wiring is \$.14 per foot. Find the cost of 175.25 feet.

175.25 x 0.14 =	175.25	2 decimal places
	<u>0.14</u>	2 decimal places
	70100	
	<u>17525</u>	
	24.5350	4 decimal places
	or	
	\$24.54	(rounded off correct to nearest cent)

Application 2.

The area of a rectangular floor plan is given by the formula

$$A = b \times h, \quad \text{where} \quad \begin{array}{l} A = \text{area} \\ b = \text{length} \\ h = \text{width} \end{array}$$

Find the area of this floor which measures 9.4m in length and 5.26m in width. Therefore

$$\begin{array}{rcl} A & = & 9.4 \times 5.26 \\ & & = 5.26 \\ A & = & 49.444 \text{ sq. meters} \\ & & \phantom{49.444 \text{ sq. meters}} \underline{9.4} \\ & & \phantom{49.444 \text{ sq. meters}} 2104 \\ & & \phantom{49.444 \text{ sq. meters}} \underline{4734} \\ & & 49.444 \quad 3 \text{ places} \end{array}$$

Drill and Practice:

Shaw, pp. 54, 55

Dressler, pp. 137, 138, 140, 141

Howett, p. 66

Smith, pp. 36-38

Sullivan, pp. 62-66

6. CONCEPT: MULTIPLICATION OF DECIMALS BY 10, 100, 1000

A spool of wire has 3.475 meters of wire. Find the total length of wire on 100 spools:

$$\begin{array}{r} 3.475 \times 100 = 3.475 \quad 3 \text{ decimal places} \\ \underline{\quad 100 \quad} \quad 0 \text{ decimal places} \\ 0000 \\ 0000 \\ \hline 3475 \end{array}$$

or $3.475 \times 100 = 3.475 \times 100 = 347.5$ on 100 spools
Briefly there are two zeros in 100, therefore move the decimal point two places to the right to obtain the product.

Rule: To multiply a decimal by 10, 100, or 1000 count the zeros in the multiplier (10, 100, or 1000) and then move the decimal point in the multiplicand the number of places to the right equal to the number of zeros.

Examples:

Multiply:

$$\begin{array}{r} 63.273 \times 10 = 63.273 = 632.73 \\ \text{one} \quad \text{one} \\ \text{zero} \quad \text{place} \end{array}$$

$$\begin{array}{r} 934.479 \times 100 = 934.479 = 93447.9 \\ \text{two} \quad \text{two} \\ \text{zeros} \quad \text{places} \end{array}$$

$$\begin{array}{r} 6.320 \times 1000 = 6.320 = 6320 \\ \text{three} \quad 3 \text{ places} \quad \text{add zero} \\ \text{zeros} \end{array}$$

Application 1.

- If ten technical hand calculators cost \$14.75 each, find the cost of 10 calculators.
 $14.75 \times 10 = 14.75$
 $= 147.5 = \underline{\$147.50}$

Application 2.

If 6.125 meters of shipping tape are needed to wrap one package, how many meters of shipping tape will be needed to wrap 1000 packages?
 $6.125 \times 1000 = 6125 = 6125. \text{ meters}$

Drill and Practice:
Dressler, p. 143

7. CONCEPT: DIVISION WITH DECIMALS

A piece of metal flatstock, 2.75 meters in length, is to be used in the manufacture of metal frames. If 5 pieces, equal in length, are to be cut from the original piece, find the length of each piece.

$$\text{Solution } 2.75 \div 5 = 5 \overline{)2.75} = 5 \overline{)2.75}$$

Remember:

divisor $\overline{) \begin{array}{c} \text{quotient} \\ \text{dividend} \end{array}}$

Decimal point in quotient is placed directly above decimal point in dividend

Now divide as with whole numbers.

$$\begin{array}{r} 55 \\ 5 \overline{)2.75} \\ \underline{25} \\ 25 \\ \underline{25} \end{array}$$

Answer 0.55 meters

Sometimes we need to divide a whole number by a larger whole number.

Example: A construction carpenter needs to divide a 9 foot plank into 12 sections. What is the width of each section?

$$\begin{array}{l} 9 \div 12 = 12 \overline{)9.} \\ \text{Remember } 9 = 9.7 \\ 12 \div 9.0 = 12 \overline{)9.0} \\ \underline{84} \\ 6 \end{array}$$

Since 12 will not divide 9, zeros may be added.

$$\begin{array}{r} 75 \\ 12 \overline{)9.00} \\ \underline{84} \\ 60 \\ \underline{60} \end{array}$$

Ans. .75 Feet

Often numbers do not divide exactly. Then we need to round off to a predetermined decimal place.

Example: Divide 32 by 45 and write the quotient correct to the nearest hundredth.

$$\begin{array}{r} .711 \\ 45 \overline{)32.000} \\ \underline{315} \\ 50 \\ \underline{45} \\ 50 \\ \underline{45} \\ 5 \end{array} = 0.711 \text{ or } 0.71$$

Notice division is carried out to one extra decimal place before rounding off.

Application 1:

The current in an electrical line is determined by the formula:

$$I = E \div R \quad \text{where} \quad \begin{array}{l} I = \text{Current (Amperes)} \\ E = \text{Voltage (Volts)} \\ R = \text{Resistance (OHMS)} \end{array}$$

Find the current when an 8.6 OHM resistance has a voltage drop of 4.25 volts.

Solution: $I = E \div R$ Correct to nearest hundredth.
 $I = 4.25 \div 8.6$

$$8.6 \overline{)4.25} = 8.6 \overline{)4.25} = 86 \overline{)42.500} = 0.49 \text{ Amperes}$$

$$\begin{array}{r} .494 \\ 86 \overline{)42.500} \\ \underline{344} \\ 810 \\ \underline{774} \\ 360 \\ \underline{344} \\ 16 \end{array}$$

Application 2:

A surface grinder removes 3.275 millimeters after 45 passes of the wheel. How much stock is removed after one pass? Determine answer correct to the nearest thousandth of a millimeter.

$$45 \overline{)3.275} = 45 \overline{)3.2750} = 0.0727 = 0.073 \text{ millimeters per pass}$$

$$\begin{array}{r} .0727 \\ 45 \overline{)3.2750} \\ \underline{315} \\ 125 \\ \underline{90} \\ 350 \\ \underline{315} \\ 35 \end{array}$$

Drill and Practice:

Shaw, pp. 78, 79

Dressler, pp. 145-147, 150-152

Howett, pp. 69-70

Smith, pp. 39-42

Sullivan, pp. 69-70

8. CONCEPT: DIVISION OF A DECIMAL BY 10, 100, 1000

The cost of producing 1000 hand calculators was \$5246.
Determine the cost of a single calculator.

$$5246 \div 1000 = 5.246 \quad \text{or } \$5.25$$

$$\begin{array}{r} 5.246 \\ 1000 \overline{) 5246.000} \\ \underline{5000} \\ 2460 \\ \underline{2000} \\ 4600 \\ \underline{4000} \\ 6000 \\ \underline{6000} \\ 0 \end{array}$$

or $5246 \div 1000$ (3 zeros)
 $= 5246. = \$5.246$ or $\$5.25$
 move decimal
 3 places to the
 left.

Rule: To divide a decimal by 10, 100, 1000 count the zeros in the divisor (10, 100, or 1000) and then move the decimal point in the multiplicand the number of places to the left equal to the number of zeros.

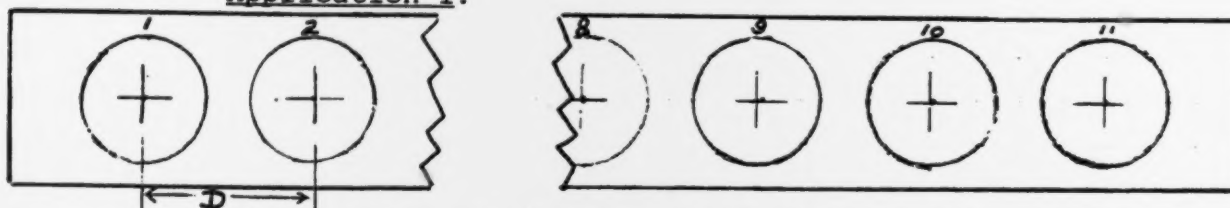
Example:

$$8.327 \div 10 = 8.327 = 0.8327$$

$$104.7 \div 100 = 1.047 = 1.047$$

$$54.9 \div 1000 = 54.9 = .0549 \quad \text{add zero on left.}$$

Application 1.



Eleven holes are to be drilled in a metal strap. If the distance from the center of hole 1 to the center of hole 11 is 23.475cm., find the distance D between the centers of any two adjacent holes.

$$23.475 \div 10 = 2.3475 = 2.3475\text{cm.}$$

Note: Eleven holes have 10 spaces.

Application 2.

The retail price of a product is \$63.50 per hundred pounds. The package labels need to show the price per pound. What price should be printed on the label?

$$63.50 \div 100 = 63.50 = .635 = \$0.635/\text{pound}$$

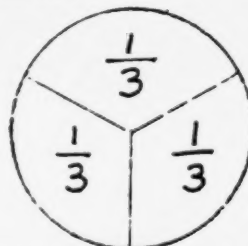
Drill and Practice:

Dressler, pp. 153, 154
 Smith, p. 39

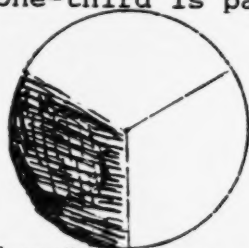
C. FRACTIONS

1. CONCEPT: THE MEANING OF FRACTIONS

If this drawing represents a circular piece of sheet metal and a tinsmith cuts it into 3 equal parts:



Each part is said to be $1/3$ or one-third of the original piece. If one-third is painted blue;



then we say that one-third or $1/3$ of the original sheet is painted. We also say that two-thirds or $2/3$ of the original sheet is not painted.

The whole circular piece of sheet metal can be represented as $3/3$ or 1 (3-thirds or one). The parts painted or unpainted can be represented as $1/3$ or $2/3$ (one-third or two thirds)

Numbers such as: $1/3$, $2/3$, $4/5$, $17/32$, or $116/5$ are called fractions. The numbers above the line 1, 2, 4, 17 or 116 are called numerators. The numbers below the line are called denominators. The numerator and denominator are called the terms of the fraction.

A fraction allows us to compare the number of objects in one set with the number of objects in another set.

Application. In a small plant, the number of production personnel is 9. The number of management personnel is 2. This gives a total of 11 employees.

We can say therefore, that the number of management personnel is a fractional part of the total number of employees or $2/11$. That is, "two elevenths" of the total number of employees is management personnel.

If we wish to compare the number of management employees with the number of production personnel then we write $2/9$, with the numerator as the first number mentioned in the comparison.

Numbers like $2/11$ and $2/9$ are called proper fractions.

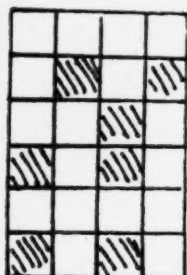
The numerator is always less than the denominator.
[Example: $2/3$, $4/9$, $5/12$, $31/64$]

If the numerator is equal to or greater than the denominator, the number is called an improper fraction.

[Example: $3/2$, $5/4$, $64/53$ $5/5$]

Note that all whole numbers can be written as improper fractions.

[Example: $9 = 9/1$]



Application 1. This diagram represents a floor which needs to be retiled. If the shaded squares are the broken tiles which need to be replaced, write a fraction which represents the portion which need to be replaced.

What number represents the total number of tiles in the floor? Answer: 24 tiles.

What number represents the number of tiles which need to be replaced? Answer: 7 tiles.

Write a number which represents the fraction of the tiles which need to be replaced. Answer: $7/24$.

Which number represents the numerator? Answer: 7.

Which number represents the denominator?

Answer: 24.

How do we read this number?

Answer: seven twenty-fourths.

Note: The fraction can also be used to indicate a division of one number by another number.

$4/5$ means $4 \div 5$

$32/8$ means $32 \div 8$

$100/100$ means $100 \div 100$ or, of course, 1

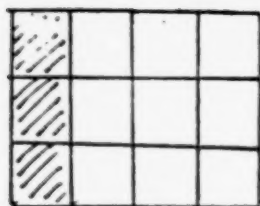
Drill and Practice:

Dressler, pp. 58, 59

Howett, p. 89

Smith, pp. 4, (1,2)

2. CONCEPT: REDUCTION OF FRACTIONS TO LOWEST TERMS



A glazier is to replace three panes of glass in a window of 12 panes.

As indicated in the diagram the missing panes can be considered as

$\frac{1 \text{ part}}{4 \text{ parts}}$ or $\frac{3 \text{ parts}}{12 \text{ parts}}$ of the total number of panes.
That is, $1/4$ has the same value as $3/12$.

The same relationship can be seen if, we divide numerator and denominator of $3/12$ by 3.

$$\frac{3 \div 3}{12 \div 3} = \frac{1}{4} \quad \text{therefore, } \frac{3}{12} = \frac{1}{4}$$

Rule: When numerator and denominator are each divided by the same non-zero number, the value of the fraction does not change.

Example: $5/10 = \frac{5 \div 5}{10 \div 5} = 1/2$

$$30/36 = \frac{30 \div 6}{36 \div 6} = 5/6$$

$$100/400 = \frac{100 \div 100}{400 \div 100} = 1/4$$

$$13/13 = \frac{13 \div 13}{13 \div 13} = 1/1 \text{ or } 1$$

$$49/35 = \frac{49 \div 7}{35 \div 7} = 7/5$$

A fraction is reduced to its lowest terms when no number other than 1 divides both numerator and denominator exactly.

Example: $2/5$, $3/8$, $5/12$, $17/50$ are in lowest terms.

$3/9$, $5/15$, $25/100$, $2/64$ are not in lowest terms and can be reduced.

Application 1. On the first day of production the number of flawed parts turned out by an automated machine tool was 75 out of a total production of 1000 parts. Write a fraction which represents the fractional part of total production which was flawed.

$$\frac{75}{1000} = \frac{75 \div 25}{1000 \div 25} = \frac{3}{40} \text{ or 3 parts out of every 40 parts are flawed.}$$

Application 2. It was determined that a fabricating part of the machine tool was causing a burr on some produced parts. This source was eliminated, and the machine tool now turned out 4 flawed parts in 1000 parts produced.

Write a fraction which represents the improved fractional part of total production which was flawed.

$$\frac{4}{1000} \quad \text{Reduce this fraction to lowest terms.}$$

$$\frac{4}{1000} = \frac{4 \div 4}{1000 \div 4} = \frac{1}{250} \text{ or one flawed part in 250 parts produced}$$

Drill and Practice:

Shaw, p. 104

Dressler, pp. 61, 62

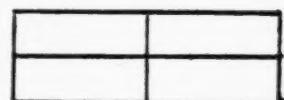
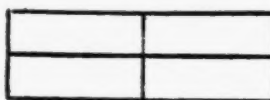
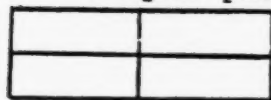
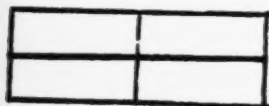
Howett, pp. 91, 92

3. CONCEPT: MIXED NUMBERS AND IMPROPER FRACTIONS

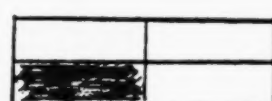
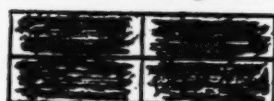
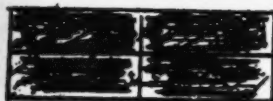
A board cut by a construction carpenter measured 3 feet, $5 \frac{3}{8}$ inches. The $5 \frac{3}{8}$ is called a mixed number. The 5 represents 5 inches and the $\frac{3}{8}$ represents $\frac{3}{8}$ of one inch. Therefore $5 \frac{3}{8}$ means 5 inches plus $\frac{3}{8}$ inches.

Sometimes it is useful to convert a mixed number to an improper fraction.

Example: Change $3 \frac{1}{4}$ to an improper fraction. Consider 4 rectangles of equal areas each of which have been divided into 4 equal parts.



Now let us shade in all parts in the first 3 rectangles and one part in the fourth rectangle.



Each shaded parts represents $\frac{1}{4}$ of a rectangle. Therefore we have 13 shaded parts, or "thirteen fourths" or $\frac{13}{4}$. Then, $3 \frac{1}{4} = \frac{13}{4}$, an improper fraction.

A shorter method is to multiply the 3 by 4, then add 1 to the product. Write this sum over 4.

$$3 \frac{1}{4} = \frac{3 \times 4 + 1}{4} = \frac{12 + 1}{4} = \frac{13}{4}$$

$$\text{Example: } 9 \frac{2}{3} = \frac{9 \times 3 + 2}{3} = \frac{27 + 2}{3} = \frac{29}{3}$$

$$11 \frac{3}{8} = \frac{11 \times 8 + 3}{8} = \frac{88 + 3}{8} = \frac{91}{8}$$

$$5 = 5 \frac{0}{1} = \frac{5 + 0}{1} = \frac{5}{1} \quad \text{Yes, whole numbers can be written as improper fractions.}$$

$$7 = 7 \frac{0}{8} = \frac{7 \times 8 + 0}{8} = \frac{56}{8}$$

Rule: To change a mixed number to an improper fraction multiply the whole number by the numerator. Add

the denominator to this product and write the result over the denominator. (To use this rule with whole numbers, rewrite the whole number as a mixed number, such as:

$$9 = 9 \frac{0}{11} = \frac{9 \times 11}{11} = \frac{99}{11} \text{ (with 11 as the desired denominator)}$$

Rule: To change improper fractions to mixed numbers: Divide the numerator by the denominator and write the remainder as the numerator of a fraction with the divisor as the denominator.

$$\text{Example: } \frac{31}{5} = 5 \overline{)31} = 6 \frac{1}{5}$$

$$\begin{array}{r} 5 \overline{)31} \\ \underline{30} \\ 1 \end{array}$$

$$\frac{17}{3} = 3 \overline{)17} = 5 \frac{2}{3}$$

$$\begin{array}{r} 3 \overline{)17} \\ \underline{15} \\ 2 \end{array}$$

Application 1. A production worker worked $8 \frac{2}{3}$ hours. Write $8 \frac{2}{3}$ as an improper fraction.

$$8 \frac{2}{3} = \frac{8 \times 3 + 2}{3} = \frac{26}{3} \text{ hours}$$

Application 2. A 9-foot plank was cut into 4 equal parts. Write the length of each part as a mixed number.

$$9 \div 4 = 4 \overline{)9} = 2 \frac{1}{4} \text{ feet}$$

$$\begin{array}{r} 4 \overline{)9} \\ \underline{8} \\ 1 \end{array}$$

Drill and Practice:

Shaw, pp. 122-123

Dressler, pp. 64, 65, 67, 68

Howett, p. 95

4. CONCEPT: CHANGING FRACTIONS TO HIGHER TERMS

We have seen that $3/12$ could be changed to $1/4$ by dividing numerator and denominator by 3, that is $\frac{3 \div 3}{12 \div 3} = \frac{1}{4}$

Conversely, it is sometimes useful to rewrite a fraction with a larger denominator. For example it is evident from the above example that:

$$1/4 = \frac{1 \times 3}{4 \times 3} = \frac{3}{12}$$

To rewrite $1/4$ with a denominator of 12, multiply numerator and denominator of the original fraction by 3.

Example:

Rewrite $3/5$ with a denominator of 10

$$3/5 = \frac{()}{10} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10}$$

Rewrite $7/8$ with a denominator of 32

$$7/8 = \frac{()}{32} = \frac{7 \times 4}{8 \times 4} = \frac{28}{32}$$

Rule: To change a fraction to an equivalent fraction with a different denominator multiply numerator and denominator with the same non-zero number.

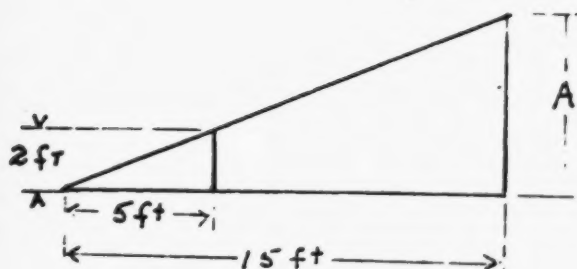
Application 1. The product manager wishes to develop an indicator which relates daily production volume to the number of defective parts produced. If 3 defective parts were produced for every 500 parts made, write this as a fraction indicating the fractional part of total production which is defective. $\frac{3}{500}$

Now write a new fraction which indicates the number of defective parts for 10,000 parts made.

$$\frac{3}{500} = \frac{()}{10000}$$

$$\frac{3}{500} = \frac{3 \times ()}{500 \times 20}$$

$$\frac{3}{500} = \frac{3 \times 20}{500 \times 20} = \frac{60}{10000} \quad \frac{\text{defective parts}}{\text{parts made}}$$

Application 2.

If a roof rises 2 feet for a run of 8 feet, determine how much it would rise for a run of 15 feet?

This can be determined by equating two fractions

$$\frac{2}{5} = \frac{A}{15} \quad \text{and then}$$

determining the value of A, the rise for a run of 15 feet.

$$\frac{2}{5} = \frac{(\quad)}{15} = \frac{2 \times 3}{5 \times 3} = \frac{10}{15}$$

therefore,

$$A = 10 \text{ feet.}$$

Drill and Practice:

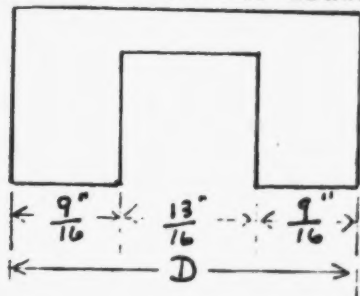
Shaw, p. 105

Dressler, pp. 69, 70

Howett, p. 105

5. CONCEPT: ADDITION OF FRACTIONS AND MIXED NUMBERS WITH THE SAME DENOMINATOR

An automated-machine operator needs to determine the distance D from the drawing of an assembly part.



$$D = \frac{9}{16} + \frac{13}{16} + \frac{9}{16} \text{ or } \frac{9+13+9}{16}$$

$$= \frac{31}{16} = 1 \frac{15}{16}$$

$$D = 1 \frac{15}{16}$$

Rule: To add fractions with the same denominator, add the numerators. Write that sum over the common denominator. Reduce if possible. If the result is an improper fraction, write as a mixed number.

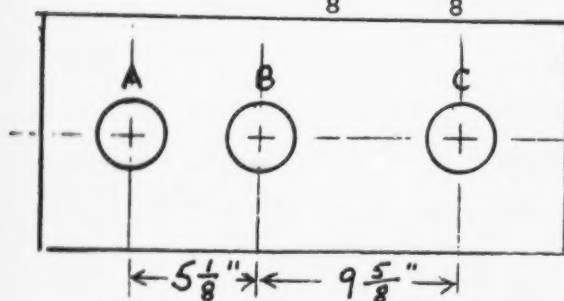
Examples:

$$\frac{1}{5} + \frac{3}{5} = \frac{1+3}{5} = \frac{4}{5}$$

$$\frac{3}{16} + \frac{1}{16} = \frac{3+1}{16} = \frac{4}{16} = \frac{4}{16} \div \frac{4}{4} = \frac{1}{4}$$

$$\frac{3}{11} + \frac{2}{11} + \frac{1}{11} = \frac{3+2+1}{11} = \frac{6}{11}$$

$$\frac{7}{8} + \frac{3}{8} = \frac{10}{8} = 1 \frac{2}{8} = 1 \frac{1}{4}$$



To determine the distance AC we add $5 \frac{1}{8}$ and $9 \frac{5}{8}$

$$5 \frac{1}{8} + 9 \frac{5}{8} = 5 \frac{1}{8}$$

$$AC = \frac{9 \frac{5}{8}}{14 \frac{6}{8}}$$

$$= 14 \frac{3}{8}$$

Rule: To add mixed numbers, add the whole numbers, add the fractions, write the result as a mixed number, then reduce to lowest terms, if possible.

Ex:

$$\begin{array}{r} 4 \frac{7}{16} + 9 \frac{3}{16} + 12 \frac{5}{16} = \\ \quad \quad \quad 4 \frac{7}{16} \\ \quad \quad \quad 9 \frac{3}{16} \\ \quad \quad \quad \underline{12 \frac{5}{16}} \\ \quad \quad \quad 25 \frac{15}{16} \end{array}$$

$$8 + 6 \frac{3}{4} + 9 \frac{1}{4} =$$

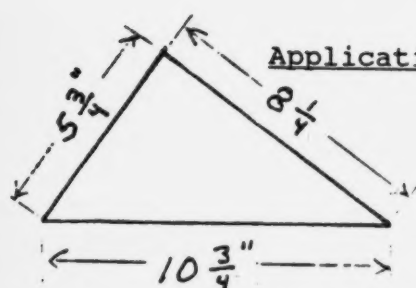
$$\begin{array}{r} 8 \\ 6 \frac{3}{4} \\ \underline{9 \frac{1}{4}} \\ 23 \frac{4}{4} = 23+1 \\ = 24 \end{array}$$

$$3 \frac{3}{32} + 5 \frac{1}{32} =$$

$$\begin{array}{r} \frac{3}{32} \\ \frac{1}{32} \\ \hline 5 \frac{32}{32} \\ 5 \frac{4}{32} = 5 \frac{4 \div 4}{32 \div 4} = 5 \frac{1}{8} \end{array}$$

$$3 \frac{14}{15} + 2 \frac{11}{15} =$$

$$\begin{array}{r} 3 \frac{14}{15} \\ \underline{2 \frac{11}{15}} \\ 5 \frac{25}{15} = 5 + 1 \frac{10}{15} \\ = 6 \frac{10}{15} = 6 \frac{2}{3} \end{array}$$



Application 1.

The perimeter of a triangle is the sum of the lengths of its sides. Find the perimeter (P) of this triangle.

$$P = 5 \frac{3}{4} + 8 \frac{1}{4} + 10 \frac{3}{4} =$$

$$\begin{array}{r} 5 \frac{3}{4} \\ 8 \frac{1}{4} \\ \underline{10 \frac{3}{4}} \\ P = 23 \frac{7}{4} = 23 + 1 \frac{3}{4} \\ = 24 \frac{3}{4} \end{array}$$

Application 2. An employee worked the following work schedule:

Monday:	7 $\frac{1}{8}$ hours
Tuesday:	8 hours
Wednesday:	9 $\frac{5}{8}$ hours
Thursday:	7 hours
Friday:	7 $\frac{1}{8}$ hours

Determine the total number of hours worked this week.

$$\begin{array}{r} \text{Weekly total} = 7 \frac{7}{8} + 8 + 9 \frac{5}{8} + 7 + 7 \frac{1}{8} \\ = 7 \frac{7}{8} \end{array}$$

$$\begin{array}{r} 8 \\ 9 \frac{5}{8} \\ 7 \\ \underline{7 \frac{1}{8}} \end{array}$$

$$\text{Weekly total } 38 \frac{13}{8} = 38 + 1 \frac{5}{8} = 39 \frac{5}{8} \text{ hours.}$$

Drill and Practice:

Shaw, pp. 134, 135

Dressler, pp. 76, 77

Howett, p. 99

6. CONCEPT: FINDING THE LOWEST COMMON MULTIPLE

Given two numbers such as 3 and 5 it is sometimes useful to find the lowest number that each of these numbers will divide into exactly. This number is called the lowest common multiple. (L.C.M.) To determine the L.C.M. lets make and complete a chart.

	x1	x2	x3	x4	x5	
3	3	6	9	12	15	multiples of 3
5	5	10	15	20	25	multiples of 5

Notice that the lowest common multiple of 3 and 5 is 15.

Example: Find the L.C.M. for 7 and 3

	x1	x2	x3	x4	x5	x6	x7	
7	7	14	21	28	35	42	49	
3	3	6	9	12	15	18	21	L.C.M. = 21

Find the L.C.M. for 5, 7, and 2.

Write the set of multiples for the largest number: 7 =

(7, 14, 21, 28, 35, 42, 49, 56, 63, 70.....)

70 is divisible exactly by 5 and 2.

Therefore, the L.C.M. = 70.

Rule: To determine the L.C.M. for two or more numbers.

- 1) If the largest number is divisible by each of the others then the largest number is the L.C.M.
- 2) If the largest number is not divisible by each of the others, then write out the set of multiples for the largest number. The lowest multiple of the largest number which is divisible by each of the other numbers is the L.C.M.

Application 1. The measurements of electric wire remaining on 3 spools are $5\frac{7}{8}$ feet, $9\frac{3}{4}$ feet and $23\frac{5}{16}$ feet. If the denominators are 8, 4, 16 find the L.C.M. (in the case of fractions, the L.C.M. is called the lowest common denominator or L.C.D.)

Multiples of 16 = (16, 32...) L.C.M.

Since 8 and 4 both divide 16 exactly, 16 is the L.C.M. or L.C.D.

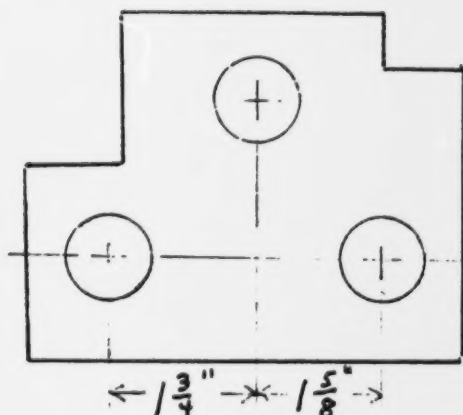
Drill and Practice:

Shaw, et.al., pp. 112-115

Dressler, p. 74

Howett, p. 104

7. CONCEPT: ADDITION OF FRACTIONS AND MIXED NUMBERS WITH DIFFERENT DENOMINATORS



The plate shown has two one-inch diameter holes whose centers are at a distance B apart. From the dimensions in the diagram, determine the distance B.

Notice that in previous problems we have been able to add fractions and mixed numbers which involve fractions with the same denominator.

This problem differs in that $1 \frac{3}{4}$ and $1 \frac{5}{8}$ have different denominators.

We approach the problem by rewriting each fraction part of each number with the same denominator. This is called the lowest common multiple (L.C.M.) of 4 and 8, or in the case of fractions, the lowest common denominator. Since 4 divides exactly into 8, 8 is the L.C.D.

Rule: Therefore, to add fractions and mixed numbers with different denominators, write each fraction with the L.C.D. as the denominator then proceed to use the rules for the subtraction of fractions with the same denominators.

Therefore, $1 \frac{3}{4} = 1 \frac{6}{8}$ rewrite with a denominator of 8

$1 \frac{5}{8} = 1 \frac{5}{8}$ no change necessary
2 $11/8$ add mixed numbers

Simplify:

$$2 \frac{11}{8} = 2 + 1 \frac{3}{8} = 3 \frac{3}{8} = B$$

Examples:

$$1) \quad \frac{3}{5} + \frac{5}{7} = \frac{3}{5} = \frac{3 \times 7}{5 \times 7} = \frac{21}{35}$$

$$\begin{array}{rcll} \text{L.C.D.} = 35 & \frac{5}{7} = & \frac{5 \times 5}{7 \times 5} = \frac{25}{35} & \\ & & \frac{21}{35} + \frac{25}{35} = 1 \frac{11}{35} & \end{array}$$

$$2) \quad 2/3 + 3/8 + 1/2 = 2/3 = \frac{(\quad)}{24} = \frac{16}{24}$$

$$3/8 = \frac{(\quad)}{24} = \frac{9}{24}$$

$$1/2 = \frac{(\quad)}{24} = \frac{12}{24}$$

$$\text{L.C.D.} = 24$$

$$\frac{16}{24} + \frac{9}{24} + \frac{12}{24} = \frac{37}{24} = 1 \frac{13}{24}$$

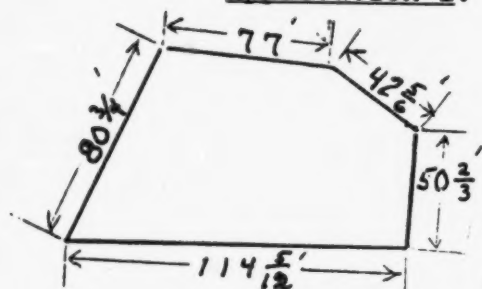
$$3) \quad 8 + 5 \frac{7}{8} + 2 \frac{13}{16} = 8 = 8$$

$$5 \frac{7}{8} = 5 \frac{14}{16}$$

$$2 \frac{13}{16} = 2 \frac{13}{16}$$

$$15 \frac{27}{16} = 15 \frac{27}{16}$$

$$15 + 1 \frac{11}{16} = 16 \frac{11}{16}$$

Application 1.

A surveyor records the lengths of the sides of an irregular piece of property as indicated in the diagram. Determine the perimeter which is the sum of the lengths of the sides.

$$50 \frac{2}{3} + 42 \frac{5}{6} + 77 + 80 \frac{3}{4} + 114 \frac{5}{12}$$

Solution:

$$\begin{array}{rcl} 50 \frac{2}{3} & = & 50 \frac{8}{12} \\ 42 \frac{5}{6} & = & 42 \frac{10}{12} \\ 77 & = & 77 \\ 80 \frac{3}{4} & = & 80 \frac{9}{12} \\ 114 \frac{5}{12} & = & 114 \frac{5}{12} \end{array}$$

L.C.D. = 12
Since 3, 6 and 4 each divides into 12 exactly

$$\begin{array}{rcl} \text{Perimeter} & = & 363 \frac{32}{12} = 363 + 2 \frac{8}{12} \\ & = & 365 \frac{8}{12} = 365 \frac{2}{3} \end{array}$$

Application 2. The weights of 3 containers of chemical reagents placed on a fork lift are $52 \frac{3}{4}$ Kg., $37 \frac{7}{8}$ Kg. and $15 \frac{3}{8}$ Kg. Find the total weight of the load on the forklift.

$$\begin{array}{rcl} \text{Total weight} & = & 52 \frac{3}{4} \text{Kg.} + 37 \frac{7}{8} + 15 \frac{3}{5} = \\ & = & 52 \frac{3}{4} = 52 \frac{35}{40} \\ & & 37 \frac{7}{8} = 37 \frac{35}{40} \\ & & 15 \frac{3}{5} = 15 \frac{24}{40} \\ & & 104 \frac{89}{40} \end{array}$$

L.C.D. = 40
Since multiples of 8 = [8, 16, 24, 32, 40]
and 40 is the lowest multiple of 8 which is divisible by both 8 and 5.

$$\begin{array}{rcl} & = & 104 + 2 \frac{9}{40} \\ \text{total weight} & = & 106 \frac{9}{40} \text{Kg.} \end{array}$$

Drill and Practice:

Shaw, pp 136-137

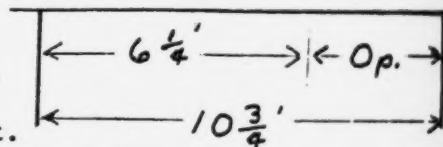
Dressler, pp. 79-81

Howett, p. 108

8. CONCEPT: SUBTRACTION OF FRACTIONS AND MIXED NUMBERS WITH SAME DENOMINATORS

Between two dividers in a production area, there is a distance of $10 \frac{3}{4}$ feet. In this space a production machine is to be installed which will use up $6 \frac{1}{4}$ feet.

Find the width of the space which will be left for the operator (Op.) To determine this, subtract $10 \frac{3}{4}' - 6 \frac{1}{4}' = 10 \frac{3}{4}$



$$\begin{array}{r} 10 \frac{3}{4} \\ - 6 \frac{1}{4} \\ \hline 4 \frac{2}{4} = 4 \frac{1}{2} \text{ feet for operator} \end{array}$$

Examples:

$$\begin{array}{r} 13 \frac{2}{3} \\ - 7 \frac{1}{3} \\ \hline 6 \frac{1}{3} \end{array} \quad \begin{array}{r} 5 \frac{2}{3} \\ - 2 \\ \hline 3 \frac{2}{3} \end{array} \quad \begin{array}{r} 8 \frac{3}{5} \\ - 8 \frac{3}{5} \\ \hline 0 \end{array}$$

However, in many cases adjustments need to be made. Notice that in the cases cited above that the fractional part of the minuend always was equal to or greater than the fractional part of the subtrahend. This not always true.

Example:

From 13 subtract $9 \frac{4}{5}$.

$$13 - 9 \frac{4}{5} = 13$$

$$\begin{array}{r} 13 \\ - 9 \frac{4}{5} \\ \hline 12 \frac{5}{5} \\ - 9 \frac{4}{5} \\ \hline 3 \frac{1}{5} \end{array}$$

Notice that a fraction part of the minuend is needed. We think of 13 as $12 + 1$. Then write 1 as $\frac{5}{5}$ with the same denominator as $\frac{4}{5}$;

Then:

$$\begin{array}{r} 12 \frac{5}{5} \\ - 9 \frac{4}{5} \\ \hline 3 \frac{1}{5} \end{array}$$

subtract in the ordinary manner.

Example:

From $24 \frac{5}{8}$ subtract $19 \frac{7}{8}$.

$$\begin{array}{r} 24 \frac{5}{8} - 19 \frac{7}{8} = 24 \frac{5}{8} = 23 + 1 \frac{5}{8} = 23 \frac{13}{8} \\ - 19 \frac{7}{8} = 19 \frac{7}{8} \\ \hline 4 \frac{6}{8} = 4 \frac{3}{4} \end{array}$$

Notice that $\frac{5}{8}$ is less than $\frac{7}{8}$.

Borrow 1 from 24 and write $24 \frac{5}{8}$ as $23 + 1 \frac{5}{8}$.

Now write $1 \frac{5}{8}$ as $\frac{13}{8}$. Then subtract in ordinary manner.

Application 1.

A machine was shut down for repairs for $5 \frac{3}{8}$ hours during an 8 hour day. For how many hours was the machine in operation?

$$\begin{array}{r} 8 - 5 \frac{3}{8} = 8 = 7 \frac{8}{8} \\ \quad \quad \quad 5 \frac{3}{8} = \underline{5 \frac{3}{8}} \\ \quad \quad \quad 2 \frac{5}{8} \text{ hours in operation} \end{array}$$

Application 2.

$\frac{3}{32}$ " was lost in the width of a piece of metal during a milling operation. If the original piece of metal was $11 \frac{1}{32}$ " in width, what was the width after milling?

$$11 \frac{1}{32} - \frac{3}{32} =$$

$$10 \frac{33}{32}$$

$$- \frac{3}{32}$$

$$10 \frac{30}{32} = 10 \frac{15}{16}$$

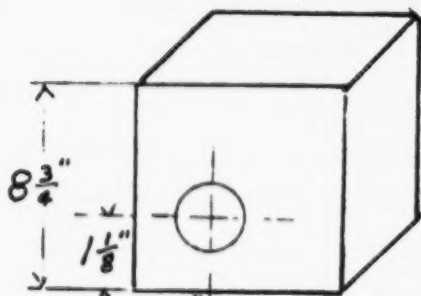
Drill and Practice:

Shaw, pp. 144-147

Dressler, pp. 83-84

Howett, pp. 100-102

9. CONCEPT: SUBTRACTION OF FRACTIONS AND MIXED NUMBERS WITH DIFFERENT DENOMINATORS



In a wooden model, the width of the block was $8 \frac{3}{4}$ ". The distance from the edge to the center of the hole was $1 \frac{1}{8}$ ". Find distance B.

$$B = 8 \frac{3}{4} - 1 \frac{1}{8} = 8 \frac{3}{4} = 8 \frac{6}{8}$$

L.C.D. = 8

Both fractions must be written with same denominator.

$$1 \frac{1}{8} = \frac{1 \frac{1}{8}}{7 \frac{5}{8}}$$

Rule: To subtract fractions and mixed numbers with different denominators, write each fraction with the L.C.D. as the denominator or each fraction, then proceed to use the rules for the subtraction of fractions with the same denominators.

Examples: (1) From $\frac{3}{5}$ subtract $\frac{1}{4}$.

$$\begin{array}{r} \frac{3}{5} - \frac{1}{4} = \frac{3}{5} = \frac{(\quad)}{20} = \frac{12}{20} \\ \text{L.C.D.}=20 \quad - \frac{1}{4} = \frac{(\quad)}{20} = \frac{5}{20} \\ \hline \frac{7}{20} \text{ Answer.} \end{array}$$

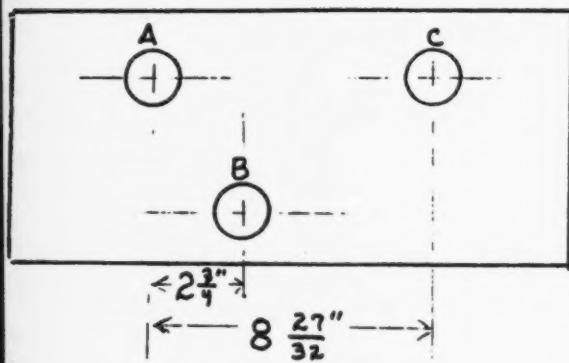
(2) From $9 \frac{15}{16}$ subtract $5 \frac{3}{8}$.

$$\begin{array}{r} 9 \frac{15}{16} - 5 \frac{3}{8} = 9 \frac{15}{16} = 9 \frac{15}{16} \\ \quad \quad \quad - 5 \frac{3}{8} = \frac{5 \frac{6}{16}}{4 \frac{9}{16}} \text{ Answer.} \end{array}$$

(3) From $30 \frac{3}{5}$ subtract $11 \frac{7}{8}$.

$$\begin{array}{r} 30 \frac{3}{5} - 11 \frac{7}{8} = 30 \frac{3}{5} = 30 \frac{24}{40} = 29 + 1 \frac{24}{40} = 29 \frac{64}{40} \\ \text{L.C.D.}=40 \quad - 11 \frac{7}{8} = 11 \frac{35}{40} = 11 \frac{35}{40} = -11 \frac{35}{40} \\ \hline \text{Since the fraction in the minuend is smaller, we must "borrow".} \end{array}$$

$$\begin{array}{r} 18 \frac{29}{40} \\ \hline \text{Answer.} \end{array}$$

Application 1.

The distance between the centers of Circle A and Circle C is $8 \frac{27}{32}$ ". The distance between the centers Circle A and Circle B is $2 \frac{3}{4}$ ". Find the distance R.

$$\begin{aligned}
 8 \frac{27}{32} - 2 \frac{3}{4} &= 8 \frac{27}{32} - 2 \frac{24}{32} = 8 \frac{27}{32} - 2 \frac{24}{32} \\
 &= 6 \frac{3}{32} = 6 \frac{1}{12} = R
 \end{aligned}$$

Application 2. After $9 \frac{7}{8}$ " is cut from a piece of steam pipe $20 \frac{5}{16}$ " in length, find the length of the remaining piece of pipe.

$$20 \frac{5}{16} - 9 \frac{7}{8} = 20 \frac{5}{16} - 9 \frac{14}{16} = 19 + 1 \frac{5}{16} = 19 \frac{21}{16}$$

$$\begin{aligned}
 9 \frac{7}{8} &= 9 \frac{14}{16} = 9 \frac{15}{16} = 9 \frac{14}{16} \\
 &= 10 \frac{7}{16}
 \end{aligned}$$

length of
remaining
piece.

Drill and Practice:

Shaw, pp. 154-155

Dressler, pp. 87-88

Howett, p. 108

10. CONCEPT: MULTIPLICATION INVOLVING FRACTIONS

5 sections of wire are to be cut from a roll. Each section is $7/8$ feet long. Find the total amount of the wire cut from the roll.

This means that the total amount cut is:

$$\frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} = \frac{7+7+7+7+7}{8} = \frac{35}{8} = 4 \frac{3}{8}$$

or more simply, 5 times $7/8 = 4 \frac{3}{8}$

Now $5 \times 7/8$ can be written as $5/1 \times 7/8$

If we find the product of the numerators and divide that number by the product of the denominators we obtain.

$$5/1 \times 7/8 = \frac{5 \times 7}{1 \times 8} = \frac{35}{8} \text{ or } 4 \frac{3}{8}$$

To multiply two fractions find the product of the numerators and divide that result by the product of the denominator. Reduce the resulting number, if possible.

Ex: Find $2/3$ of 6. Note: " $2/3$ of 6 means $2/3 \times 6$ "

$$2/3 \times 6 = 2/3 \times 6/1 = \frac{2 \times 6}{3 \times 1} = \frac{12}{3} = 4 \text{ Answer}$$

Ex: Find $3/4$ of $7/9$.

$$3/4 \times 7/9 = \frac{3 \times 7}{4 \times 9} = \frac{21}{36} = \frac{7}{12} \text{ Answer}$$

Ex: Find $6 \frac{2}{3} \times 3/5 = 20/3 \times 3/5 = \frac{20 \times 3}{3 \times 5} = \frac{60}{15} = 4/1 = 4$

Write mixed numbers as improper fractions.

Example: Find $8 \frac{2}{3} \times 5/13$

$$\frac{26}{3} \times \frac{5}{13} = \frac{26 \times 5}{3 \times 13} = \frac{\cancel{26}^2 \times 5}{3 \times \cancel{13}_1} = \frac{2 \times 5}{3 \times 1} = \frac{10}{3} = 3 \frac{1}{3}$$

To simplify some products divide any numerator and any denominator by the same common factor.

Example:

$6 \frac{7}{8} \times 3/5 \times 3 \frac{1}{5}$

$$\frac{55}{8} \times \frac{3}{5} \times \frac{16}{5} = \frac{\cancel{55}^{11} \times 3 \times \cancel{16}_2}{8 \times \cancel{5}_1 \times 5} = \frac{11 \times 3 \times 2}{1 \times 1 \times 5} = \frac{66}{5} = 13 \frac{1}{5}$$

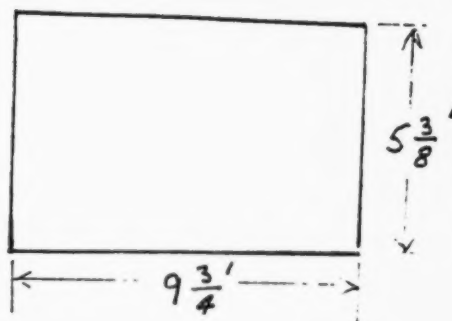
Application 1.

The area of a floor space is to be determined from this drawing. Using the formula Area = length x width, find the area if length = $9\frac{3}{4}'$ and width = $5\frac{3}{8}'$.

Solution:

$$\begin{aligned}\text{area} &= \text{length} \times \text{width} \\ &= 9\frac{3}{4} \times 5\frac{3}{8} \\ &= \frac{39}{4} \times \frac{43}{8} = \frac{39 \times 43}{4 \times 8} = \frac{1677}{32} = 52\frac{13}{32}\end{aligned}$$

Square feet
(area is given
in square units)

Application 2.

The data from a production department indicates that 3 parts produced are defective. If 6500 parts are made, what number of parts might be expected to be defective?

Solution: $\frac{3}{500}$ of the parts produced are defective

Therefore: $\frac{3}{500}$ of 6500 parts should be defective

$$\frac{3}{500} \text{ of } 6500 \text{ means } \frac{3}{500} \times 6500 = \frac{3}{500} \times \frac{6500}{1}$$

$$= \frac{3 \times 6500}{500 \times 1} = \frac{3 \times 13}{1 \times 1} = 39 \text{ defective parts}$$

Drill and Practice:

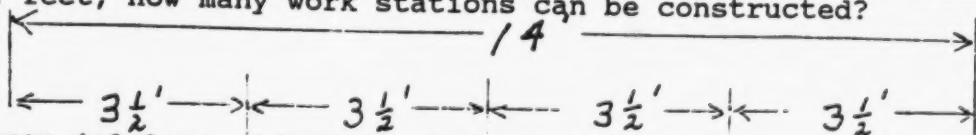
Shaw, pp. 166-169

Dressler, pp. 90-93, 97-100

Howett, pp. 110-112

11. CONCEPT: DIVISION INVOLVING FRACTIONS

A production area is to be divided into work stations which are $3\frac{1}{2}$ feet wide. If the width of the production area is 14 feet, how many work stations can be constructed?



Since $4 \times 3\frac{1}{2} = 4 \times \frac{7}{2} = \frac{4 \times 7}{1 \times 2} = 14$ feet, 4 work stations

can be constructed. This means that $14 \div 3\frac{1}{2}$ must equal 4.

We can obtain the 4 by rewriting:

$14 \div 3\frac{1}{2}$ as $14 \div \frac{7}{2}$ or $14/1 \times \frac{2}{7} = \frac{28}{7} = 4$ work stations

Rule: To divide two mixed numbers or fractions write the mixed numbers as improper fractions, invert the divisor and multiply. (whole numbers can be written as improper fractions, ex: $7 = 7/1$)

Examples:

$$1) \quad 8 \div \frac{3}{4} = \frac{8}{1} \times \frac{4}{3} = \frac{8 \times 4}{1 \times 3} = \frac{32}{3} = 10 \frac{2}{3}$$

$$2) \quad \frac{7}{8} \div 2 = \frac{7}{8} \div \frac{2}{1} = \frac{7 \times 1}{8 \times 2} = \frac{7 \times 1}{8 \times 2} = \frac{7}{16}$$

$$3) \quad \frac{5}{6} \div \frac{2}{3} = \frac{5}{6} \times \frac{3}{2} = \frac{5 \times 3}{6 \times 2} = \frac{5}{4} = 1 \frac{1}{4}$$

$$4) \quad 8\frac{3}{4} \div 2\frac{7}{8} = \frac{35}{4} \div \frac{23}{8} = \frac{35}{4} \times \frac{8}{23} = \frac{35 \times 8}{4 \times 23} = \frac{70}{23} = 2 \frac{14}{23}$$

Rule:

Alternative Method:

To divide a number by a fraction multiply the number by the reciprocal of the divisor.

The reciprocal of a number is the number inverted; that is the reciprocal is obtained by interchanging the numerator and denominator of the number when written as a fraction.

Ex: The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$
 " " of $\frac{7}{8}$ is $\frac{8}{7}$
 " " of 9 is $\frac{1}{9}$ (write 9 as $\frac{9}{1}$)
 " " of $8\frac{2}{3}$ is $\frac{3}{26}$ (write $8\frac{2}{3}$ as $\frac{26}{3}$)

Therefore:

$$\frac{4}{5} \div \frac{3}{8} = \frac{4}{5} \times \frac{8}{3} = \frac{32}{15} \text{ or } 2 \frac{2}{15}$$

Applications.

- 1) An automated milling machine completes a part in $5 \frac{2}{3}$ minutes. How many parts will it complete in 68 minutes?

$$68 \div 5 \frac{2}{3} = \frac{68}{1} \div \frac{17}{3} = \frac{68}{1} \times \frac{3}{17} = \frac{\overset{4}{\cancel{68}} \times 3}{1 \times \cancel{17}} = \frac{4 \times 3}{1 \times 1} = \frac{12}{1}$$

- 2) A roll of wire contains $28 \frac{1}{2}$ feet. How many $4 \frac{3}{4}$ ' lengths can be cut from the roll?

$$28 \frac{1}{2} \div 4 \frac{3}{4} = \frac{57}{2} \div \frac{19}{4} = \frac{57}{2} \times \frac{4}{19} = \frac{\overset{3}{\cancel{57}} \times \overset{2}{\cancel{4}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{19}}} = \frac{3 \times 2}{1 \times 1} = \frac{6}{1} = 6 \text{ lengths}$$

Drill and Practice:

Shaw, pp. 176-179

Dressler, pp. 102, 103, 106,

108, 110, 112

Howett, p. 117

12. CONCEPT: SOLUTION OF PROBLEMS INVOLVING FRACTIONS
MULTIPLICATION AND DIVISION WITH FRACTIONS

Finding the fractional part of a number:

Ex: 3 parts of every 500 parts manufactured were found to be defective in department A. How many parts would be expected to be defective in 3000 parts?

Solution: $3/500$ represents the fraction part of the total number produced which are defective. Therefore, $3/500$ of 3000 = $3/500 \times 3000$

$$= 3/500 \times 3000/1 = \frac{3 \times 3000}{500 \times 1} = \frac{9000}{500} = 18$$

defective parts.

Finding the fractional part that one number is of another:

Ex: 15 employees were late for work on Monday out of a total of 615 employees. What fractional part of the total employees were late?

$$\text{fraction} = \frac{\text{late employees}}{\text{total}} = \frac{15}{615} = \frac{1}{41}$$

Finding a number when a fractional part of the number is known:

Ex: $3/4$ of the employees of a large corporation are in the United States. The corporation reports 48000 employees in the United States. How many employees are there worldwide?

Solution:

[number of world-wide employees] $\times 3/4 = 48,000$
 This means that

$$\text{number of world-wide employees} = 48000 \div 3/4$$

$$= \frac{48000}{1} \times \frac{4}{3} = \frac{48000 \times 4}{1 \times 3} = 64000 \text{ employees world-wide}$$

Drill and Practice:

Dressler, pp. 58, 59, 113,
 114, 117

Howett, pp. 117, 118, 120-125

13. CONCEPT: CALCULATING WITH FRACTIONS AND DECIMALS

It is sometimes convenient to write fractions as decimals, or decimals as fractions with a particular denominator.

Ex: Convert $7 \frac{5}{8}$ to a decimal

$7 \frac{5}{8}$ means $7 + \frac{5}{8}$ or $7 + .625 = 7.625$
 $.625$ is obtained by dividing 5 by 8 which can be quickly obtained from a table or from a hand calculator.

$18 \frac{15}{32}$ means $18 + \frac{15}{32} = 18 + 0.46875 = 18.46875$

$\frac{15}{32}$ can be quickly determined from a hand calculator or from tables.

If a calculator is used, the steps taken are:

15	Display
$\div$	Operation
32	Display
=	Operation
0.5625	Display (Answer)

Ex: Convert:

8.37 inches to a mixed number correct to the nearest 64th.

$$\frac{37}{100} \times 64 = \frac{37}{100} \times \frac{64}{1} = 23.68 \text{ (64ths)}$$

$$= \frac{24}{64}$$

$$8.37 = 8 \frac{24}{64} \text{ inches.}$$

Convert 31.823 to a mixed fraction correct to nearest 8th.

$$.823 \times 8 = 4.984 \text{ eighths or } 5 \text{ eighths} = \frac{5}{8}$$

Therefore, $31.823 = 31 \frac{5}{8}$

Ex: Find $\frac{3}{4}$ of 9.24

$$\text{Solution: } \frac{3}{4} \times 9.24 = \frac{3 \times 9.24}{4} = \frac{27.72}{4} = 6.93$$

$$\text{or } \frac{3}{4} \times 9.24 = \frac{3 \times 9.24}{4} = \frac{3 \times 2.31}{1} = 6.93$$

Comparison of Fractions and Decimals:Which is greater?

$$\frac{57}{64} \quad \text{or} \quad \frac{7}{8}$$

Write both with the same denominator:

$$\frac{57}{64} = \frac{56}{64} \quad \text{Greater: } \frac{57}{64}$$

Which is greater?

$$\frac{57}{64} \quad \text{or} \quad .8$$

Write both as decimals with the same number of decimal places:

$$.8906 \quad .8000 \quad \text{Greater: } \frac{57}{64}$$

Which is greater?

$$.053 \quad \text{or} \quad .503 \quad \text{Align decimal points.}$$

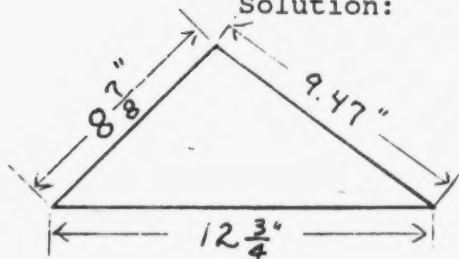
$$.053$$

$$.503$$

$$\text{Greater: } .503$$

Application 1. Find the perimeter of the triangle:

Solution:



$$\begin{aligned} \text{Perimeter} &= 8\frac{7}{8} + 9.47 + 12\frac{3}{4} \\ &= 8.875 + 9.47 + 12.75 \\ &= 8.875 \\ &\quad 9.47 \\ &\quad \underline{12.75} \\ &= 31.095 \text{ Inches} \end{aligned}$$

Application 2. The width of a rectangular metal plate is 3.75 feet. The length is $12\frac{7}{8}$ feet. Find the area of the plate.

[Area of a Rectangle = length x width]

$$\begin{aligned} \text{Solution: Area} &= 12\frac{7}{8} \times 3.75 \\ &= 12.875 \times 3.75 = 112.65625 \text{ Sq. Ft.} \end{aligned}$$

Drill and Practice:

Shaw, pp. 6-7

Dressler, pp. 157-158

14. CONCEPT: FIND THE AVERAGE OF A SET OF NUMBERS

The counter on a production machine registered as follows:

<u>Day</u>	<u>No. of Units Produced</u>
Mon.	2341
Tues.	2437
Wed.	2550
Thurs.	1944
Fri.	2478

Average: $\frac{\text{the sum of all readings}}{\text{number of readings}}$

$$\text{Average} = \frac{11750}{5} = 2350 \text{ units produced/day}$$

Ex: Find the average of $18.6 + 9.8 + 7.3$

$$\text{Average} = \frac{18.6+9.8+7.3}{3} = \frac{35.7}{3} = 11.9 \text{ average}$$

Find the average of $8\frac{3}{4}$, $7\frac{1}{8}$, $10\frac{5}{16}$, $11\frac{1}{2}$

$$\text{Average} = \frac{8\frac{3}{4} + 7\frac{1}{8} + 10\frac{5}{16} + 11\frac{1}{2}}{4} \quad \text{L.C.D.}=16$$

$$= \frac{8\frac{12}{16} + 7\frac{2}{16} + 10\frac{5}{16} + 11\frac{8}{16}}{4}$$

$$= \frac{36\frac{27}{16}}{4} = \frac{972}{16} \times \frac{1}{4} = \frac{972 \times 1}{16 \times 4}$$

$$= \frac{243 \times 1}{16 \times 1} = \frac{243}{16} = [15.1875]$$

or 15.2 average

Application: The diameters of 10 metal cylinders were measured and recorded as:

3.236cm	3.239cm
3.235cm	3.240cm
3.236cm	3.236cm
3.237cm	3.235cm
3.236cm	3.237cm

Find the average diameter?

$$\text{Average} = \frac{\text{Sum of all readings}}{\text{No. of readings}} = \frac{32.367}{10} = 3.2367\text{cm} = 3.237\text{cm}$$

Drill and Practice:

Shaw, p. 224

Dressler, pp. 235-237

D. PERCENTS

1. CONCEPT: AN UNDERSTANDING OF PERCENT

Percent means "the number of parts out of 100 parts."

A 6% unemployment rate means that 6 persons out of every hundred persons qualified to work is unemployed.

% is the symbol for "percent"

18% means 18 parts out of 100 parts or 18/100

31% means 31 parts out of 100 parts or 31/100

5.4% means 5.4 parts out of 100 parts or 5.4/100
or 54 out of 1000 or 54/1000

100% means 100 out of 100 or 100/100

120% means 120 out of 100 or 120/100

Ex: If sales for this month are 120% of sales last month, then for every \$100 worth sold last month \$120 have been sold this month.

$7/8\% = 7/8$ of 1 part out of 100

or $\frac{7}{8} = \frac{7}{8} \times \frac{100}{100} = \frac{7}{8} \times 100 = \frac{7}{8} \times 800$

$8 \frac{3}{4}\%$ means $8 \frac{3}{4}$ parts out of 100 parts or

$8 \frac{3}{4}$ or $8 \frac{3}{4} \div 100$
 $\frac{35}{4} \times \frac{1}{100} = \frac{35}{400}$

Drill and Practice:

Dressler, pp. 166-167

Howett, pp. 139-140

2. CONCEPT: CHANGING PERCENTS TO DECIMALS

46% of all machine parts were subcontracted outside of the plant. Write: 46% as a decimal

$$46\% = \frac{46}{100} = 46 \div 100 = \underline{46.} = .46$$

Examples:

$$\begin{aligned} 59\% &= \underline{59.} = .59 \\ 81.4\% &= \underline{81.4} = .814 \\ 9\% &= \underline{9} = .09 \\ .6\% &= \underline{.6} = .006 \\ 3/4\% &= .00 \text{ } 3/4 \text{ (3/4\% means 3/4 of 1\%)} \\ 165\% &= \underline{165.} = 1.65 \end{aligned}$$

Rule: To convert a percent to a decimal, write the percent without the percent symbol and then move the decimal point two places to the left.

Application 1.

A truck is loaded to 95% of its capacity. Write 95% as a decimal. $95\% = \underline{95.} = .95$

Application 2.

If 25% is added to the number of square yards for flooring to account for waste, write 25% as a fraction. $25\% = \underline{25.} = .25$

Drill and Practice:

Shaw, 244-246

Dressler, pp. 168-169

Howett, p. 141

3. CONCEPT: TO CHANGE A DECIMAL TO A PERCENT

.65 of a company's fleet of delivery trucks is rented.
Write .65 as a percent.

$$65/100 = 65\%$$

$$\text{or } .65 = 65\% = 65\%$$

Rule: To change a decimal to a percent write the number of the percent, move the decimal point two places to the right and add the percent symbol.

Examples:

$$.07 = .07\% = 7\%$$

$$.13 = .13\% = 13\%$$

$$.125 = .125\% = 12.5\%$$

$$1.02 = 1.02\% = 102\%$$

$$.37 \frac{1}{2} = .37 \frac{1}{2}\% = 37 \frac{1}{2}\%$$

$$.2 = .20\% = 20\%$$

$$.003 = .003\% = .3\%$$

$$.00 \frac{1}{4} = .00 \frac{1}{4}\% = \frac{1}{4}\% \text{ [} \frac{1}{4}\% \text{ means } \frac{1}{4} \text{ of the hundredths decimal place therefore no decimal appears]}$$

Application 1.

The slope of a roadway was recorded as .07.
Write this slope as a percent.

$$.07 = .07\% = 7\%$$

Application 2.

During July .273 of the production costs in an assembly department went for overhead expenses.
Write .273 as a percent.

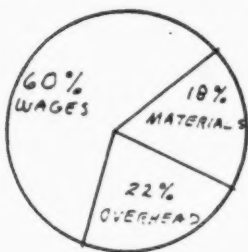
$$.273 = .273\% = 27.3\%$$

Drill and Practice:

Shaw, p. 246

Dressler, pp. 170-171

Howett, p. 142

4. CONCEPT: TO CHANGE A PERCENT TO A FRACTION

In a circle graph for a production department, 22% was spent on overhead, 18% was spent on materials, 60% was spent on wages.

Write each of these percents as common fractions.

$$22\% = \frac{22}{100} \quad \text{or} \quad \frac{11}{50}$$

$$18\% = \frac{18}{100} \quad \text{or} \quad \frac{9}{50}$$

$$60\% = \frac{60}{100} \quad \text{or} \quad \frac{30}{50} \quad \text{or} \quad \frac{3}{5}$$

$$\text{Adding: } \frac{11}{50} + \frac{9}{50} + \frac{30}{50} = \frac{50}{50} \quad \text{or} \quad 1$$

Note: In a circle graph, the total of the percents = 100%.

Rule: To write a percent as a common fraction, drop the percent symbol and write the number over 100, then reduce if possible.

Examples:

$$93\% = \frac{93}{100}$$

$$2\% = \frac{2}{100} = \frac{1}{50}$$

$$8.4\% = \frac{8.4}{100} \quad \text{or} \quad \frac{8.4 \times 10}{100 \times 10} = \frac{84}{1000} = \frac{21}{250}$$

Multiply numerator and denominator by 10.

$$9.47\% = \frac{9.47}{100} = \frac{9.47 \times 100}{100 \times 10} = \frac{947}{10000}$$

$$6 \frac{2}{3}\% = \frac{6 \frac{2}{3}}{100} = 6 \frac{2}{3} \times \frac{100}{100} = \frac{20}{3} \times \frac{100}{1} = \frac{20 \times 1}{3 \times 100}$$

$$= \frac{20}{3 \times 100} = \frac{1}{15}$$

Application 1.

Overseas sales of earthmoving equipment increased 225% during July.

Write 225% as a fraction.

$$225\% = \frac{225}{100} = \frac{9}{4} \text{ or } 2 \frac{1}{4}$$

That is, to say overseas sales increased $2 \frac{1}{4}$ times.

Application 2.

Shipping costs for machine tools were 11.35% of total sales. Write 11.35% as a common fraction.

$$11.35\% = \frac{11.35}{100} = \frac{11.335 \times 100}{100 \times 100} = \frac{1135}{10000} = \frac{227}{2000}$$

Drill and Practice:

Shaw, p. 255

Dressler, pp. 171-173

Howett, p. 145

5. CONCEPT: TO CHANGE A FRACTION, MIXED NUMBER OR A WHOLE NUMBER TO A PERCENT

A kiln is operating at an efficiency rate of $7/8$. Write $7/8$ as a percent.

Method 1: Convert the fraction to a decimal. Then change the decimal to a percent.

$$7/8 = 8 \overline{) 7.000} = .875 = .875 \times 100\% = 87.5\%$$

$$\begin{array}{r} .875 \\ 8 \overline{) 7.000} \\ \underline{64} \\ 60 \\ \underline{56} \\ 40 \end{array}$$

Method 2: Divide the denominator into 100, then multiply this result by both numerator and denominator.

$$7/8 = \frac{11}{100} = \frac{12.5 \times 7}{100} = \frac{87.5}{100} = 87.5\%$$

$$\begin{array}{r} 12.5 \\ 8 \overline{) 100.} \\ \underline{8} \\ 20 \\ \underline{16} \\ 40 \end{array}$$

Ans. 87.5%

Method 3: Multiply the fraction by 100%.

$$7/8 = 7/8 \times 100 = \frac{700}{8} = 87.5\%$$

Example:

Write as a percent:

$$3/4 = 3/4 \times 100 = 300/4 = 75\%$$

$$17/25 = \frac{17 \times 4}{25 \times 4} = \frac{68}{100} = 68\% \quad \text{Choose 4 to make denominator equal 100.}$$

$$9/32 = 32 \overline{) 9.000} = 28\%$$

$$\begin{array}{r} .281 \\ 32 \overline{) 9.000} \\ \underline{64} \\ 260 \\ \underline{256} \\ 40 \\ \underline{36} \end{array}$$

$$1 \frac{1}{20} = 21/20 = \frac{21 \times 5}{20 \times 5} = \frac{105}{100} = 105\%$$

$$8 = 8/1 = \frac{8 \times 100}{1 \times 100} = \frac{800}{100} = 800\%$$

Application 1.

During the month of August, overhead costs in the Accounting Department decreased by $\frac{3}{8}$. Write $\frac{3}{8}$ as a percent.

$$\frac{3}{8} \times 100\% = \frac{300\%}{8} = 8 \overline{)300.0} = 37.5\%$$

$$\begin{array}{r} 37.5 \\ 8 \overline{)300.0} \\ \underline{24} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

Application 2.

A test technician rejected 3 out of every 120 chasses tested.

Write his rejection rate as a fraction.
 $\frac{3}{120}$ answer.

Write his rejection rate as a percent.

$$\frac{3}{120} = \frac{3}{120} \times 100\% = \frac{300\%}{120} = \frac{30\%}{12} = \frac{5\%}{2} = 2 \frac{1}{2}\% \text{ or } 2.5\%$$

Drill and Practice:

Shaw, p. 247

Dressler, pp. 175-176

Howett, p. 143

6. CONCEPT: FINDING THE PERCENT OF A NUMBER

A small corporation spends 65% of its total costs for wages. If the total cost per month is \$56,300, find the total monthly outlay for wages.

Solution: Find 65% of \$56,000.

$$\begin{aligned}\text{Expenditure for wages} &= 65\% \times 56,300 \\ (\text{convert } 65\% \text{ to decimal}) &= .65 \times 56,300 \\ &= \$36,595 \text{ per month}\end{aligned}$$

Examples:

- 1) Find 3% of 84. =
 $.03 \times 84 = 2.52$
- 2) Find 8.37% of 112 =
 $.0837 \times 112 = 9.3744$
- 3) Find $32 \frac{3}{4}\%$ of 842 correct to nearest hundredth.
 $.32 \frac{3}{4} \times 842$
 $= 275.76$

$$\begin{array}{r} 842 \\ \times .32 \frac{3}{4} \quad \text{multiply by 3} \\ \hline 4 \underline{2526} \quad \text{divide by 4} \\ 631 \frac{1}{2} \\ 1684 \\ \hline 2526 \\ \hline 275.75 \frac{1}{2} \end{array}$$

or

$$\begin{aligned}&.3275 \times 842 \\ &= 275.755 \\ &= 275.76\end{aligned}$$

or

$$\begin{aligned}&\frac{32 \frac{3}{4}}{100} \times 842 \\ &= 32 \frac{3}{4} \div 100 \times 842 \\ &\frac{131}{4} \times \frac{1}{100} \times \frac{842}{1} \\ &= \frac{131 \times 842}{4 \times 100} = \frac{110302}{400} = 275.755 = 275.76 \\ &\quad \quad \quad = 275.76\end{aligned}$$

Application 1.

39% of paper stock in a print shop was used during the first week of the month. If 1200 reams of paper were available on the first day of the month. Find the number of reams used during the first week.

Solution: 39% of 1200
= $.39 \times 1200$
= 468 reams

Application 2.

Sales tax paid on the purchase of a new sprinkler system which costs \$15,200 was 8%. Calculate the sales tax.

Solution: 8% of \$15,200
= $.08 \times 15200$
Sales tax = \$1216

Drill and Practice:

Shaw, 266-269

Dressler, pp. 178-180

Howett, p. 151

7. CONCEPT: FINDING THE PERCENT WHICH COMPARES ONE NUMBER TO ANOTHER NUMBER

Complaints were received by a shipping department that 7 packages arrived damaged. If 70 packages were shipped, what percent arrived damaged?

Solution: Write a fraction which indicates what fractional part of the total shipment arrived damaged.

$$\frac{\text{No. of damaged packages}}{\text{Total No. packages shipped}} = \frac{7}{10}$$

Now rewrite 7/10 as a percent:

$$\frac{7}{10} = \frac{7}{10} \times 100\% = \frac{700\%}{10} = 70\%$$

Examples:

1) 13 is what percent of 20?

$$\frac{13}{20} = \frac{13 \times 5}{20 \times 5} = \frac{65}{100} = 65\%$$

2) 21 is what percent of 25?

$$\frac{21}{25} = \frac{21}{25} \times 100\% = \frac{21}{25} \times \frac{100\%}{1} = \frac{21 \times 100}{25 \times 1} = \frac{84}{1} = 84\%$$

3) 8 is what percent of 9?

$$\frac{8}{9} = 9 \overline{) 8.00} = .88 \quad \frac{8}{9} = 88 \frac{8}{9} \%$$

$$\begin{array}{r} .88 \\ 9 \overline{) 8.00} \\ \underline{72} \\ 80 \\ \underline{72} \\ 8 \end{array}$$

or

$$\frac{8}{9} = 9 \overline{) 8.000} = .89 = 89\% \text{ correct to nearest whole number.}$$

$$\begin{array}{r} .888 \\ 9 \overline{) 8.000} \\ \underline{72} \\ 80 \\ \underline{72} \\ 80 \\ \underline{72} \end{array}$$

Application 1. The product goal of a small subcontractor was 24,000 switch assemblies per year. Actually 22,490 were manufactured. What percent of the goal was achieved? [to nearest tenth of a percent].

Solution: Write as a fraction.

No. of assemblies actually made
Total manufacturing goal

$\frac{22,490}{24,000}$ Now convert this fraction to
 a percent.

$$\frac{22,490}{24,000} \times 100\% = \frac{22490 \times 100\%}{24000 \times 1} = \frac{2249000}{24,000}$$

= [93.708333]% (calculator readout)
 = 93.7% correct to nearest tenth of a percent.

Application 2. A large manufacturing plant hired
 526 new employees in 1986; in 1987,
 422 new employees were hired. In
 comparison with the number of persons
 hired in 1986 what percent represents
 the number of new employees in 1987?

Solution: Write as a fraction:

No. employees hired (1986)
No. employees hired (1987)

$$= \frac{422}{526}$$

Rewrite this fraction as a percent [correct to
 nearest whole
 number]

$$= \frac{422}{526} \times 100\% = \frac{42200\%}{526} = [80.228137]\% \\ = 80\%$$

Drill and Practice:

Shaw, pp. 278-279

Dressler, pp. 181-183

Howett, pp. 151, 153, 155-157

8. CONCEPT: FINDING A NUMBER WHEN A PERCENT OF IT IS KNOWN

An automated machine has been operating at 65% of its total capacity. If 390 units are processed in one day, what is the maximum daily capacity of the machine?

Solution: Write the problem as a number sentence:
 65% of maximum daily capacity = 390 units
 or $.65 \times \text{M.D.C.} = 390.$
 Therefore $\text{M.D.C.} = 390 \div .65$
 Maximum Daily Capacity = 600 units.

Rule: To find a number when a percent of it is known, divide the number by the decimal which represents the percent.

Example:

- 1) 84 is 12% of what number?

Solution: The number = $84 \div 12\%$
 $84 \div .12 = 700$

- 2) 113.2 is 48% of what number (correct to nearest tenth)

Solution: The number = $113.2 \div 48\%$
 $113.2 \div .48$
 $= 235.83333$
 $= 235.8$

Application 1. A machine part was ordered from a supplier for \$840.25. This price was 80% of the list price. What was the list price?

840.25 is 80% of list price
 $840.25 = .80 \times \text{L.P.}$
 Therefore $\text{L.P.} = 840.25 \div .80$
 $= [1082.6795] \text{read out}$
 List Price = \$1082.68

Application 2. In a small plant, policy is to keep overhead expenses under 40% of total sales revenue. If the overhead is \$19,100, find the minimum total sales revenue needed.

Solution: \$19,100 is 40% of total sales revenue
 or $19,100 = .40 \times \text{TSR}$
 Therefore $\text{TSR} = 19,100 \div .40$
 Total Sales Revenue = \$47.750
 (Minimum)

Drill and Practice:

Shaw, pp. 266-267, 276-277

Dressler, p. 185

Howett, pp. 151, 153, 155-157

9. CONCEPT: CALCULATING THE PERCENT INCREASE OR DECREASE

The floor area for the Auditing Department was 1800 sq. ft. The area has been expanded to 2250 sq. ft. Find the percent increase in area.

$$\text{Percent Increase} = \frac{(\text{New area} - \text{Original area})}{\text{Original Area}} \times 100\%$$

$$= \frac{2250 - 1800}{1800} \times 100\%$$

$$= \frac{450}{1800} \times 100\% = \frac{45000\%}{1800} = 25\%$$

To find the percent increase or decrease of a quantity use the formula method:

$$\text{Percent increase} = \frac{\text{New Value Minus Original Value}}{\text{Original Value}} \times 100$$

Application 1. The value of a manufacturing plant was \$1,250,000. One year later its value was appraised at \$950,000. Calculate the percent decrease in appraised value.

$$\text{Percent decrease} = \frac{1,250,000 - 950,000}{1,250,000} \times 100\%$$

$$= \frac{300,000}{1,250,000} \times \frac{100\%}{1} = \frac{30000000}{1,250,000}$$

$$= 24\%$$

Drill and Practice:

Shaw. p. 280-281

Dressler, p. 189

Howett, p. 159

E. MEASUREMENT

1. CONCEPT: UNDERSTANDING THE UNITS OF LENGTH

We say that a work area is 75 feet in width or that a caliper measures the width of a piece of circular stock as 1.312 centimeters or that a car was test-driven for 32 kilometers. How do these different measurement units relate?

One system of measurement is called the English or U.S. Standard System.

The units involved are:

- 1 mile = 5280 feet
- 3 feet = 1 yard
- 1 foot = 12 inches
- 1 yard = 36 inches

Another system of measure is called the metric system.

- 1 kilometer = 1000 meters
- 1 meter = 100 centimeters
- 1 centimeter = 10 millimeters
- 1 meter = 1000 millimeters

Sometimes the two systems need to be related.

- 1 meter = 39.37 inches
- 1 inch = 2.54 centimeters

Application 1.

Convert 14 inches to centimeters
 $14 \times 2.54 = 35.56$ centimeters

Convert 6 meters to inches
 $6 \times 39.37 = 238.22$ inches

Convert 74.5 centimeters to inches
 $74.5 \div 2.54 = 29.3$ "

Convert 937cm to meters
 937 centimeters = 937. = 9.37 meters

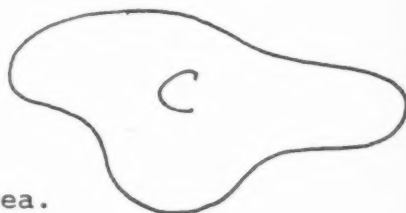
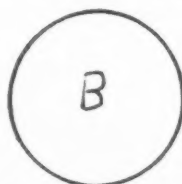
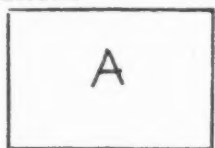
Convert 4327 millimeters to meters
 4327 millimeters = 4327. = 4.327 meters

Drill and Practice:

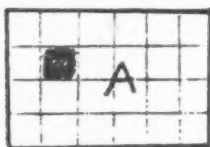
Shaw, p. 38-39(1-32), 70-71, 80-81
 Dressler, p. 196 (1-17)

2. CONCEPT: UNDERSTANDING UNITS OF AREA

The area is a measure of the surface enclosed by boundary lines.



In each case A, B or C represents the area.



If the length of this figure is 6 units and the width is 4 units, each shaded unit represents 1 square unit of area. By counting these units we arrive at 24 square units. By a similar method we could estimate the areas B and C.

Areas are measured in square units. For example:

The area of A = 24 square feet if length and width were in feet.

The area of B could be 17 square meters.

The area of C could be 42 square centimeters.

U.S. System of Area Units:

1 square mile = $(5280)^2 = 27,878,400$ square feet

1 square mile = $1760^2 = 3,097,600$ square yards

1 square yard = $3^2 = 9$ square feet

1 square yard = $36^2 = 1296$ square inches

1 square foot = $12^2 = 144$ square inches

Metric System of Area Units

1 square meter = $100^2 = 10,000$ square centimeters

1 square centimeters = $10^2 = 100$ square millimeters

Application 1. A work area which measures 17 ft. x 32 ft. is to be carpeted. If the carpet is ordered by the square yard, what is the equivalent area in square yards?

$$A = 17 \times 31 = 527 \text{ square feet}$$

$$A = 527 \text{ sq. yd.} = \frac{1 \text{ sq. yd.}}{27 \text{ sq. ft.}} = \frac{527}{27} = 19 \frac{14}{27} \text{ sq. yd.}$$

Application 2. A micro technique is being developed to apply solder on points which are 1 square millimeter in area. If

425 such points will be processed, what will be the total area covered in square centimeters?

1 x 425 sq. millimeters

$$425 \text{ sq. millimeters} \times \frac{1 \text{ sq. cm}}{100 \text{ sq. mil.}} = \frac{425}{100} = 4.25 \text{ sq. centimeters}$$

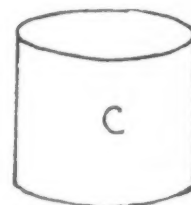
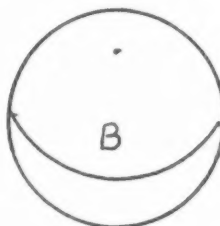
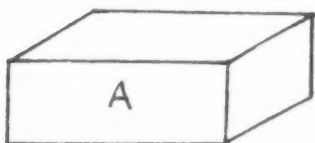
Drill and Practice:

Shaw, p. 170 (7-15)

Dressler, p. 436-437

3. CONCEPT: UNDERSTANDING UNITS OF VOLUME

Volume is a measure of space that an object occupies:



The volume of A could be 114 cubic yards.
 The volume of B could be 552 cubic centimeters.
 The volume of C could be 0.72 cubic meters.

Metric Units

Cubic Millimeters
 Cubic Centimeters
 Cubic Meters
 Cubic Kilometers

U.S. Standard Units

Cubic Inch
 Cubic Foot
 Cubic Yard
 Cubic Mile

To determine the number of cubic feet in a cubic yard:
 Consider:

3 ft. = 1 yard (measure of length)
 Therefore 3^3 cubic feet in 1 cubic yard
or 27 cubic feet in 1 cubic yard

In the U.S. Standard System:

1 cubic mile = $(5280)^3$ cubic feet
 1 cubic mile = $(1760)^3$ cubic yard
 1 cubic yard = $3^3 = 27$ cubic feet
 1 cubic yard = $(36)^3 =$ cubic inches
 1 cubic foot = $12^3 = 1728$ cubic inches

In the Metric System:

1 cubic meter = $(100)^3 = 1000000$ cubic centimeters
 1 cubic centimeter = $10^3 = 1000$ cubic millimeters

Application 1. A footing has a volume of 54 cu. ft.
 How many cubic yards of concrete will
 fill the form?

$$54 \text{ cu. ft.} = 54 \text{ cu. ft.} \times \frac{1 \text{ cu. yd.}}{27 \text{ cu. ft.}} = 2 \text{ cu. yd.}$$

92

There will be fewer cubic yards for the number
 of cubic feet so divide by 27.

Application 2. 243 cubic centimeters of acid was
needed in a laboratory process.
Convert this amount to cubic
millimeters

$$\begin{array}{rcl} 243 \text{ cc} \times \frac{1000 \text{ cu. millimeters}}{1 \text{ cc}} & = & 243 \times 1000 \\ & & = 243.000. \\ & & = 243000 \text{ cu. mil.} \end{array}$$

Drill and Practice:

Shaw, p. 188 (7-15)

Dressler, pp. 448-450, 455-456

4. CONCEPT: UNDERSTANDING LIQUID MEASURE OR CAPACITY

Liquid measure is a method of measuring volume of a substance in liquid form, or the capacity of a container to contain a liquid.

Example: 37 liters of commercial solvent
 14 gallons of gasoline
 20 milliliters of sulfuric acid

Many substances in liquid form are measured using the metric system.

1 kiloliter = 1000 liters

1 liter = 1000 milliliters

Others are still measured in the U.S. Standard or English system.

1 gallon = 4 quarts

1 quart = 2 pints

1 pint = 2 cups

1 cup = 8 ounces (liquid)

To relate the two systems:

1 quart (liquid) = .95 liters

1 liter = 1.1 quart (liquid)

Application 1. 3.4 liters of dilute sulphuric acid were stored in a special cabinet.
 Write this amount in milliliters.

1 liter = 1000 milliliters

Therefore:

3.4 liters = 3.400. = 3400 milliliters

Application 2. 37 quarts of milk were ordered by a dietician for the plant cafeteria.
 Write this amount as gallons.

1 gallons = 4 quarts

37 quarts x $\frac{1 \text{ gallon}}{4 \text{ quarts}}$ = $\frac{37 \times 1}{4}$

= 3 $\frac{3}{4}$ gallons

Drill and Practice:

Shaw, p. 89

Dressler, pp. 207-208 (16-22)

5. CONCEPT: UNDERSTANDING MEASURES OF WEIGHT OR MASS

In practice we may say that a chemical reagent sample weighs 1.6 kilograms or that a medication is in tablets of 25 milligrams or a pickup truck carries 1.5 tons.

The metric system is used in an increasing number of situations.

1 kilogram = 1000 grams
1 gram = 1000 milligrams

The U.S. Standard System or English system remains in some areas.

16 ounces = 1 pound
2000 pounds = 1 ton

To relate the two systems:

1 kilogram = 2.2 pounds
1 gram = .0352 ounces

Application 1. A sample of residue was taken from a smokestack. It weighed 492 grams. Rewrite 492 grams as kilograms.

492 grams = 492. = .492 kilograms

Application 2. A refuse plant collected 15,220 pounds of ash. Rewrite this amount as tons.

$$15220 \text{ pounds} \times \frac{1 \text{ ton}}{2000 \text{ pounds}} =$$

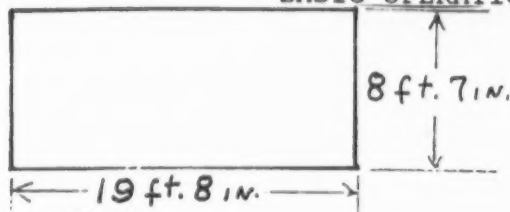
$$= \frac{15220}{2000} = 7.610 = 7.61 \text{ tons}$$

Drill and Practice:

Shaw, p. 88

Dressler, p. 208 (23-30), p. 199-200

6. CONCEPT: USING MIXED UNITS OF MEASUREMENT IN THE BASIC OPERATIONS



Find the perimeter of a rectangular floor area with the mixed dimensions as indicated in the drawing.

Addition

$$\begin{aligned}
 \text{Perimeter} &= 2 \times (19 \text{ ft. } 8 \text{ in.}) + 2 \times (8 \text{ ft. } 7 \text{ in.}) \\
 &= 38 \text{ ft. } 16 \text{ in.} + 16 \text{ ft. } 14 \text{ in.} \\
 &= 38 \text{ ft. } 16 \text{ in.} \\
 &+ \underline{16 \text{ ft. } 14 \text{ in.}} \\
 &54 \text{ ft. } 20 \text{ in.} \quad \text{or } 54 \text{ ft. } + 1 \text{ ft. } 8 \text{ in.} \\
 &= 55 \text{ ft. } 8 \text{ in.}
 \end{aligned}$$

Multiplication

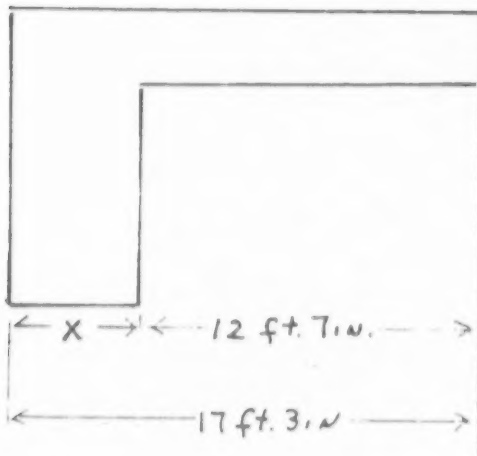
Find the area of the above rectangle

$$\begin{aligned}
 \text{Area} &= (19 \text{ ft. } 8 \text{ in.}) \times (8 \text{ ft. } 7 \text{ in.}) \\
 &= (19 \times 12 + 8 \text{ in.}) \times (8 \times 12 + 7) \\
 &= (228 + 8) \times (96 + 7) \\
 &= 236 \times 103 = 24308 \text{ square inches}
 \end{aligned}$$

$$\begin{aligned}
 \text{Area} &= 24308 \quad 144 = 168.80556 \text{ readout} \\
 &= 168.8 \text{ square ft.}
 \end{aligned}$$

Subtraction

$$x = (17 \text{ ft. } 3 \text{ in.}) - (12 \text{ ft. } 7 \text{ in.})$$

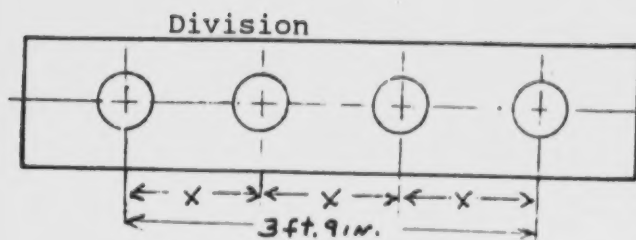


$$\begin{aligned}
 &17 \text{ ft. } 3 \text{ in.} \\
 &\underline{12 \text{ ft. } 7 \text{ in.}}
 \end{aligned}$$

Beginning with smallest unit. We cannot subtract 7 in. from 3 in. Therefore, borrow 1 ft. from 17 ft. This allows us to add 12 inches to 3 inches.

$$\begin{aligned}
 &17 \text{ ft. } 3 \text{ in.} \quad 16 \text{ ft. } 15 \text{ in.} \\
 &\underline{12 \text{ ft. } 7 \text{ in.}} \quad \underline{12 \text{ ft. } 7 \text{ in.}}
 \end{aligned}$$

$$x = 4 \text{ ft. } 8 \text{ in.}$$



Find the distance between the centers of holes to be drilled if the three holes are equally spaced.

$$x = \frac{3 \text{ ft. } 9 \text{ in.}}{3} = \frac{3 \text{ ft.}}{3} + \frac{9 \text{ in.}}{3} = 1 \text{ ft. } 3 \text{ in.}$$

Now if the total distance were 8 ft. 4 in.

$$x = \frac{8 \text{ ft. } 4 \text{ in.}}{3} =$$

Start with the largest unit (8 ft.). 8 is not divisible by 3, therefore remove a number of feet to give a number of feet divisible by 3. That is 8 ft. 4 in. = 6 ft. 28 in.

$$x = \frac{8 \text{ ft. } 4 \text{ in.}}{3} = \frac{6 \text{ ft. } 28 \text{ in.}}{3} = 2 \text{ ft. } 9 \frac{1}{3} \text{ in.} = 2 \text{ ft. } 9 \text{ in.}$$

Drill and Practice:
Dressler, pp. 209-216

F. RATIO

1. CONCEPT: AN UNDERSTANDING OF RATIO

If 977 parts out of 1000 parts examined pass inspection we can compare these numbers by saying that the ratio of the numbers passing inspection to the total number inspected by writing this fraction:

$$\frac{977}{1000}$$

or

$$977:1000$$

When two numbers are compared in this manner, it is called a ratio; and is read, "nine hundred seventy-seven is to one thousand"

To compare the number of parts passing to that number of parts which did not pass we would write:

$$\frac{977}{133}$$

or

$$977:133$$

This is read "nine hundred seventy-seven is to one hundred thirty-three".

Examples: Compare 5 and 4

$$"5 \text{ is to } 4" = \frac{5}{4} \text{ or } 5:4$$

Compare 20 and 30

$$"20 \text{ is to } 30" = \frac{20}{30} \text{ or } 20:30$$

However, like any fraction

$$\frac{20}{30} \text{ can be reduced, } \frac{20}{30} = \frac{2}{3} \text{ or } 2:3$$

To sum up, "20 is to 30" as "2 is to 3"

Equal ratios are in the form of equal fractions.

$$\text{That is: } \frac{20}{30} = \frac{2 \times 10}{3 \times 10} = \frac{2}{3}$$

Application 1. 72 out of 981 employees were absent on a work day.

Write a comparison which indicates the ratio of employees absent to the total number of employees.

$$\frac{72}{981} = \frac{8}{109} \text{ or } 8:109$$

Drill and Practice:

Shaw, op. cit., p. 202

Dressler, op. cit., pp. 301-302

2. CONCEPT: UNDERSTANDING RATIOS AS DECIMALS AND PERCENT

The ratio of width to length of a metal plate was said to be .7. Rewrite .7 as a fraction and as a percent.

$$.7 = \frac{7}{10} = \frac{70}{100} = 70\%$$

$$\text{Ratio } .7 = \frac{7}{10} \text{ or } 7:10 = 70\%$$

Examples:

Write 2:5 as a fraction, decimal and percent.

$$2:5 = \frac{2}{5} = 5 \overline{)2.0}^{\cdot 4} = \frac{40}{100} = 40\%$$

Write 35% as a ratio.

$$35\% = \frac{35}{100} = \frac{7}{20} \text{ or } 7:20$$

Application 1. Consumer response is important to most successful manufacturers. It is estimated that the ratio of dissatisfied consumers who respond (by letter or phone) to the total number of dissatisfied consumers is 1:50. Write this ratio as a decimal and then as a percent.

$$\frac{1}{50} = 50 \overline{)1.00} = .02$$

$$\frac{1}{50} = .02 = 2\% \text{ This means that 2\% of concerned consumers actually contacts the manufacturer.}$$

Drill and Practice:

Shaw, op. cit., p. 278

Dressler, op. cit., pp. 304, 306

G. GRAPHS AND CHARTS

1. CONCEPT: THE BAR GRAPH

A graph is one way to visualize information. Not only are the viewers able to see the relative sizes of the numbers involved but they are also able to see relationships and trends within the data depicted.

A six month report from the delivery department includes the following information:

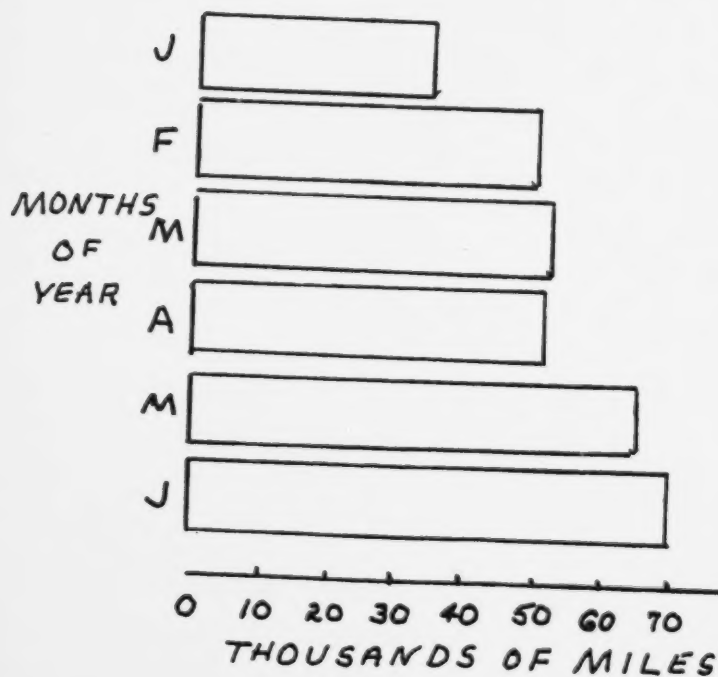
Total No. of miles traveled (6 trucks)

Jan.	44,209 mi.
Feb.	50,127 mi.
Mar.	52,120 mi.
Apr.	51,007 mi.
May	65,274 mi.
June	68,922 mi.

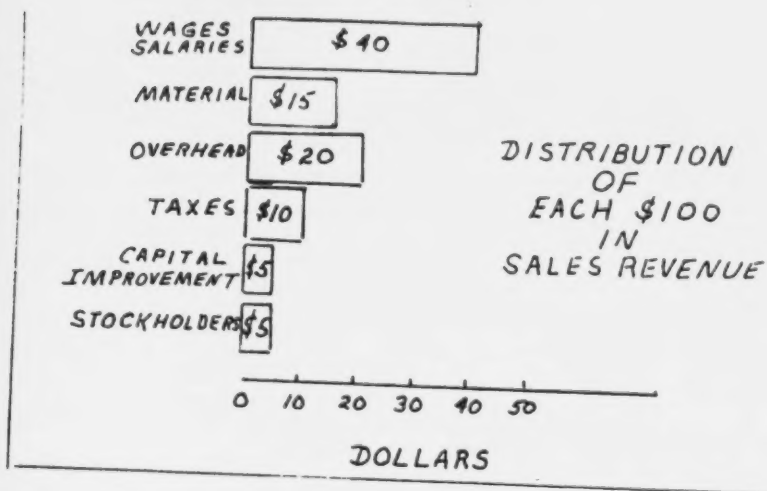
To illustrate this information during a meeting to plan the budget for the next six months a bar graph was drawn. [1 centimeter = 10,000 mi.]

To depict visually the general information each number in the data was rounded off to the nearest thousand.

34,209	34000 or	3.4 cm.
50,127	50000 or	5.0 cm.
52,120	52000 or	5.2 cm.
51,007	51000 or	5.1 cm.
65,274	65000 or	6.5 cm.
68,922	69000 or	6.9 cm.



SIX MONTHS
TOTAL MILEAGE
REPORT

Application 1.

In a report to its employees and stockholders a company reported how each \$100 in sales was used.

- (a) How many dollars out of \$100 went for wages and salaries? \$40.
- (b) How many more dollars went for taxes than to the stockholders. $\$10 - \$5 = \$5$

Drill and Practice:

Shaw, pp. 238-239, 460

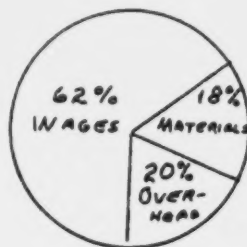
Dressler, pp. 470-472

Howett, p. 194-196

2. CONCEPT: THE CIRCLE GRAPH

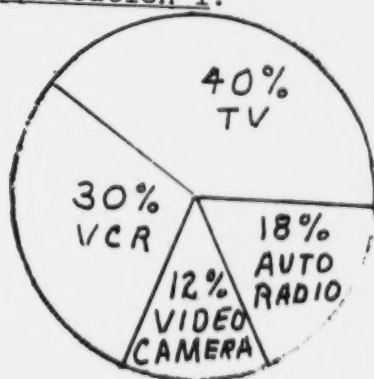
The circle graph is useful in illustrating the relative size of several components in comparison with a total. The size of the section of the circle is dependent upon the number it represents.

The budget for a production department was \$1,275,000. For the previous year the expenditures were illustrated in a circle graph.



1. What is the largest expense in the budget?
2. What percent was spent on materials?
3. What percent was spent on overhead and materials?
4. If the trend continues what amount should be allocated for overhead if the budget total is 1,275,000?
 $20\% \text{ of } 1,275,000 =$
 $.20 \times 1,275,000 = \$255,000$

Application 1.



The circle graph at the left represents the relative total sales revenue of electronic equipment produced by a corporation.

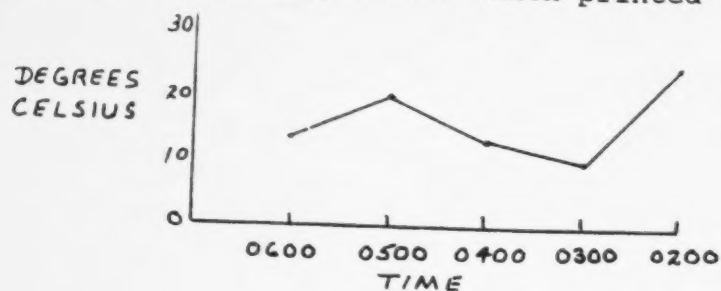
- 1) Which item makes up the greatest percentage of the total sales revenue? T.V.
- 2) Which item makes up the smallest percentage of the total sales revenue? Video Cameras.

Drill and Practice:

Shaw, p. 256-257
 Dressler, p. 477-482
 Howett, p. 189-193

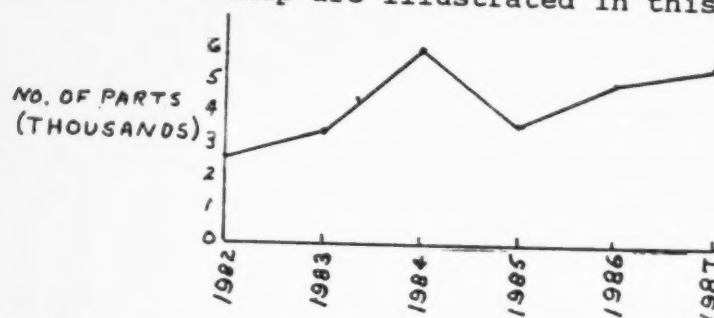
3. CONCEPT: THE LINE GRAPH

Room temperature in an electronic assembly department was monitored by a recording device which printed this record.



- 1) During the hours of 2 AM and 6 AM, which time registers the lowest temperature?
- 2) Read the lowest temperature.
- 3) Read the highest temperature.
- 4) At what hour did that occur?

Application Production of machine parts for a small shop are illustrated in this line graph.



- 1) What year had the highest production? 1984.
- 2) What year had the lowest production? 1982.
- 3) Estimate the production for 1985. 3500 parts.

Drill and Practice:

Shaw, p. 248-249

Dressler, p. 474-477

4. CONCEPT: READING A SPREAD SHEET

A spread sheet is a printout of data in rows and columns which relates information to such variables as time, temperature, payrates, personnel, etc. There are spread sheets of many different types.

Example:

Department Payroll Account

<u>Employee</u>	<u>Rate</u>	Regular <u>Weekly Pay</u>	Overtime <u>Hours</u>	Overtime <u>Pay</u>	Weekly <u>Total</u>
Ashton	7.00	280	4	42.00	322.00
Butte	7.50	300	4	45.00	345.00
Conrad	6.50	260	0	0.00	260.00
Fargo	8.30	332	8	99.60	431.60
Graham	7.00	280	2	21.00	301.00

How many employees worked overtime? 4.
 What is the total number of overtime hours worked?
 $4 + 4 + 0 + 8 + 2 = 18$ hours.
 Who received the highest total pay? Fargo.
 Who received the lowest total pay? Conrad.

Application. Department Budget Comparison 1987.

	1987 <u>Budget</u>	1987 <u>Actual</u>	<u>Variance</u>
Personnel	635,000	635,000	5000
Overtime	65,000	77,609	(2609)
Equipment	80,000	79,600	400
Supplies	12,400	13,650	(250)
Education	8,000	9,150	(1150)
Utilities	15,750	21,400	(5650)

Under Variance, data within parentheses () indicate the amount overspent.

List the items that stayed within budgeted amount.
 Personnel, Equipment.

List the items that were overspent.
 Overtime, Supplies, Education, Utilities.

For Utilities what percent of amount budgeted is the variance. (overspent amount.)

$$\frac{\text{variance} \times 100}{\text{amount budgeted}} = \frac{5650 \times 100}{15750} = [35.873016] \text{ read out} \\ = \underline{35.9\%}$$

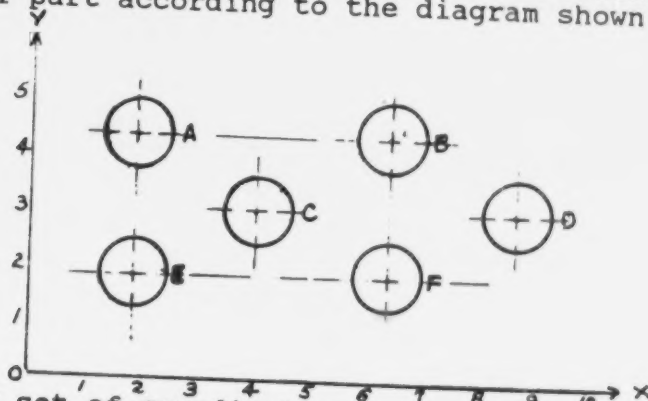
Drill and Practice:

Shaw, p. 11, 462

Dressler, p. 381

5. CONCEPT: PROGRAMMING IN THE XY PLANE

A machinist wishes to program the drilling of holes in a metal part according to the diagram shown here.



An XY set of coordinates has been superimposed over the drawing. The line OX is called the X-axis. The line OY is called the Y-axis. O is called the origin.

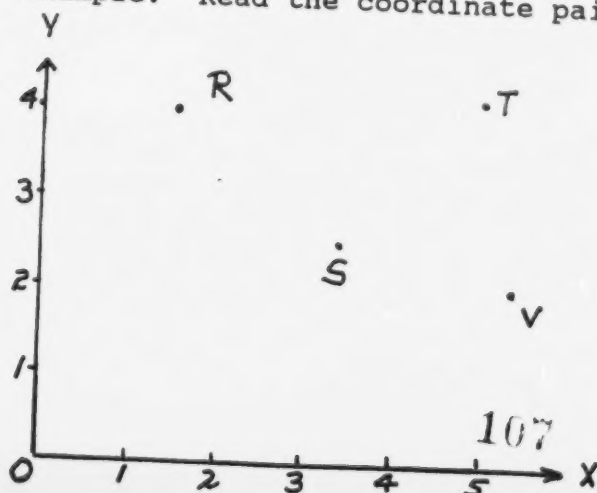
Consider the center of Hole C. From the diagram we read that C is located by C (4.0,3.0). This was obtained by reading the intersection of OX and a line through C perpendicular to OX, 4.0. Then we read the intersection of OY and a line through C perpendicular to OY, 3.0. Thus, C (4.0,3.0) indicates that C is located by moving 4.0 units to the right of the origin on the X-axis (OX) and 3.0 units up from the origin on the Y axis (OY).

From this diagram the location of each of the six points would be read as:

A (1.8,4.3)	D (8.6,3.0)
B (6.4,4.3)	E (1.8,1.8)
D (4.0,3.0)	F (6.4,1.8)

Each pair of numbers is called an ordered pair.

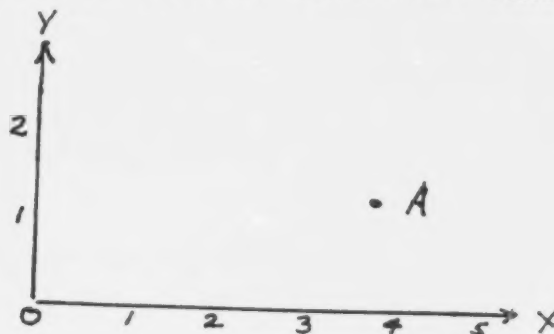
This data can now be entered into an automatic drill.
Example: Read the coordinate pairs for each point.



R (1.5,5)
S (3.4,2.5)
T (5.0,4.2)
V (6.4,2)

Application

Given the instructions that a 1-inch hole is to be drilled at A (1.25, 3.75) locate the center of this hole.



Drill and Practice:
Shaw, p. 380-381

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MODULE 2

ALGEBRA I,
GEOMETRY I &
STATISTICS

INTRODUCTION

Module II contains an introduction to Algebra, Geometry and Statistics. It follows closely the New York State Curriculum for Sequential Mathematics I. The format of Sequential Mathematics I includes algebra and geometry in a spiral configuration in which strands of learning are introduced on an elementary level and then expanded upon as the course develops.

There are no clear boundaries between arithmetic, algebra and geometry. As concepts and skills are developed they are merged into the stream of mathematics instruction.

Module III will include those topics in Sequential Mathematics II and III which were not developed fully in Module II. Module III will complete Algebra II, Geometry II and Trigonometry to the threshold of Pre-Calculus. Probability and logic are not included.

- I. INTRODUCTION
- II. ALGEBRA, GEOMETRY AND STATISTICS
- A. Introduction to Algebra
1. An Understanding of Sets
 2. The Number Line
 3. The Absolute Value of a Number
 4. The Addition of Signed or Directed Numbers
 5. The Subtraction of Signed or Directed Numbers
 6. The Multiplication of Signed or Directed Numbers
 7. The Division of Signed or Directed Numbers
 8. The Rational Numbers - The Operations of Addition and Subtraction
 9. The Rational Numbers - The Operations of Multiplication and Division
 10. The Order of Operations
 11. Powers and Directed Numbers
 12. The Order of Operations with Grouping Signs and Powers
 13. The Algebraic Expression
 14. Introduction to the Vocabulary of Algebra
 15. The Evaluation of Algebraic Expressions and Formulas
 16. The Linear Equation and its Solution
 17. The Properties of Equality
 18. The Solution of Linear Equations by Addition
 19. The Solution of Linear Equations by Multiplication
 20. Using Both Addition and Multiplication to Solve Equations
 21. Solving Equations with Parentheses
 22. Solving Equations with Variables in Both Members of Equations
 23. Translating Word Sentences into Algebraic Expressions
 24. Problem Solving with Equations
 25. Problem Solving with Formulas
 26. Solving Literal Equations and Formulas
 27. An Understanding of Inequality and its Symbols
 28. Solving Inequalities by Addition or Multiplication and Graphing the Solution Set
 29. Solving Inequalities by Addition and Multiplication
- B. An Introduction to Geometry
1. The Point, the Line and the Plane
 2. An Understanding of Distance, the Line Segment and the Ray
 3. The Measurement of an Angle and the Names of Special Angles
 4. Understanding Complementary Angles, Supplementary Angles, and Vertical Angles
 5. Understanding Parallel Lines and Intersecting Lines
 6. The Sum of the Measures of the Angles of a

- Triangle is 180 Degrees
- 7. Understanding the Relationships Between the Interior and Exterior Angles of a Triangle
- 8. Polygons and the Perimeter of a Polygon
- 9. Congruent Triangles
- 10. The Isosceles Triangle and its Properties
- 11. Triangles Classified According to Angles
- 12. The Use of the Statement - Corresponding Parts of Congruent Triangles are Congruent
- 13. The Parallelogram and its Properties
- 14. Methods of Recognizing when a Quadrilateral is a Parallelogram
- 15. The Trapezoid and its Properties
- 16. The Rectangle, the Rhombus and the Square, and their Properties
- 17. The Area of a Rectangle and the Area of a Square
- 18. The Area of a Parallelogram
- 19. The Area of a Triangle
- 20. The Area of a Trapezoid
- 21. The Volume of a Prism and a Pyramid

C. A Further Development of Algebra

- 1. Factors, Powers, Exponents and Bases
- 2. An Understanding of Polynomials
- 3. Addition and Subtraction of Polynomials
- 4. Multiplication Involving Monomials Which Contain Exponents
- 5. Multiplication of a Polynomial By A Monomial
- 6. Multiplication of Two Polynomials
- 7. Division of Monomials which Contain Exponents
- 8. Division of a Polynomial by a Monomial
- 9. Prime Factors, Common Factors, Greatest Common Factors, and Common Monomial Factors
- 10. A Short Method of Multiplying Two Binomials (FOIL)
- 11. Factoring a Polynomial of the Form Ax^2+Bx+C
- 12. Special Products and Factoring
- 13. The Solution of Quadratic Equations
- 14. Rational Numbers, Changing Fractions to Decimals
- 15. Rational Numbers, Changing Decimals to Fractions
- 16. Square Root
- 17. Multiplication and Division of Radicals
- 18. Expressing Radicals in Simplest Form
- 19. Simplifying Radical Expressions Involving Addition and Subtraction
- 20. Multiplication of Two Binomials Containing Radicals
- 21. The Quadratic Formula

D. Ratio and Proportion

1. An Understanding of Ratio
 2. The Use of Ratio in an Algebraic or Geometric Setting
 3. Proportion
 4. Percent and the Solution of Problems Involving Percent
- E. A Further Development of Geometry
1. Similar Polygons - Definition and Properties
 2. If a Line is Parallel to One Side of a Triangle and Intersects the Other Two Sides, A Second Triangle is Formed which is Similar to the Original Triangle
 3. The Pythagorean Theorem
 4. The Vocabulary of the Circle
 5. The Arcs of a Circle and Central Angles
 6. The Inscribed Angle of a Circle
 7. The Inscribed Polygon and the Circumscribed Circle
 8. The Circumference of a Circle
 9. The Area of a Circle
 10. The Ratio of the Circumferences of Two Circles and the Ratio of Their Radii
 11. The Sphere, Cylinder, and Cone: Vocabulary and Volume
- F. Statistics
1. Statistics - Definition and Vocabulary
 2. The Mean, the Median, and the Mode
 3. The Quartile and the Percentile
 4. Random Samples and Simulation
- G. The Mathematics of Linear Equations
1. Programming in the XY Plane
 2. The Distance Between Two Points on the Same Horizontal or Vertical Line
 3. The Graph of a Linear Equation
 4. The Slope of a Line
 5. The Slope - Intercept Form of a Linear Equation
 6. The Graph of a Linear Inequality in Two Variables
 7. The Graphic Solution of a System of Linear Equations
 8. Solving a System of two Linear Equations Using the Substitution Method
 9. Solving a System of Two Linear Equations Using the Addition-Subtraction Method
 10. Graphing Systems of Linear Inequalities

INTRODUCTION

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INTRODUCTION TO ALGEBRA

1. CONCEPT: AN UNDERSTANDING OF SETS

A set is a collection of things. A set can merely exist in the mind of a person or it can have an actual reality in the workplace.

Examples:

Set A = {General Motors, Kodak, Xerox}
 Set B = {Ramon, Marie, Thang, John}
 Set C = {1/2, .05, 103, 14, 6 1/2}
 Set D = {Numbers on a Data Sheet}

Sets can be lists, such as sets A,B, or C, or sets can be defined by a description as in Set D. Sets can be named (A,B,C or D) and braces {} are often used to enclose the items in a set called elements.

A set with no elements is called the empty set or the null set. Symbols $\emptyset$ or {}.

The sets A,B,C,D are called finite sets since each has a definite number of elements.

An infinite set would be one with an unlimited number of elements like {positive whole numbers} or {1,2,3,4,5...}.

Application: On the worksite, in the shop or the laboratory, people are continually dealing with sets of numerical information called data. Statistics is the mathematics of numerical data.

If we were determining the life span of an auto battery, the population is the set of all batteries manufactured during 1987. The sample is the set of auto batteries actually tested during 1987.

Drill and Practice:

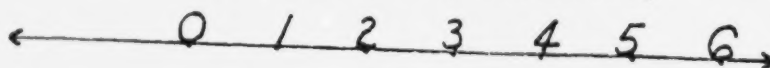
Rising, p. 1-3

Dressler, p. 42-43

2. CONCEPT: THE NUMBER LINE

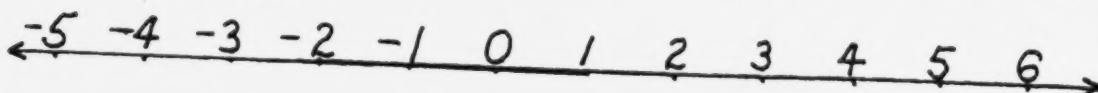
Consider the set of whole numbers as we normally understand them.

Whole Numbers = $\{0, 1, 2, 3, 4, 5, \dots\}$



These numbers can be indicated on a number line where the standard unit is the distance between 0 and 1.

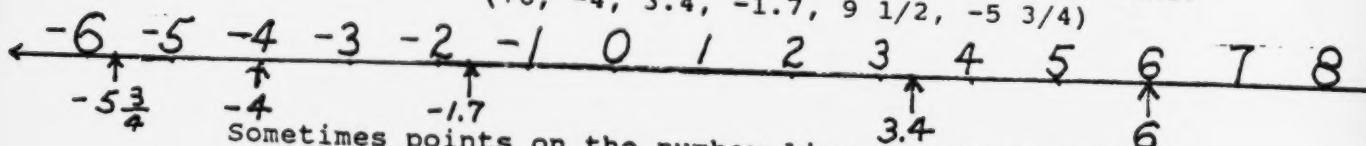
By extending the numbers to the left of zero we can locate other positions which can be identified by the symbols -1, -2, -3, -4, etc.



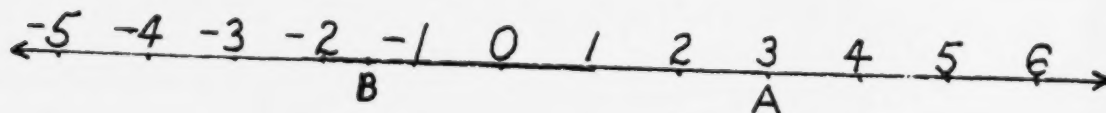
-6 is read "negative 6", 3 is read "positive 3". Negative signs must be shown. Positive signs need not be shown, that is, 5 and +5 have the same meaning. Numbers extend infinitely (without limit) in both directions. This set is called the set of integers (that is, the positive whole numbers, the negative whole numbers, and zero.) The line is called the Real Number Line.

Actually the Real Number Line is not limited to the representation of the integers. Every point on the real number line represents a number. Conversely, every real number is represented by a point on the line. A real number set includes all whole numbers, decimals and fractions.

Example: Locate these numbers on the number line:
 $(+6, -4, 3.4, -1.7, 9 \frac{1}{2}, -5 \frac{3}{4})$



Sometimes points on the number line are labeled with a letter. The number associated with that point is said to be the coordinate of that point.



The coordinate of A is 3. Therefore, we write $A(3)$. The coordinate of B is $-1 \frac{1}{2}$ or $B(-1 \frac{1}{2})$.

Note, in some textbooks the number -4 on the number line is written as $\bar{4}$.

$\bar{4}$ or -4 can have several meanings:

- 1) the position $\bar{4}$ or -4 on the number line
- 2) a movement of 4 units to the left
- 3) the subtraction of 4

0 can mean:

- 1) the position 0
- 2) a movement of 0 units

+8 or +8 can mean:

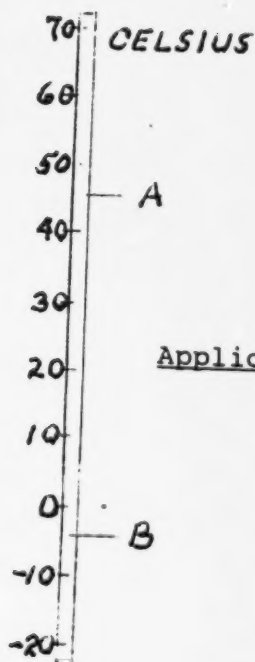
- 1) the position +8 or +8
- 2) a movement of 8 units to the right
- 3) the addition of 8

These numbers are called signed numbers or directed numbers.

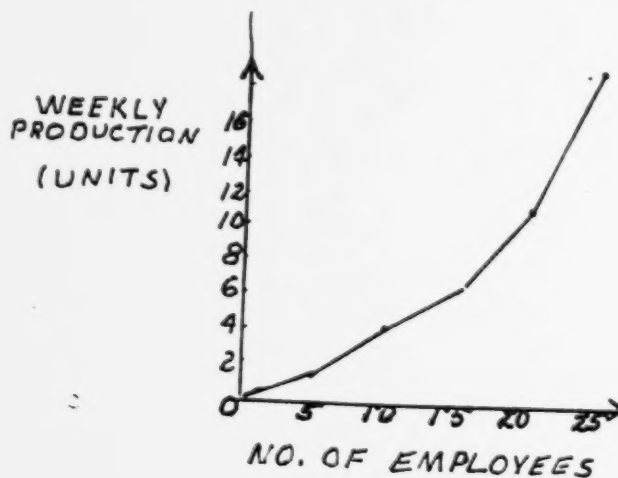
Application 1. A thermometer is a familiar example of a real number line. Positive numbers, negative numbers and zero are represented.

Read temperature indicated at A.
Approximately 35°C .

Read temperature indicated at B.
Approximately -5°C .



Application 2. Many number lines which are limited to zero and the positive integers are used in line graphs.



Drill and Practice:

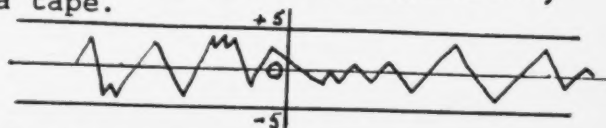
Rising, p. 5,6

Dressler, p. 186

Smith, p. 117-118

3. CONCEPT: THE ABSOLUTE VALUE OF A NUMBER

A marker, indicating variations from a controlled temperature, recorded those variations on a tape.



On the vertical scale the numbers +5 and -5 represent two points, each of which has the same distance from 0 (the origin). Numbers which have the same distance from the origin are said to have the same absolute value.

Example: The distance of +5 from the origin is 5 units or +5.

The distance of -5 from the origin is 5 units or +5.

That is, opposites have the same absolute value. Except for 0, the absolute value of any number is positive. The absolute value of 0 is 0.

The symbol of absolute value is a pair of vertical bars.

Example:

$$\begin{array}{ll} | +7 | = +7 & | -3 | + | +2 | = (+3) + (+2) = +5 \\ | +11 | = +11 & \\ | -4 | = +4 & | -3 + (+2) | = | -1 | = +1 \\ | 0 | = 0 & \end{array}$$

Application: Write the absolute value of 12 degrees below zero. $| -12^{\circ} | = +12^{\circ}$

Write the absolute value of a tolerance of ± 0.0052 .

± 0.0052 means between +.0052 and -.0052.

$$| +.0052 | = +.0052, \quad | -.0052 | = +.0052.$$

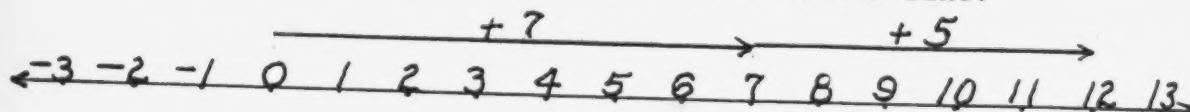
Drill and Practice:
Dressler, p. 193-194

4. CONCEPT: THE ADDITION OF SIGNED OR DIRECTED NUMBERS

When we write $7 + 5$ we normally mean to add the 7 and 5. The result is 12. From what we learned in our discussion of the number line, $7 + 5$ could also be written as

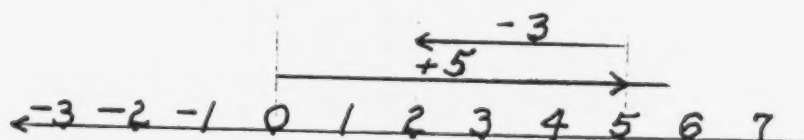
$+7 + +5$ where $+7$ and $+5$ are directed or signed numbers. And the $+$ sign between $+7$ and $+5$ is a sign of operation (addition).

To illustrate this operation on the number line:

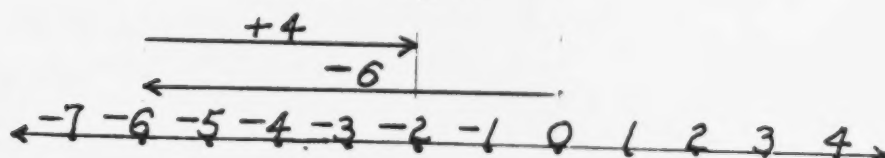


That is, $+7 + +5 = +12$
 seven five the position
 units units $+12$
 right right

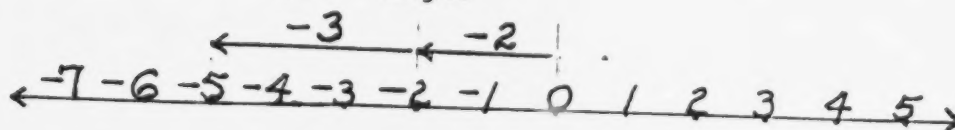
Three other examples illustrate what happens when different combinations of directed numbers are combined on the number line. This operation is called addition of signed or directed numbers.



$(+5) + (-3) = +2$
 five three position
 units units $+2$
 right right



$-6 + +4 = -2$
 six four position
 units units -2
 left right



$-2 + -3 = -5$
 two three position
 units units -5
 left left

Now try without the diagrams:

$$\begin{aligned} +8 + +6 &= +14 \\ -3 + -2 &= -5 \\ -8 + +9 &= +1 \\ -16 + +10 &= -6 \end{aligned}$$

Rule: To add
To combine
or To find the sum of } two signed numbers,

- (a) If the numbers have the same sign, add their absolute values and affix to that result the common sign.
- (b) If the numbers have opposite signs, subtract their absolute values and affix to that result the sign of the larger absolute value.

For a thorough understanding of the addition operation, certain properties of addition should be considered.

Properties of the Addition Operation

1. Commutative Property: Numbers can be added in any order.
Ex: $(+71) + (-35) = +36$
or $(-35) + (+71) = +36$
2. Associative Property: Numbers can be grouped in any order.
 $(-7) + (+3) + (-2) = [(+3) + (-2)] + (-7)$
 $= (+1) + (-7) = -6$
or
 $(-7) + (+3) + (-2) = (-7) + [(+3) + (-2)]$
 $= (-7) + (+1) = -6$

The Addition Property of Zero

The sum of zero and any number is the number. Zero is called the identity element for addition.

$$\begin{aligned} \text{Ex: } (-7) + 0 &= -7 \\ 0 + (+2) &= +2 \end{aligned}$$

The Additive Inverse

The additive inverse of a number is its opposite. The sum of a number and its additive inverse is zero.

$$\text{Ex: } (+8) + (-8) = 0, \quad (-11) + (+11) = 0$$

Zero is its own additive inverse. $0 + 0 = 0$

Examples: Addition

$$\text{Add: } 8 - 3 = +8 + -3 = +5$$

$$\text{Add: } -2 + -4 = -6$$

$$\text{Add: } (-7) + (+11) = +4$$

$$\text{Add: } 6 + (-6) = 0$$

$$\text{Add: } (+1.732) + (-0.843) = +0.889$$

$$\text{Add: } -8 + +6 + -1 + +13 =$$

$$-8 + -1 + +6 + +13 =$$

$$(-8 + -1) + (+6 + +13) =$$

$$-9 + +19 =$$

$$+10$$

Change the order
(commutative property
of addition) so that
negative numbers and
positive numbers can

be grouped (associative property of
addition), then add.

Application 1.

In a steering control test, a car veered -13.7cm (left) for the first 100 meters, +1.4cm (right) for the second 100 meters, to be followed by readings of +7.4cm, -2.1cm and -4.3cm. For the 500 meters total, what was the net result if all readings were combined?

Solution:

$$\text{or } (-13.7) + (+1.4) + (+7.4) + (-2.1) + (-4.3)$$

$$[(-13.7) + (-2.1) + (-4.3)] + [(+1.4) + (+7.4)]$$

$$-13.7$$

$$-2.1$$

$$-4.3$$

$$-20.1$$

$$+1.4$$

$$+7.1$$

$$+8.5$$

$$= (-20.1) + (+8.5) = -11.6$$

$$\text{or } 11.6\text{cm.}$$

$$\text{to the left}$$

$$-20.1$$

$$8.5$$

$$-11.6$$

Application 2.

During a 6-month period, production costs in a manufacturing department were above or below the average production costs as indicated.

<u>Month</u>	<u>Variation from Average Monthly Production Cost</u>
April	+ \$9,320
May	+ 1,413
June	- 1,522
July	- 2,227
August	+ 1,644
September	+ 2,499

What is the combined 6 month variation? Above or below the average?

$$\left\{ \begin{array}{l} \text{Combined} \\ \text{Six-month} \\ \text{Variation} \end{array} \right\} = +9320 + (+1,413) + (-1522) + (-2277) + (2499)$$

+9320	-1522	13232
+1413	<u>-2227</u>	<u>-3749</u>
<u>+2499</u>		
\$ (13232) +	(-3749) =	+9481

Answer: \$9481 above the average

Drill and Practice:
 Rising, p. 7-10
 Dressler, p. 199-203

5. CONCEPT: THE SUBTRACTION OF SIGNED OR DIRECTED NUMBERS

Rule: To subtract two numbers, add the opposite of the number being subtracted.

Example: $9-12 = 9+(-12) = -3$ or -3 .
 $(-3)-(-4) = 3+(+4) = -1$
 $(+6)-(+7) = +6 + (-7) = -1$

In practice, rewrite the problem, change the sign of the subtrahend to its opposite then proceed according to the rules of addition.

Note: There is no commutative or associative property for subtraction.

Ex: $6-3$ does not equal $3-6$
 Since $6+(-3) = 3$ and $3 + (-6) = -3$

Therefore subtraction is not commutative that is, we cannot reverse the position of the two numbers.

Ex: $9-5-4$ means $(9-5) - 4$
 or $[9+(-5)] - 4$
 $4-4 = 0$

not $9-(5-4)$
 $9-[5-(+4)]$
 $9-1 = 8$

Associative property does not hold for subtraction.

Application 1. At ignition radiator temperature in a car dropped from 10°F on one day to -8° on the following day. Through how many degrees did the temperature fall?

Temperature fall = Final reading - Original reading
 $= -8^{\circ} - (+10^{\circ}) = -8 + (-10) = -18^{\circ}$

This is a subtraction problem since we are seeking a difference in temperature.

Application 2. The reading on a vernier micrometer was 0.969 inches when a cylinder diameter was measured. If the drawing dimension was $.968+.002$, was the micrometer reading within the tolerance allowed?

Solution: Maximum measurement $= 0.968+.002$
 $= 0.970$ inches
 Minimum measurement $= 0.968-.002$
 $= 0.968+(-0.002)$
 $= 0.966$ inches

Yes. 0.969 is between 0.970 and 0.966

Drill and Practice:

Dressler, p. 203, 207

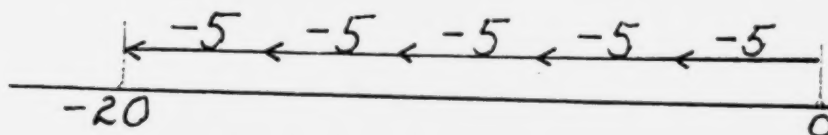
Rising, p. 11, 12

Smith, R.D., p. 119-120

6. CONCEPT: MULTIPLICATION OF SIGNED OR DIRECTED NUMBERS

If $4 \times 5 = 20$, then we can write that:
 $(+4) \times (+5) = +20$

Rule: The product of two positive numbers is positive.
 We can also write that
 $+4$ times $(-5) = -20$ or $(+4) \times (-5) = -20$



Rule: The product of a positive number and a negative number is negative.

If $(+4) \times (-5) = -20$
 then $(+3) \times (-5) = -15$
 $(+2) \times (-5) = -10$
 $(+1) \times (-5) = -5$
 $0 \times (-5) = 0$
 therefore, following the pattern
 $(-1) \times (-5) = +5$
 $(-2) \times (-5) = +10$

Rule: The product of two negative numbers is positive.

Symbols for multiplication: $9 \cdot -2$ means $(+9) \times (-2)$

$(-8)(-6)$ means $(-8) \times (-6)$

The times sign $\times$ is often not used, to avoid confusion with the letter x .

Example:

$$\begin{aligned} (-6)(-5) &= +30 \\ (+10)(-2) &= -20 \\ (-8)(-9)(+2) &= +144 \\ (-1)(-2)(-3)(-5)(+6) &= +180 \\ (-3) \cdot +2 \cdot 0 &= 0 \end{aligned}$$

Properties of the Multiplication Operation

Commutative Property: Numbers can be multiplied in any order.

$$(+3) \cdot (-4) = -12 \text{ and } (-4) \cdot (+3) = -12$$

Associative Property: For multiplication, numbers can be grouped in any order.

$$\begin{aligned} (-6) \cdot (-3) \cdot (+5) &= [(-6) \cdot (-3)] \cdot (+5) \\ &= (+18) \cdot (+5) = +90 \end{aligned}$$

$$\begin{aligned} \text{or } (-6) \cdot (-3) \cdot (+5) &= (-6) \cdot [(-3) \cdot (+5)] \\ &= (-6) \cdot (-15) = +90 \end{aligned}$$

Multiplication Property of Zero

$$\text{Since } (+9) \cdot 0 = 0$$

$$\text{and } (-2) \cdot 0 = 0$$

The product of a number and zero is zero.

The Multiplicative Inverse (or the reciprocal) of a number.

$$+5 \cdot (+1/5) = (5/1)(1/5) = \frac{5 \cdot 1}{1 \cdot 5} = 5/5 = 1$$

$$-2 \cdot (-1/2) = (-2/1)(-1/2) = \frac{(-2)(-1)}{1 \cdot 2} = \frac{+2}{+2} = 1$$

$$\frac{-3}{4} \cdot (-4/3) = (-3/4)(-4/3) = \frac{(-3)(-4)}{4 \cdot 3} = \frac{+12}{12} = 1$$

The multiplicative inverse (or reciprocal) of a number is another number such that the product of the two numbers is 1.

The Identity Property for Multiplication

The product of any number and 1 equals the number.

$$\text{Ex: } 7 \times 1 = 7 \quad (-3) \times 1 = -3 \quad \text{Note also, } 13 = 13 \times 1$$

The Distributive Property: Multiplication is distributive over addition.

$$\begin{aligned} \text{Ex: } 3 \cdot (4+5) &= 3 \cdot 4 + 3 \cdot 5 \\ &= 12 + 15 = 27 \end{aligned}$$

$$\text{or } 3(7) = 27$$

$$\text{In general: } a(c+d) = a \cdot c + a \cdot d$$

$$\begin{aligned} \text{Ex: } 6(X+2) &= 6 \cdot X + 6 \cdot 2 = 6x + 12 \\ a(r-q) &= ar - aq \end{aligned}$$

The distributive property of multiplication allows us to eliminate parentheses from an expression.

$$\text{Ex: } 3(a+6) = 3a + 18$$

Also, the distributive property allows us to combine terms with identical variables.

$$8s + 7s = s(8+7) = s \cdot 15 = 15s$$

$$\text{Therefore, } 9m - 4m = (9-4)m = 5m$$

$$\text{or } 30y - 14y = 16y$$

30y and -14y are called similar terms since their literal coefficients are identical.

6x and 6y are not similar

-13x²y and 10xy² are not similar.

4x²y³ and -5x²y³ are similar.

Application.

19

In calibrating a fuel control regulator the temperature in a reactor was observed to be 3.7° below the average for the last ten days. How many "degree-days" were indicated and was this figure positive or negative?

$$\begin{aligned}\text{Degree-Days} &= \text{No. of days} \times \text{Temp. Variation from Average} \\ &= 10 \times (-3.7^{\circ}) = -37 \text{ degree-days}\end{aligned}$$

Drill and Practice:

Rising, p. 12-15

Dressler, p. 207-212

Smith, p. 119

7. CONCEPT: THE DIVISION OF SIGNED OR DIRECTED NUMBERS

Since $(+5)(+2) = +10$, it follows that

$$(+10) \div (+2) = +5$$

Rule: The quotient of two positive numbers is positive.
Again, if $(-8)(-4) = +32$ it follows that

$$(+32) \div (-4) = -8$$

Rule: The quotient of a positive number and a negative number is negative.

$$\text{Further, since } (-9)(+3) = -27$$

$$\text{then, } (-27) \div (-9) = +3$$

Rule: The quotient of two negative numbers is positive.
Special considerations of the division operation are:

1. Division is not commutative

$$(+12) \div (-3) = -4 \quad \text{but} \quad (-3) \div (+12) = \frac{-3}{+12} = \frac{-1}{4}$$

2. Division is not associative.

Compare:

$$\begin{array}{lcl} +27 \div [(-9) \div (+3)] & \text{with} & [+27 \div (-9)] \div (+3) \\ +27 \div (-3) & & (-3) \div (+3) \\ = -9 & & = -1 \end{array}$$

3. Division by 0 is undefined

Since, if $5 \div 0 = \text{some number } k$,
then $k \times 0$ should equal 5. This is impossible.

Examples:

$$(-30) \div (-5) = +6$$

$$(-8) \div (+2) = -4$$

$$(+20) \div (-8) = \frac{+20}{-8} = -2 \frac{4}{8} = -2 \frac{1}{2}$$

Application 1.

A truck worth 16,000 was depreciated over a period of 5 years. This can be represented as a total change in value of -\$16,000. The depreciation per year would be $(-16,000) \div 5$ or -\$3200.

Application 2.

The annual loss in sales revenue for a small machine shop was \$19,429. What is the average monthly loss?

Solution: $(-19,429) \div (+12) = -\$1619.08$
-1619.08 indicates the monthly loss.

Drill and Practice:

Rising, p. 15, 16

Dressler, p. 212-213

Smith, p. 119-120

8. CONCEPT: THE RATIONAL NUMBERS - THE OPERATIONS OF ADDITION AND SUBTRACTION

Definition: A rational number is a number which can be written in the form $\frac{a}{b}$ where a and b are integers and b is not zero.

Examples:

Write 5 as a rational number $\frac{5}{1}$

$\frac{3}{5}$ is a rational number.

$6\frac{2}{3}$ can be written as $\frac{20}{3}$

.7 is a rational number since $.7 = \frac{7}{10}$

16.75 is a rational number
since $16.75 = 16 \frac{75}{100} = 16 \frac{3}{4} = \frac{67}{4}$

All whole numbers, fractions, mixed numbers and repeating decimals are rational numbers.

To add or subtract rational numbers with the same or different denominators rewrite the fractions with the same denominators, if necessary, then proceed according to the rules of addition and subtraction of fractions.

$$\frac{9}{10} - \frac{5}{10} = \frac{9-5}{10} = \frac{4}{10} = \frac{2}{5}$$

$$\frac{6}{11} - \frac{7}{11} = \frac{6-7}{11} = \frac{-1}{11}$$

$$\frac{-5}{8} + \frac{3}{8} = \frac{-5+3}{8} = \frac{-2}{8} = \frac{-1}{4}$$

$$\frac{-3}{16} + \frac{5}{8} = \frac{-3}{16} + \frac{10}{16} = \frac{-3+10}{16} = \frac{7}{16}$$

16 is the lowest common denominator.

$$\begin{array}{r} -5 \frac{3}{4} - (-8 \frac{1}{5}) = -5 \frac{3}{4} = -5 \frac{15}{20} + 8 \frac{4}{20} = +7 \frac{24}{20} \\ +8 \frac{1}{5} = +8 \frac{4}{20} -5 \frac{15}{20} = -5 \frac{15}{20} \\ \hline \phantom{-5 \frac{3}{4}} +2 \frac{9}{20} \end{array}$$

Example:

Combine

$$\begin{aligned}
 & -1.732 + (+8.97) + (-0.392) \\
 & = [-1.732 + (-0.392)] + (+8.97) \\
 & = -2.124 + (+8.97) \\
 & = +6.846
 \end{aligned}$$

Application 1.

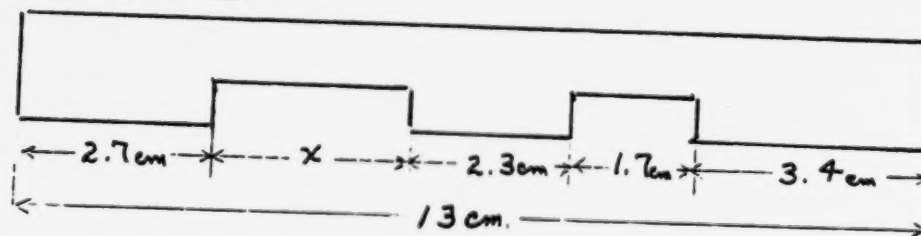
Temperature variations in a batch process device were recorded as follows:

<u>Time</u>	
12:00 Noon	+1.7°C
1:00 P.M.	-2.4°C
2:00 P.M.	-0.7°C
3:00 P.M.	+0.5°C

What is the average variation depicted by these four readings?

$$\begin{aligned}
 \text{Average} &= \frac{\text{sum of readings}}{\text{no. of readings}} = \frac{+1.7 - 2.4 - 0.7 + 0.5}{4} = \frac{-0.9}{4} \\
 &= -0.225^{\circ}\text{C}
 \end{aligned}$$

Application 2.



From the drawing, using the dimensions as indicated, find the length of x.

$$\begin{aligned}
 \text{Solution: } x &= 13 - 2.7 - 2.3 - 1.7 - 3.4 \\
 x &= 13 - 10.1 \\
 x &= 2.9\text{cm.}
 \end{aligned}$$

Drill and Practice:

Rising, p. 17-23

Smith, p. 122

9. CONCEPT: THE RATIONAL NUMBERS - THE OPERATIONS OF MULTIPLICATION AND DIVISION

To multiply two fractions or mixed numbers divide the product of the numerators by the product of the denominators, then simplify.

Rule: $\frac{a}{b} \times \frac{c}{d} = \frac{axc}{bxd}$ where a, b, c, d are integers.

To divide two fractions or mixed numbers multiply the dividend by the multiplicative inverse of the divisor, then simplify.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{axd}{bxc}$$

Examples:

$$\frac{(-2)}{3} \frac{(-3)}{10} = \frac{(-2)(-3)}{3(10)} = \frac{(-2)(-1)}{1(10)} = \frac{+1}{10}$$

$$(-6 \frac{2}{3}) \div \frac{3}{4} = \frac{(-20)}{3} \frac{(4)}{3} = \frac{-20 \times 4}{3 \times 3} = \frac{-60}{9} = -6 \frac{6}{9}$$

$$= -6 \frac{2}{3}$$

$$\frac{(-3)}{4} \div 5 = \frac{-3}{4} \times \frac{1}{5} = \frac{-3 \times 1}{4 \times 5} = \frac{-3}{20}$$

Note: $-\frac{3}{4}$ or $-\frac{3}{4}$ means $-\frac{3}{4}$ or $\frac{3}{-4}$ not $\frac{-3}{-4}$

Example: Find the product.

$$(-1.7)(-20.6) = +35.02$$

$$\begin{array}{r} -20.6 \\ \times -1.7 \\ \hline 1442 \\ 206 \\ \hline +35.02 \end{array}$$

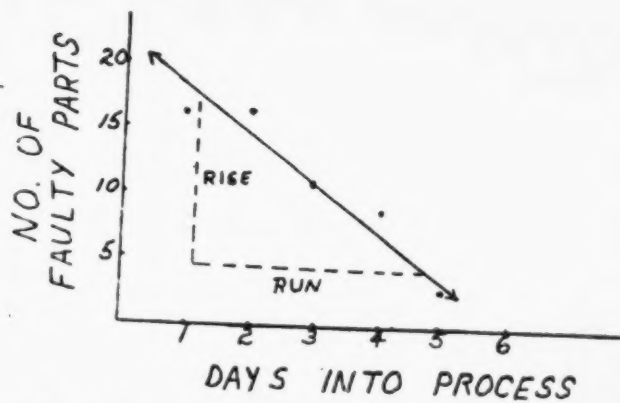
Example: Divide $-8.6471 \div 0.42$ correct to nearest tenth

$$-8.6471 \div 0.42 = 0.42 \overline{) 20.58} \text{ or } -20.6$$

Note signs can be omitted in this step.

$$\begin{array}{r} 20.58 \\ 8 \overline{) 20.58} \\ \underline{16} \\ 47 \\ \underline{42} \\ 50 \\ \underline{42} \\ 80 \\ \underline{84} \\ 35 \end{array}$$

Now place the proper sign.

Application..

The quality control engineering staff drew a "best fit" line through a series of points which represented data.

The "slope" of this line was determined by dividing the "rise" by the "run"

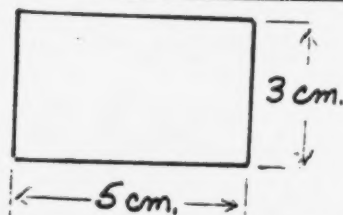
If the "rise" equaled -11.4 and the "run" equaled +2.6
 The slope of the line = $\frac{-11.4}{2.6} = -4.4$

Drill and Practice:

Rising, p. 22, 23

Smith, p. 122, 123

10. CONCEPT: THE ORDER OF OPERATIONS



The perimeter of a metal plate can be determined from this formula:
 Perimeter (P) = $2L + 2W$
 Where L = Length
 W = Width

To determine the perimeter of the plate in the drawing we use $L = 5\text{cm.}$ and $W = 3\text{cm.}$

$$P = 2L + 2W \text{ which means } P = 2 \cdot L + 2 \cdot W$$

$$\text{or } P = 2 \cdot 5 + 2 \cdot 3$$

Notice that two operations are involved. Performing the multiplications first we obtain $P = 10 + 6$.
 Now adding we obtain $P = 16\text{cm.}$

This introduces us to an order of operations.

1. Do all multiplications and divisions.
2. Follow with all additions and subtractions.

Examples:

Find the value of each:

$$1. \quad 4 \cdot 5 - 6 \cdot 3 + 2 = 20 - 18 + 2$$

$$= 20 + 2 - 18 = 4$$

$$2. \quad 9 - 7 \cdot 3 - 20 = 9 - 21 - 20 = 9 - 41 = -32$$

$$3. \quad 6(-4) + 7(-3) - 4(-2) =$$

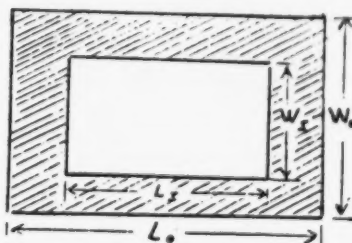
$$-24 - 21 + 8 = -45 + 8$$

$$= -37$$

Note:

$-4(-2)$ can be treated as $+(-4)(-2) = +8$.

Applications:



The area of a floor area represented by the shaded part of this drawing can be found by subtracting the area of the inner rectangle from the area of the outer rectangle or
 shaded area = $L_o \times W_o - L_i \times W_i$ where

L_o = length of outer rectangle
 W_o = width of outer rectangle
 L_i = length of inner rectangle
 W_i = width of inner rectangle
 L_i = 12 meters
 W_i = 5 meters.

If $L_o = 21$ meters
 $W_o = 11$ meters

$$\begin{aligned}\text{Then shaded area} &= 21 \times 11 - 12 \times 5 \\ &= 231 - 60 = 171 \text{ square meters}\end{aligned}$$

Drill and Practice:

Rising, p. 23-25

Dressler, p. 17

Smith, p. 23-24, 43-46

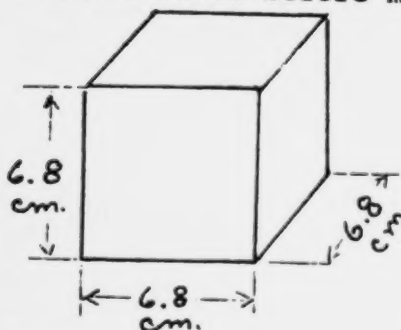
Note: If there is a repetition (within the same term) of multiplication and/or division the order is from left to right.

Example: $(-6) \div (-3) \cdot (+2) = (+2)(+2) = +4$

Not $(-6) \div (-3) \cdot (+2) = (-6) \div (-6) = +1$

11. CONCEPT: POWERS AND DIRECTED NUMBERS

A metal block before machining is in the form of a cube.
Each side measures 6.8cm.



$$\text{Volume} = (6.8)(6.8)(6.8)$$

$$\text{or } V = (6.8)^3 = 300.832 \text{ cu. cent.}$$

$$(6.8)^3 = 300.832$$

6.8 is called the base

3 is called the exponent

300.832 is called the power

Notice $(6.8)^3$ means $6.8 \times 6.8 \times 6.8$

and reads "6.8 to the third power or 6.8 cubed"

Therefore

5^7 means $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$

5^7 reads "five to the seventh power"

3^5 reads "three to the fifth power"

6^2 reads "six to the second power or six squared".

Evaluate:

$$(-4)^2 = (-4)(-4) = +16$$

$$(-1)^3 = (-1)(-1)(-1) = -1$$

$$(+2)^4 = +16$$

Note: An even power of a negative number is positive.
An odd power of a negative number is negative.

(-1) to any power equals either -1 or $+1$.

Example: $(-1)^{101} = -1 \cdot (-1)^n = -1$ when n is odd
 $(-1)^{2000} = +1 \cdot (-1)^n = +1$ when n is even

Note: $(-3)^3 = (-3)(-3)(-3) = -27$

Note: $-3^3 = (-1) \cdot 3^3 = (-1) \cdot 3 \cdot 3 \cdot 3 = -27$

But: $(-3)^4 = (-3)(-3)(-3)(-3) = +81$

$$-3^4 = (-1) \cdot 3 \cdot 3 \cdot 3 \cdot 3 = (-1) \cdot 81 = -81$$

observe

Note: The exponent applies only to the base above which it is written.

Application: Ex: $3 \cdot 5^2 = 3 \cdot 25 = 75$
 $(3.5)^2 = 3^2 \cdot 5^2 = 9 \cdot 25 = 225$
 $(-4)^2 = +16$
 $-4^2 = (-1) \cdot (4)^2 = -16$

Drill and Practice:

Rising, p. 120, 123

Dressler, p. 22, 24, 226, 227

Smith, p. 120, 123

Sullivan, p. 76-77

12. CONCEPT: THE ORDER OF OPERATIONS WITH GROUPING SIGNS AND POWERS

The formula for the conversion of Fahrenheit degrees to Celsius degrees is

$$C^{\circ} = \frac{5}{9} (F^{\circ} - 32) \quad \text{where } C^{\circ} = \text{Celsius Degrees} \\ F^{\circ} = \text{Fahrenheit Degrees}$$

Find C° if $F^{\circ} = 70^{\circ}$

$$\text{Substituting } C = \frac{5}{9} (70 - 32)$$

This expression includes a grouping sign called a parentheses. We perform whatever operation (in order) within the parentheses first.

$$\text{Then } C = \frac{5}{9} (38)$$

Now multiply or divide.

$$C = \frac{5}{9} \cdot \frac{38}{1} = \frac{5 \cdot 38}{9 \cdot 1} = \frac{190}{9} = 21.1^{\circ}\text{C}$$

The bar or vinculum is another grouping symbol.

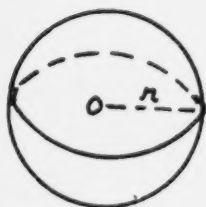
Example:

$$\text{Evaluate } \frac{5(-3) + (-2)(+1)}{6 - (-4)}$$

1. Evaluate the numerator.
2. Evaluate the denominator.
3. Then divide.

$$\frac{-15 + (-2)}{6 + (+4)} = \frac{-17}{+10} = -1.7$$

A formula sometimes contains a power such as the volume of a steel pellet in the shape of a sphere.



$$V = \frac{4}{3} \pi r^3$$

Find volume if
 $r = 1.2\text{cm}$
 $[\pi = 3.14]$

Therefore

$$V = \frac{4}{3} \cdot 3.14 \cdot (1.2)^3 \quad \text{Raise 1.2 to the third power first.}$$

$$V = \frac{4}{3} \cdot \frac{3.14}{1} \cdot \frac{1.728}{1} = \frac{4}{3} \cdot \frac{3.14}{1} \cdot \frac{1.728}{1}$$

$$V = 8.26392 \text{ or } 8.26 \text{ cubic centimeters}$$

Order of Operations

1. Evaluate all powers first.
2. Perform all multiplication and divisions.
3. Perform all additions and subtractions
4. If a grouping sign is involved, perform all operations within the group first then proceed with the order of operations.

Examples:

$$\begin{aligned}
 1) \quad & 9+6(-3) + 4(2 - 8 \cdot 3) \\
 & 9+6(-3) + 4(2 - 24) \\
 & 9+6(-3) + 4(-22)
 \end{aligned}$$

Perform operations
within parentheses
first.

$$\begin{aligned}
 & 9 - 18 - 88 \\
 & \quad - 9 - 88 \\
 & \quad \quad - 97
 \end{aligned}$$

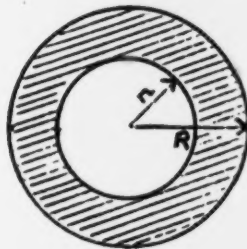
$$\begin{aligned}
 2) \quad & 6^2 \cdot 4 - 8 + 4 \cdot 3^4 \\
 & 36 \cdot 4 - 8 + 4 \cdot 81 \\
 & 144 - 8 + 324 \\
 & 136 + 324 = \underline{\underline{460}}
 \end{aligned}$$

Evaluate powers.
Mult./Divide.
Add/Subtract.

$$\begin{aligned}
 3) \quad & \frac{3^2 - 2^2}{9 - (-6)}
 \end{aligned}$$

Evaluate numerator
and denominator
first.

$$\begin{aligned}
 & \frac{9 - 4}{9 + 6} \\
 & \frac{5}{15} = \frac{1}{3}
 \end{aligned}$$

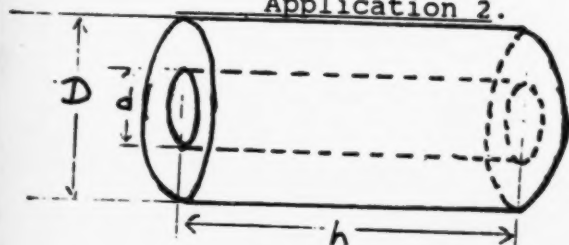
Application 1.

The area of a washer is indicated by the shaded part of this drawing.
Shaded area = $\pi R^2 - \pi r^2$
Where $R = 2.1\text{cm}$ and $r = 1.4\text{ cm}$
and $\pi = 3.14$

To find the area,

$$\begin{aligned}
 \text{Shaded area} &= 3.14 (2.1)^2 - 3.14 (1.4)^2 \\
 &= 3.14 (4.41) - 3.14 (1.96) \\
 &= 13.8474 - 6.1544 \\
 &= 7.6950 \text{ square centimeters}
 \end{aligned}$$

Application 2.



The volume of a hollow cylinder can be determined by the formula

$$V = \frac{1}{4}\pi \cdot h \cdot (D^2 - d^2)$$

Where h = length of cylinder

D = outside diameter

d = inside diameter

If h = 10.3cm, D = 1.8cm and d = 0.7cm. Find the volume of the cylinder [use $\pi = 3.14$]

$$V = \frac{1}{4} \cdot 3.14 \cdot 10.3 \cdot [1.8^2 - 0.7^2]$$

$$V = \frac{1}{4} \cdot 3.14 \cdot 10.3 \cdot [3.24 - .49]$$

$$V = \frac{1}{4} \cdot \frac{3.14}{1} \cdot \frac{10.3}{1} \cdot \frac{2.75}{1} = \frac{3.14 (10.3) (2.75)}{4}$$

= 22.23264 or 22.2
cubic centimeters

Drill and Practice:

Rising, p. 25, 26, 29, 30

Dressler, p. 19, 20, 21

Smith, p. 123

13. CONCEPT: THE ALGEBRAIC EXPRESSION



The design phase of a chassis produced several sketches in which the dimensions were unknown at this point. The sketch of a metal plate is shown here where letters (lower case) were used to represent the unknown dimensions. When letters are used to represent numbers we have an algebraic expression. For example "a" is an algebraic expression.

Other examples are $c+c$ or $a+b$ or x or $b-d$. Do you see which dimension these expressions represent?

$c+c$ can also be written as $2c$. $2c$ can mean $2xc$. When a number and letter are multiplied, the x sign or the dot need not be used. This is also common when two or more letters are multiplied. Ex: Area of Rectangle = LW
 LW means $L \times W$ or $L W$.

If b represents a dimension
 $5b$ represents 5 times that dimension
 $b+6$ is six more than b
 $b-1$ is one less than b
 $3b+2$ means 2 more than 3 times b

Note: b means $1b$ or $1 \cdot b$.

Applications: Formulas using in electricity, electronics, machine technology design are algebraic expressions.

Electricity: $E = IR$
 E = voltage
 I - amperes
 R = ohms

Electronics: $x_C = \frac{1}{2\pi fC}$
 $\pi = 3.1416$
 f = frequency
 C = capacitance

Machine Technology: $N = \frac{12 \cdot C}{\pi D}$
 C = cutting speed
 $= 3.1416$
 D = diameter
 N = revolution per minute

Design: $V = \frac{4}{3}\pi r^3$
 V = volume of sphere
 $= 3.1416$
 r = radius of sphere

Drill and Practice:
Dressler, p. 52-58
Rising, p. 73

14. CONCEPT: INTRODUCTION TO THE VOCABULARY OF ALGEBRA

Algebraic expressions and formulas give us mathematical messages and are a form of mathematical shorthand.

Example: A machinist determines the number of revolutions per minute from this formula

$$N = \frac{12C}{3.1416D}$$

Since all of the operations indicated are either multiplication or division there is no order of operations. Therefore it could mean to multiply 12 by C, multiply 3.1416 by D and divide the two results. Are there other ways to interpret this formula?

Example:

$a+b - c$	the sum of a and b minus c.
$ab - c$	the product of a and b <u>minus</u> c.
$ab - cd$	the difference between the product (ab) and product (cd).

A term: The part of an algebraic expression which involves only multiplication or division.

Example: $3ab$ is an expression of one term
 $3ab$ means $3 \cdot a \cdot b$.

5 is a term since 5 means $5 \cdot 1$.

x is a term since x means $1x$ or $1 \cdot x$.

Example:

How many terms are there in each expression?

$m - k+p$	3 terms
$m - kp$	2 terms
$4r - l$	2 terms
$-4pqrt$	1 term

Sometimes exponents are used to indicate powers in algebraic expressions.

Example: $4a^2b = 4 \cdot a \cdot a \cdot b$
 $-5q^3t = (-5) \cdot q \cdot q \cdot q \cdot t$ or $(-1) \cdot 5 \cdot q \cdot q \cdot q \cdot t$

How many terms are there in these expressions?

$-8m^3n^2t$	1 term
$8 - m^3n^2t$	2 terms
$8 - m^3 - n^2t$	3 terms

What does $(-4d)^3$ mean? What does $-4d^3$ mean?

$$\begin{aligned} (-4d)^3 &= (-4d) \cdot (-4d) \cdot (-4d) \cdot (-4d) \\ -4d^3 &= -4 \cdot d \cdot d \cdot d \end{aligned}$$

Consider $7m^3$

7 = numerical coefficient
m = base
3 = exponent

Application: A formula for the volume of a hollow cylinder $V = \pi \cdot h \cdot (R^2 - r^2)$ can be rewritten as $V = \pi R^2 h - \pi r^2 h$.

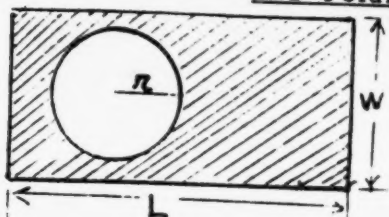
How many terms are there in this formula? Two terms.
What are they? $\pi R^2 h$ and $-\pi r^2 h$.

Drill and Practice:

Dressler, p. 99-100

Rising, p. 228-229

15. CONCEPT: THE EVALUATION OF ALGEBRAIC EXPRESSIONS AND FORMULAS



In some previous examples we have evaluated certain expressions and formulas. Let's try to formalize this procedure.

Ex: Using the formula,

$$\text{Shaded Area} = \text{Area Rectangle} - \text{Area Circle},$$

we decide that

$$\text{Shaded Area} = LW - \pi r^2$$

Find the shaded area if $L = 13.1\text{cm}$, $W = 4.9\text{cm}$
 $r = 1.2\text{cm}$, $\pi = 3.14\text{cm}$

Procedure

1. Rewrite formula: $\text{Shaded Area} = LW - \pi r^2$
2. Substitute numbers for corresponding letters. Introduce parentheses for clarity.
 $= 13.1(4.9) - 3.14(1.2)^2$
3. Perform indicated operations.
 $= 13.1(4.9) - 3.14(1.44)$
 $= 64.19 - 4.5216$
 $= 59.6684$ or 59.7
square cent.

Example:

Evaluate $9x - 10$ if $x = (-3)$
 $9(-3) - 10$
 $-27 - 10 = -27$

Evaluate $6 - 2y$ if $y = 4$
 $6 - 2(4)$
 $6 - 8 = -2$

Evaluate $\frac{x-3y}{x+y}$ if $x = -1$
 $y = 4$

$$\frac{-1-3(4)}{-1+4} = \frac{-1-12}{+3} = \frac{-13}{3} = -4 \frac{1}{3}$$

Evaluate $c^2 - 4d^2$ if $c = 10$, $d = -2$
 $10^2 - 4(-2)^2$
 $100 - 4(4)$
 $100 - 16$
 84

Application 1:

Find area of trapezoid, using:

$$A = \frac{1}{2} h (b_1 + b_2)$$

$$\text{if } h = 10.1\text{m}$$

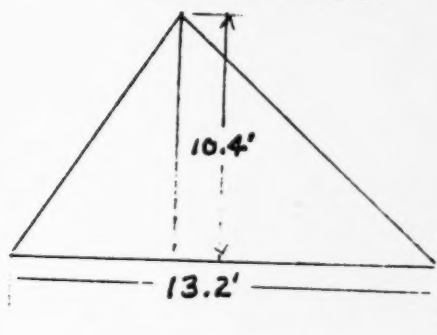
$$A = \frac{1}{2} \cdot (10.1)(12.3 + 17.4)$$

$$b_1 = 12.3\text{m}$$

$$b_2 = 17.4\text{m}$$

$$A = \frac{1}{2} \cdot \frac{10.1}{1} \cdot \frac{(29.7)}{1}$$

$$A = 297 = 148.5 \text{ square meters}$$

Application 2:

The area of a triangle is given as

$$\text{Area} = \frac{1}{2} b \cdot h$$

Find the area, if

$$b = 13.2'$$

$$h = 10.4'$$

$$\text{Area} = \frac{1}{2} h \cdot b$$

$$\text{Area} = \frac{1}{2} (13.2)(10.4)$$

$$= \frac{1}{2} \cdot \frac{13.2}{1} \cdot \frac{10.4}{1}$$

$$\text{Area} = 68.84 \text{ square ft.}$$

Drill and Practice:

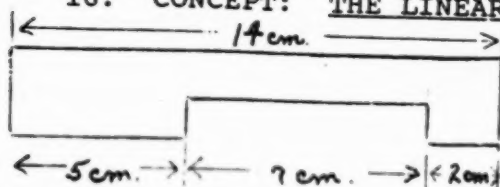
Dressler, p. 61-64, 66-72

Rising, p. 75

Smith, p. 45-46

Sullivan, p. 92-97

16. CONCEPT: THE LINEAR EQUATION AND ITS SOLUTION SET



It is obvious from this diagram that $5\text{cm} + 7\text{cm} + 2\text{cm} = 14\text{cm}$ or $5 + 7 + 2 = 14$, a well known fact of arithmetic.

$5 + 7 + 2 = 14$ is an equation, which is a statement that two expressions are equal ($=$). $5 + 7 + 2$ is the left member of the equation, and 14 is the right member of the equation.

Other examples of equations are:

$$c + 9 = 13 \text{ or } 3x + 4y = 20$$

which contain the variables c , x and y . Lets consider the type of equation typified by $c + 9 = 13$. This statement is true only when $c = 4$. (4) is said to be the solution set of $c + 9 = 13$. If the 4 was chosen from the set of positive integers, that is $\{1, 2, 3, 4, 5, \dots\}$, then the set of positive integers is said to be the replacement set or domain of $c + 9 = 13$.

Example:

given: domain = $\{2, 4, 6, 8\}$

find the solution set of $x + 7 = 13$

Solution set = 6 or $\{6\}$

Usually we do not limit the domain, so unless otherwise stated the domain will be the set of real numbers.

Solve for x :

$$x + 10 = 11$$

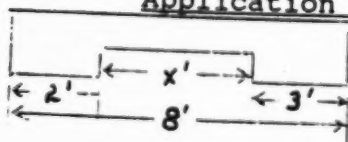
$x = 1$ by inspection.

Solve for x

$$5x - 1 = 29$$

$x = 6$ by inspection
since $5 \cdot 6 - 1 = 29$.

Application 1.



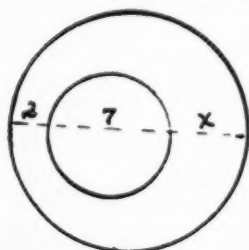
From this drawing

$$2 + x + 3 = 8.$$

By inspection

$x = 3$ to make the statement true.

Application 2.



From this drawing

$$2 + 7 + x = 14$$

By inspection

$x = 5$ since $2 + 7 + 5 = 14$.

Drill and Practice:

Rising, p. 78-79

Dressler, p. 76

Smith, p. 145, 146

17. CONCEPT: PROPERTIES OF EQUALITY

For an understanding of the concept of equality in a mathematics context, there are three properties which should be explained.

1. The reflexive property of equality states that any number equals itself.

Example: $6 = 6$

$$a = a$$

$$6.732 = 6.732$$

This obvious observation is useful in explaining concepts which are far more complex.

2. The symmetric property of equality states that for any two numbers p and q
if $p = q$, then $q = p$.
3. The transitive property of equality states that for any three numbers, p , q , r ,
if $p = q$ and $q = r$, then $p = r$.

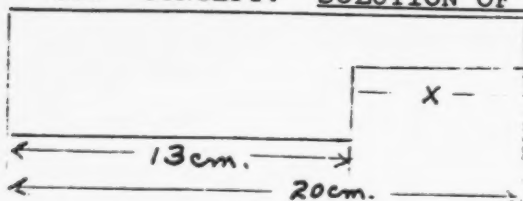
Sometimes this property is called substitution.

Ex: if $x + y = 6$ and $a = x + y$
then $a = 6$.

Drill and Practice:

Dressler, p. 96

18. CONCEPT: SOLUTION OF LINEAR EQUATIONS BY ADDITION



Since $x + 13 = 20$ is evident from this drawing, we can find the solution set by inspection. However, since the solution of

many equations is not obvious, a more systematic method would be valuable.

Now $x + 13 = 20$ and $x + 2 = 9$ and $x = 7$ are all linear equations, for which the solution set is $\{7\}$.

Equations which have the same solution set are called equivalent equations.

The addition property of equality states that, if the same number is added to both members of an equation, an equivalent equation is produced.

So, if $x + 13 = 20$ and since the additive inverses (opposites) $(+13)$ and (-13) when added equal zero.

Then, $x + 13 - 13 = 20 - 13$
or $x + 0 = 7$
or $x = 7$

Add -13 to each member.

An equivalent equation whose solution set is $\{7\}$.

Examples:

$$\begin{aligned} 1) \quad x - 20 &= 23 \\ x - 20 + 20 &= 23 + 20 \\ \text{therefore; } x &= 43 \end{aligned}$$

Add $+20$ to both sides solution set.

$$\begin{aligned} 2) \quad 8 + b &= 12 \\ 8 - 8 + b &= 12 - 8 \\ b &= 4 \end{aligned}$$

Add -8 to both sides solution set.

$$\begin{aligned} 3) \quad x - 9 &= -3 \\ x - 9 + 9 &= -3 + 9 \\ x &= 6 \end{aligned}$$

Add $+9$ to both sides solution set.

$$\begin{aligned} 4) \quad -1.732 + x &= 1.141 \\ -1.732 + 1.732 + x &= 1.141 + 1.732 \\ x &= 2.873 \end{aligned}$$

$$\begin{array}{r} +1.732 \\ +1.141 \\ \hline +2.873 \end{array}$$

$$\begin{aligned} 5) \quad 7 + y &= 2 \\ 7 - 7 + y &= 2 - 7 \\ y &= -5 \end{aligned}$$

$$6) \quad -8 \frac{2}{3} + m = 5 \frac{3}{4}$$

$$-8 \frac{2}{3} + 8 \frac{2}{3} + m = 5 \frac{3}{4} + 8 \frac{2}{3}$$

$$m = 14 \frac{5}{12}$$

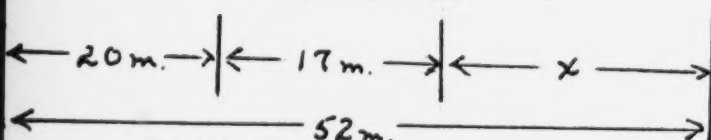
$$5 \frac{3}{4} = 5 \frac{9}{12}$$

$$8 \frac{2}{3} = 8 \frac{8}{12}$$

$$\begin{array}{r} 5 \frac{9}{12} \\ + 8 \frac{8}{12} \\ \hline 13 \frac{17}{12} \end{array}$$

$$= 14 \frac{5}{12}$$

Application 1.



To find x:

$$\text{Write } 20 + 17 + x = 52$$

$$\text{Then } 37 + x = 52$$

$$37 - 37 + x = 52 - 37$$

$$x = 15m$$

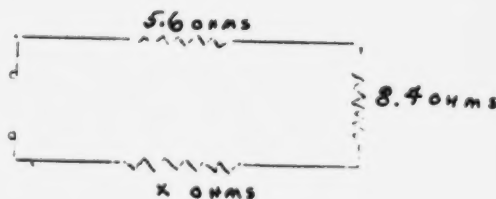
A drawing representing a work area is shown with one dimension missing.

Note: Calculations can be arranged in a horizontal form if it seems more convenient.

$$\begin{array}{r} \text{Such as } 37 + x = 52 \\ -37 \quad -37 \\ \hline x = 15m. \end{array}$$

Application 2.

In an electrical circuit three resistances are connected in series. If the total resistance is 23.2 ohms and two known resistances are 5.6 ohms and 8.4 ohms, find the number of ohms in the unknown resistance.



$$23.2 = 5.6 + 8.4 + x$$

$$\text{then, } 23.2 = 14.0 + x$$

$$\begin{array}{r} 23.2 = 14.0 + x \\ -14.0 = -14.0 \\ \hline 9.2 = x \end{array}$$

$$9.2 = x$$

$$\text{therefore, } x = 9.2 \text{ ohms}$$

Drill and Practice:

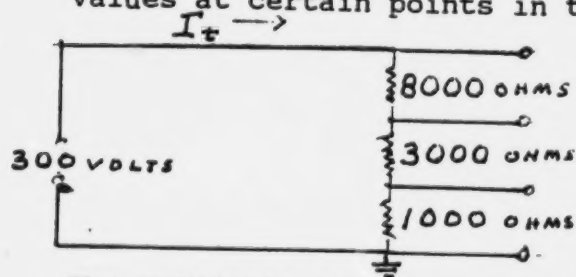
Rising, p. 81

Dressler, p. 256-257

SMith, p. 145-146

19. CONCEPT: SOLUTION OF LINEAR EQUATIONS BY MULTIPLICATION

A voltage divider circuit produces voltages for different values at certain points in the circuit.



Find I_t , the total current supplied,

Solution:

$$E = I_t \cdot R$$

$$R = (8000 + 3000 + 1000) \text{ ohms}$$

$$E = 300 \text{ volts}$$

$$300 = I_t \cdot 12,000$$

$$\text{or } 300 = 12000 I_t$$

The multiplication property of equality states that if each member of an equation is multiplied by the same non-zero number, an equivalent equation is produced.

$$\text{So, if } 300 = 12000 \cdot I_t$$

Then, $12000 \cdot I_t = 300$ (symmetric prop. of equality) and
 $(1/12000)(12000 \cdot I_t) = (300)(1/12000)$ (multiply both

$$(1/12000)(12000)I_t = \frac{300}{12000}$$

sides by the reciprocal of 12000)

$$1 \cdot I_t = .025 \text{ amps}$$

$$I_t = .025 \text{ amps}$$

Examples:

Solve for x:

$$9x = 36$$

$$(1/9) \cdot 9x = 36 \cdot (1/9)$$

$$1x = 36/9$$

$$x = 4$$

Multiply both members with the reciprocal of 9.

Solve for x:

$$-13x = 39$$

$$(-1/13) \cdot (-13x) = (39) \cdot (-1/13)$$

$$1x = \frac{39}{-13}$$

$$-13$$

$$x = -3$$

Solve for q:

$$\frac{q}{5} = 3$$

$$\frac{5 \cdot q}{1 \cdot 5} = 3 \cdot \frac{5}{1}$$

$$q = 15$$

In some cases a corollary to the multiplication property allows division of both members by the same number.

Solve: $.432y = 15.68$

$$\frac{.432y}{.432} = \frac{-15.68}{.432}$$

$$y = -36.2$$

Solve for k

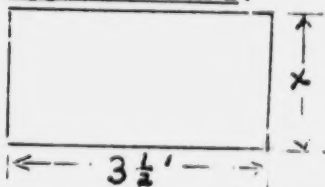
$$\frac{3}{4}k = 21$$

$$\frac{4}{3} \cdot \frac{3}{4}k = \frac{21}{1} \cdot \frac{4}{3}$$

$$1k = 28$$

The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$

Application 1.



The area of this rectangular sheet of sheet metal is $8 \frac{3}{4}$ square feet. If the length is $3 \frac{1}{2}$ feet find x, the width.

Area = LW where L = length
W = width

Therefore,

$$8 \frac{3}{4} = \left(3 \frac{1}{2}\right) \cdot W$$

$$\frac{35}{4} = \frac{7}{2} W$$

$$\frac{2}{7} \cdot \left(\frac{35}{4}\right) = \frac{2}{7} \cdot \left(\frac{7}{2} W\right)$$

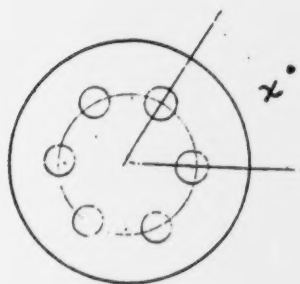
$$\frac{2 \cdot 35}{7 \cdot 4} = 1W$$

$$\frac{5}{2} = W$$

$$\text{or } W = 2 \frac{1}{2} \text{ feet}$$

Rewrite mixed numbers as improper fractions.

Multiply both members by the reciprocal of $\frac{7}{2}$

Application 2.

Find x , the number of degrees which separates the radii which are drawn through the centers of 6 holes.

Remember: 360° represent one complete rotation of a single point on the circle.

Therefore, $6x = 360$

$$\frac{1}{6} 6x = 360 \frac{1}{6}$$

$$1x = 60$$

$$x = 60^\circ$$

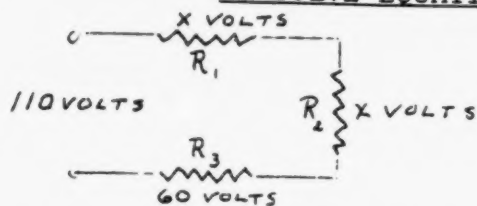
Drill and Practice:

Rising, p. 85

Dressler, p. 258

Smith, p. 147

20. CONCEPT: USING BOTH ADDITION AND MULTIPLICATION TO SOLVE EQUATIONS



If three resistors are in series across 110 volts, a voltmeter measures 60 volts across R_3 .

If $R_1 = R_2$ find the voltage drop across R_1 and R_2 .

Solution: $x + x + 60 = 110$

$2x + 60 = 110$ (Dist. property, combine similar terms)

$2x + 60 - 60 = 110 - 60$ Addition Prop. of equality
 $2x = 50$

$\frac{1}{2} \cdot (2x) = (50) \cdot \frac{1}{2}$

Mult. each member by the reciprocal of 2

$x = 25$ volts.

Examples:

Solve: $3x - 5 = 10$

$$\begin{array}{r} 3x - 5 = 10 \\ + 5 \quad + 5 \\ \hline \end{array}$$

$3x = 15$

$\frac{1}{3} \cdot (3x) = (15) \cdot \frac{1}{3}$

$x = 5$

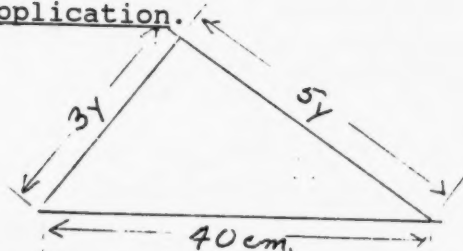
$\frac{x}{5} + 1 = 9$

$\frac{x}{5} = 8$

$\frac{5}{1} \cdot (\frac{x}{5}) = (8) \cdot \frac{5}{1}$

$x = 40$

Application.



The perimeter of this piece of sheet metal is 100cm. Find the length of two sides if the length of one side is 40cm.

Solution: $3y + 5y + 40 = 100$

$8y + 40 = 100$ (Dist. Prop.)

$\begin{array}{r} 8y + 40 = 100 \\ - 40 \quad - 40 \\ \hline 8y = 60 \end{array}$

$\frac{1}{8} \cdot (8y) = (60) \cdot \frac{1}{8}$

$y = 7.5$ cm

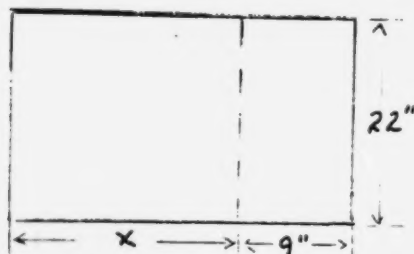
Now, one side is represented by $3y$ which means that if $y = 7.5$, $3y = 3(7.5) = 22.5\text{cm}$. If the other side is $5y$, then $5(7.5) = 37.5\text{cm}$.

Drill and Practice:

Rising, p. 87

Dressler, p. 260-261

21. CONCEPT: SOLVING EQUATIONS WITH PARENTHESES



The perimeter of a workbench surface is 100 inches. If one area is to be 9 inches wide, find the width of the other area, x .

$$\begin{aligned} \text{Perimeter of} \\ \text{rectangle} &= 2L + 2W \end{aligned}$$

$$100 = 2(x+9) + 2(22)$$

$$100 = 2(x+9) + 44$$

$$100 = 2x + 18 + 44$$

$$100 = 2x + 62$$

$$\begin{array}{r} -62 \\ 38 = 2x \end{array}$$

$$38 = 2x$$

$$\frac{1}{2}(38) = (2x) \cdot \frac{1}{2}$$

$$x = 19\text{in.}$$

Examples:

$$5(c-4) - 3 = 8$$

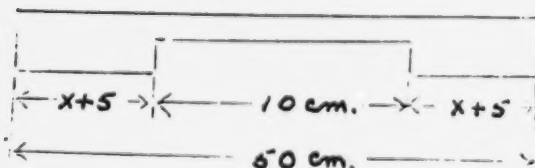
$$5x - 20 - 3 = 8$$

$$5x - 23 = 8$$

$$\begin{array}{r} + 23 \\ 5x = 31 \end{array}$$

$$\begin{array}{r} 5x = 31 \\ (1/5)(5x) = (31)(1/5) \\ x = 6 \frac{1}{5} \end{array}$$

Application.



Solution:

$$2(x+5) + 10 = 50$$

$$2x + 10 + 10 = 50$$

$$2x + 20 = 50$$

$$\begin{array}{r} -20 \\ 2x = 30 \end{array}$$

$$2x = 30$$

$$(1/2)(2x) = 30 (1/2)$$

$$x = 15\text{cm}$$

Distributive Prop.

Addition Prop. of Equality

Multiplication Prop. of Equality

$$(2/3)(m-1) = 4 \frac{1}{2}$$

$$(2/3)m - 2/3 = 9/2$$

$$\begin{array}{r} + 2/3 \\ (2/3)m = 31/6 \end{array}$$

$$(2/3)m = 31/6 (\text{how?})$$

$$(3/2)(2/3)m = (31/6)(3/2)$$

$$1m = 31/4$$

$$m = 7 \frac{3}{4}$$

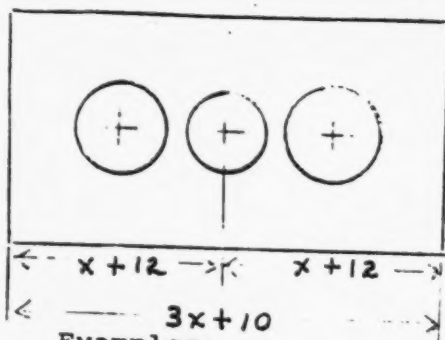
A bracket in the form shown at the left has two sections at either end with a length represented by $x+5$. If the center section is 10cm and the total length is 50cm find the length of x .

Drill and Practice:

Rising, p. 88

Dressler, p. 265

22. CONCEPT: SOLVING EQUATIONS WITH THE VARIABLE IN TERMS IN BOTH MEMBERS OF THE EQUATION



Examples:

$$9x + 5 = 4x - 25$$

$$\begin{array}{r} -5 \\ \hline 9x = 4x - 30 \end{array}$$

$$9x = 4x - 30$$

$$\begin{array}{r} -4x \\ \hline 5x = -30 \end{array}$$

$$5x = -30$$

$$(1/5)(5x) = (30)(1/5)$$

$$x = 6.$$

From this drawing
 $3x + 10 = x + 12 + x + 12.$

$$3x + 10 = 2x + 24$$

Note x appears on both members of the equation.

$$3x + 10 - 10 = 2x + 24 - 10$$

$$3x = 2x + 14$$

$$3x - 2x = 2x - 2x + 14$$

$$1x = 14 \text{ cm}$$

$$3m - 4 + m = 5m - 2$$

$$4m - 4 = 5m - 2$$

$$\begin{array}{r} +4 \\ \hline 4m = 5m + 2 \end{array}$$

$$4m = 5m + 2$$

$$\begin{array}{r} -5m \\ \hline -1m = +2 \end{array}$$

$$-1m = +2$$

Note: $-1m = (-1)m$

therefore since we want the value of m or $+1m$

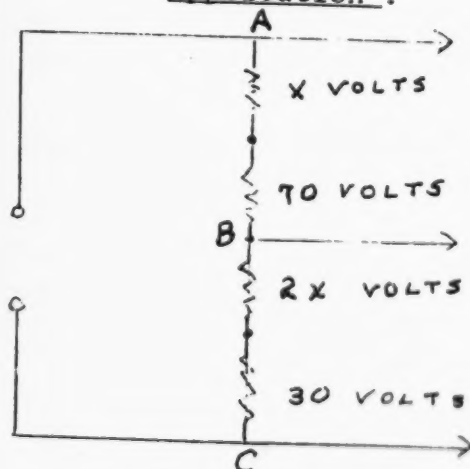
merely multiply (or divide) both members by -1 .

$$(-1)$$

$$(-1)(-1m) = (+2)(-1)$$

$$1m = -2$$

Application.



In a series circuit the voltage across AB was designed to equal the voltage across BC therefore,

$$x + 70 = 2x + 30$$

$$\begin{array}{r} -70 \\ \hline x = 2x - 40 \end{array}$$

$$x = 2x - 40$$

$$\begin{array}{r} -2x \\ \hline -1x = -40 \end{array}$$

$$-1x = -40$$

Therefore $x = 40$ volts

An alternative method of solving equations, when there are several terms in each member, is called transposition.
Solving the above equation by transposition we obtain

$$x + 70 = 2x + 30$$

$$x - 2x = 70 + 30.$$

Notice that we collected those terms containing x on one side of the equal sign and those terms not containing x on the other side of the equal sign. We did this by rewriting the original equation, moving appropriate terms to the opposite side of the equation while changing the term to its opposite. Now proceed,

$$-1x = -40$$

$$x = 40 \text{ volts divide by } (-1)$$

Example:

$$\begin{aligned} \text{Transposing: } 6m - 20 + 4m &= 32 + 5m - 8m. \\ 6m + 4m - 5m + 8m &= 32 + 20 \\ 13m &= 52 \\ (1/13)(13m) &= (52)(1/13) \\ m &= 4 \text{ Ans.} \end{aligned}$$

Drill and Practice:

Rising, p. 88

Dressler, p. 262-263

Smith, p. 154-160

23. CONCEPT: TRANSLATING WORD SENTENCES INTO ALGEBRAIC EQUATIONS

The sales manager of a small machine tool company stated, "our sales are up \$350,000 over last year's sales". These words are familiar, but there is another way of thinking about this same information.

We, at least not yet, do not know the amount of last year's sales.

Algebra allows us to discuss data, even though we don't know what that data is.

The algebraic method allows us to say "Let x represent the dollar value of last year's sales. Then $x+350,000$ represents the dollar value of this year's sales."

x and $x+350,000$ are algebraic expressions.

To help translate words into algebraic expressions and then into equations, let us consider common words and their algebraic translation.

<u>Words</u>	<u>Algebraic Translation</u>
The sum of a and b	$a + b$
6 increased by m (or m more than 6)	$6 + m$ or $m + 6$ (Addition is commutative)
The difference between x and y	$x - y$ or $y - x$ Be careful because subtraction is <u>not</u> commutative.
$2\frac{3}{4}$ less than x	$x - 2\frac{3}{4}$ <u>not</u> $2\frac{3}{4} - x$
30 decreased by q	$30 - q$ <u>not</u> $q - 30$
The product of 100 and z	$100z$
The quotient of 45 and t	$45 \cdot t$ or $t \cdot 45$ Be careful because division is <u>not</u> commutative
9 more than 5 times t	$9 + 5t$ or $5t + 9$
$\frac{5}{8}$ of r	$\frac{5}{8} \cdot r$ or $\frac{5r}{8}$

Sometimes "a number" is included in the expression. "A number" can be translated as "x" or some other letter.

<u>Words</u>	<u>Algebraic Translation</u>
The sum of a number and 11	$x + 11$
5 less than $\frac{3}{4}$ of a number	$(\frac{3}{4})x - 5$
15 times a number	$15x$

These are all algebraic expressions. However, once a verb (is, are, becomes, etc.) is used we have a complete sentence. A sentence can often be written as an algebraic equation.

<u>Sentence</u>	<u>Equation</u>
The sum of x and 2 <u>is</u> 8	$x + 2 = 8$ Note: The verb "is" is translated as the equal sign (=).
The product of a number and seven is 21	Let n = the number $7 \cdot n = 21$ <u>or</u> $7n = 21$
6 less than the product of 3 and x is 18	$3x - 6 = 18$

Application.

The sales manager continues his outline of sales records. He says that next year's sales should be 3 times the first year's sales.
So, let x = 1st year's sales
 $x + 350,000$ = 2nd year's sales
 $3x$ = 3rd year's sales.

These are merely algebraic expressions. It is not until he relates these 3 unknown quantities with another fact that we can write an equation. "The total dollar sales for the 3 periods should be \$4,275,000."

Now we write the equation.

$$\begin{array}{rclcl}
 x & + & x+350,000 & + & 3x & = & \$4,275,000 \\
 \text{sales} & + & \text{sales} & + & \text{sales} & = & \text{total} \\
 \text{for} & & \text{for} & & \text{for} & & \text{sales} \\
 \text{first} & & \text{second} & & \text{third} & & \\
 \text{year} & & \text{year} & & \text{year} & &
 \end{array}$$

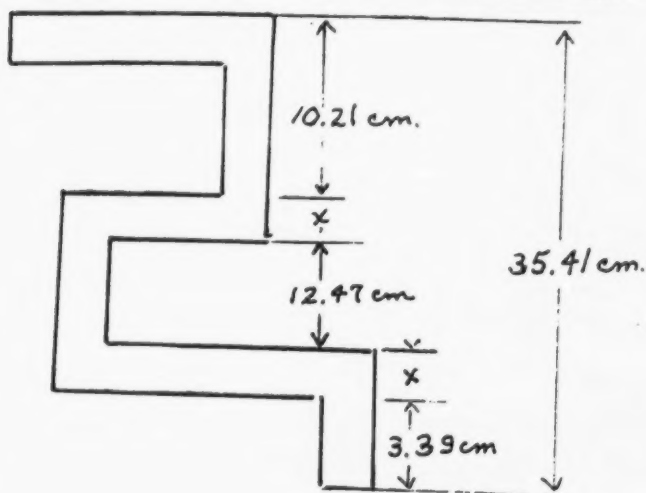
Drill and Practice:

Rising, p. 89-93

Dressler, p. 261 (26-32)

263 (39-46), 265 (33-34)

24. CONCEPT: PROBLEM SOLVING WITH EQUATIONS



Find the value of x in this drawing.

To write an equation we must first:

1. Study the drawing carefully. Do not be in a hurry.
2. Locate x in the drawing in as many locations that it appears.
3. Look for relationships between x and the other numbers and expressions in the drawing.

After a through examination of the drawing it might seem to you that the distance from the top of the drawing to the bottom of the drawing is 35.41cm. It is also evident that this same distance could be represented by $10.21 + x + 12.47 + x + 3.39$. Since both representations are equal we can write

$$10.21 + x + 12.47 + x + 3.39 = 35.41$$

Now lets solve the equation.

$$2x + 26.07 = 35.41$$

$$\begin{array}{r} - 26.07 \\ \hline 2x \end{array} = \begin{array}{r} 35.41 \\ - 26.07 \\ \hline 9.34 \end{array}$$

$$2x = 9.34$$

$$1/2(2x) = (9.34) \cdot 1/2$$

$$x = 4.67\text{cm.}$$

Some problems may be stated verbally without any diagram. The monthly cost for the rental of 7 delivery trucks was \$13,223. Find the monthly rental cost for each truck. Read the problem carefully. Decide what question is being asked. Let x (or n) equal the value of the quantity which is being sought. Using this letter and the number relations in the problem write an equation.

Let x = the monthly rental for one truck.

$$\text{Then } 7 \cdot x = 13,223$$

$$\text{or } 7x = 13,223$$

Solve the equation.

$$(1/7) \cdot (7x) = (13,223) \cdot (1/7)$$

$$x = \$1,889 \text{ per truck.}$$

To summarize:

1. Read the problem and/or study the drawing carefully.
Reread and restudy more than once.
2. Decide what quantity is asked for
3. Choose a letter (if one is not already provided) to represent the quantity sought.
4. Write an equation using the relationships in the problem and/or from the drawing.
5. Solve the equation.
6. Check the equation.
7. Check the solution to see if your results make sense in the problem or drawing. (Sometimes we write the wrong equation.)

Drill and Practice:

Rising, p. 95-96

Dressler, p. 261, 263, 265

Smith, p. 138-140

25. CONCEPT: PROBLEM SOLVING WITH FORMULAS

A machinist wishes to determine the cutting speed when a steel shaft is turned on a lathe. The shaft is 1.175 inches in diameter and the lathe is turned at 250 r.p.m. From the machinists handbook he chooses the formula

$$C = \frac{3.1416 D N}{12}$$

where D = diameter
N = revolutions per
minute

Substituting
numbers for
letters

$$C = \frac{3.1416(1.175)(250)}{12}$$

$$C = [76.90375] \text{ feet/minute*}$$

$$C = 76.9 \text{ feet/minute}$$

correct to nearest
tenth of a foot

*calculator
read out

Often the formulas may be used directly as found in laboratory manuals or trade manuals. As cited in the problem above. Make certain that the rules of the order of operation are observed.

Drill and Practice:

Dressler, p. 66-73

Smith, p. 44-46

Sullivan, p. 92-97

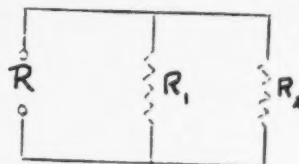
26. CONCEPT: SOLVING LITERAL EQUATIONS AND FORMULAS

A formula is an equation which describes a relationship between two or more variables which are usually specific to an occupation, a field of science, or mathematics.

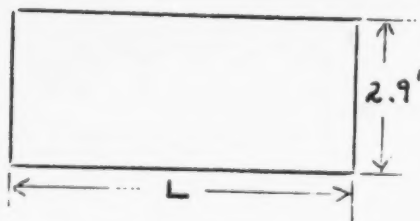
Example:

Resistances in parallel are related by the following formula

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$



Where R is the combined resistance of R_1 and R_2 . Sometimes formulas are more useful if they at first are rearranged; that is solved for a different variable.



A piece of angle iron is to be welded in place about the perimeter of a rectangular metal chassis. The perimeter is 24.8 ft. The width of the

chassis is 2.9 ft. Find the length of the pieces to be cut for the length. The standard formula for perimeter of a rectangle is $P = 2L + 2W$.

Where

P = Perimeter

L = Length

W = Width

This problem can be solved by either of two methods.

Method I

Substitute in the original formula

$$P = 2L + 2W$$

$$24.8 = 2L + 2(2.9)$$

Simplify.

$$24.8 = 2L + 5.8$$

Now solve for L using methods for solving linear equations

$$24.8 = 2L + 5.8$$

$$\begin{array}{r} 24.8 \\ -5.8 \\ \hline 19 \end{array} \quad \begin{array}{r} \\ -5.8 \\ \hline \end{array}$$

$$19 = 2L$$

$$(1/2)(19) = (2L)(1/2)$$

$$9.5\text{ft} = L$$

$$\text{or } L = 9.5\text{ft.}$$

Method II

This is especially useful if the solution must be repeated many times.

Write down the original formula

$$P = 2L + 2W$$

Solve this formula for L "in terms of" the other variables P and W . That is, treat L as "the" variable and P and W as "known values"

$$P = 2L + 2W$$

$$\begin{array}{r} P \\ -2W \\ \hline P-2W \end{array} \quad \begin{array}{r} \\ -2W \\ \hline 2L \end{array}$$

Not similar terms,
therefore sum is
indicated.

$$\frac{P-2W}{2} = \frac{2L}{2}$$

Divide
members
by 2, or
multiply by 1/2

$$\frac{P-2W}{2} = L$$

$$\text{or } L = \frac{P-2W}{2}$$

Now substitute

$$L = \frac{24.8 - 2(2.9)}{2}$$

$$L = \frac{24.8 - 5.8}{2} = \frac{19.0}{2} = 9.5 \text{ ft.}$$

To practice solving formulas we sometimes invent equations which look like formulas. These are called literal equations. A literal equation is one which in addition to the usual representation of variables..x, y or z (the end of the alphabet, also contain letters a, b, c... the beginning of the alphabet). The a, b, and c are considered "known" quantities. The x, y, and z, are considered the variables.

Examples:

$$3x = b$$

$$(1/3)(3x) = (b)(1/3)$$

$$[x = b/3] \quad [x \text{ has been solved for "in terms of b"}]$$

$$ay + 5 = c$$

$$\underline{- 5 = 5}$$

$$ay = c - 5$$

$$(1/a)(ay) = (1/a)(c-5)$$

$$y = \frac{c-5}{a}$$

[y has been
solved for
in terms of
a and c]

$$13z + c = 4z + d$$

$$\begin{array}{r} -c \quad -c \\ 13z = 4z + d - c \\ -4z \quad -4z \\ 9z = d - c \\ z = \frac{d-c}{9} \end{array}$$

$$13z = 4z + d - c$$

$$9z = d - c$$

$$z = \frac{d-c}{9}$$

Divide both members by 9.

$$5t + x = 12t - ax$$

$$\begin{array}{r} -5t \quad = -5t \\ x = 7t - ax \\ +ax \quad +ax \\ x + ax = 7t \end{array}$$

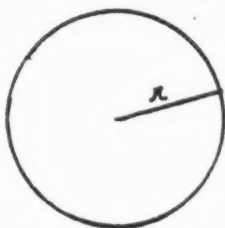
Notice that x and (ax) are not similar terms. Therefore we use commutative property

$$x(1+a) = 7t \quad [\text{Now divide by } 1+a]$$

$$\frac{x(1+a)}{(1+a)} = \frac{7t}{1+a}$$

$$\text{or } x = \frac{7t}{1+a}$$

Application 1. Now let's try the methods on formulas. The formula for the circumference of a circle.



$$C = 2\pi r$$

Solve for r

$$C = 2\pi r$$

$$\frac{1}{2} \cdot (C) = (2\pi r) \cdot \frac{1}{2}$$

$$\frac{C}{2\pi} = r \quad \text{or} \quad r = \frac{C}{2\pi}$$

where

C = circumference

r = radius length

$$= 3.1416$$

(a constant)

Application 2.

The formula for the total cost in the tire production plant

$$C = 5V + F$$

Where C = total cost of production

V = the number of tires produced

F = overhead cost

Solve this formula for V .

$$C = 5V + F$$

$$\begin{array}{r} -F \quad -F \\ C - F = 5V \\ 5 \quad 5 \end{array}$$

$$\frac{C-F}{5} = \frac{5V}{5}$$

$$\frac{C-F}{5} = V \quad \text{or} \quad V = \frac{C-F}{5}$$

Drill and Practice:

Rising, p. 99

Dressler, p. 266-267

Smith, p. 157-159, 160-161

Sullivan, p. 98-108

27. CONCEPT: AN UNDERSTANDING OF INEQUALITY AND ITS SYMBOLS

In the manufacturing of a carburetor a certain part must pass between two departments. Each department has a certain number of hours available for the manufacture of the part.

Dept. A has 150 hours available.

Dept. B has 200 hours available.

This means that the number of hours in Dept. A must be less than (or equal to) 150 hours;

And the number of hours in Dept. B must be less than (or equal to) 200 hours.

The idea "is less than" is expressed by the symbol $<$; that is no. of hours (Dept. A) < 150 .

Since the number of hours can also equal 150, we use this symbol $\leq$: "is less than or equal to".

These symbols allow us to express the idea of inequality.

$<$	is less than
$>$	is greater than
$\leq$	is less than or equal to
$\geq$	is greater than or equal to
$\neq$	is not equal to

These relationships can be seen by locating numbers on the real number line.



Compare: 5 and 3	5 is greater than 3.	$5 > 3$
4 and 0	4 is greater than 0	$4 > 0$
1 and -2	1 is greater than -2	$1 > -2$
-1 and -5	-1 is greater than -5	$-1 > -5$

a is greater than b, if a is to the right of b on the number line.

or		
Compare: 5 and 3	3 is less than 5,	$3 < 5$
4 and 0	0 is less than 4,	$0 < 4$
1 and -2	-2 is less than 1,	$-2 < 1$
-1 and -5	-5 is less than -1,	$-5 < -1$

b is less than a, if b is to the left of a on the number line.

Examples: Write with symbols:

17 is less than 23.	$17 < 23$
-2 is less than 0	$-2 < 0$
51.4 is greater than 20.4	$51.4 > 20.4$
81.2 is not equal to 17.1	$81.2 \neq 17.1$
	or $17.1 \neq 81.2$

-4 is equal to
or less than 6

$-4 \leq 6$
(is this true?)

Some properties of inequalities are important.

1. The Order or Comparison Property.

Given 2 real numbers, a and b only.

One of these statements can be true.

(1) $a > b$	Ex: Consider 5, -4
(2) $a < b$	$5 > -4$ true
(3) $a = b$	$5 < -4$ false
	$5 = -4$ false

2. The Transitive Property of Inequalities

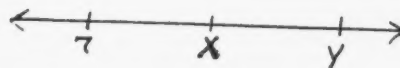
Given 3 real numbers, a, b and c.

If $a < b$ and $b < c$, then $a < c$

or $a > b$ and $b > c$ then $a > c$

Example: If $x > 7$ and $y > x$
then $y > 7$

Observe on the number line



x must be to the right of 7
y must be to the right of x
therefore $y > 7$

3. Addition Property of Inequalities

If $5 > 3$ and 1 is added to each member of the inequality

$$\begin{array}{r} 1=1 \\ 6>4 \end{array}$$

or if a, b and c are real numbers and if $a > b$
then $a + c > b + c$

Ex: $9 < 20$

$$\begin{array}{r} -2 \quad -2 \\ 7 < 18 \end{array}$$

If a, b, c are real numbers and if $a < b$, then
 $a + c < b + c$

4. Multiplication Property of Inequalities.

If a, b, and c are real numbers and c is positive,
and if $a > b$, then $ac > bc$
and if $a < b$, then $ac < bc$

If a, b, and c are real numbers and c is negative
and if $a < b$, then $ac > bc$
and if $a > b$, then $ac < bc$

Examples:

If $15 > 7$
Then $15 + 2 > 7 + 2$
 $17 > 9$

If $-3 < 1$
then $-3 + 2 < 1 + 2$
 $-1 < 3$

$$\begin{array}{l}
 13 > 5 \\
 (6)(13) > (6)(5) \\
 78 > 30
 \end{array}$$

$$\begin{array}{l}
 -23 < 4 \\
 (-1)(-23) > (-1)(4) \\
 23 > -4
 \end{array}$$

Note: The inequality is reversed if both members are multiplied by -1.

Applications.

The production goal of a department was 1500 auto batteries during September and 1200 during October. To compare these two figures, we can say

$$1500 > 1200$$

batteries batteries

If both months goals were reduced by 100 batteries we can say

$$1500 - 100 > 1200 - 100$$

or $1400 > 1100$

batteries batteries

If both monthly goals were cut in half we can say

$$(1500)(1/2) > (1200)(1/2)$$

$$750 > 600$$

batteries batteries

If each member of statement $750 > 600$ were multiplied by (-1) we would obtain the opposites

$$-750 < -600$$

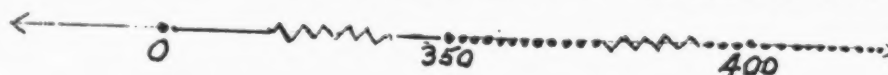
and the direction of the symbol would be reversed.

Drill and Practice:

Rising, p. 108-109, 111
Dressler, p. 276

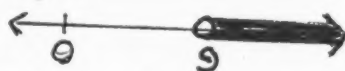
28. CONCEPT: SOLVING INEQUALITIES BY ADDITION OR MULTIPLICATION AND GRAPHING THE SOLUTION SET

A production department is to turn out at least 350 car radiators per week. This information can be written as an inequality, where x is the number of radiators made in one week, $x \geq 350$. Since the inequality represents a set of numbers, the integers (No. of cars must be an integer) equal to or is greater than 350. We can graph them on the number line.



Each dot represents an integer. The arrowhead implies that the set is infinite (although this is impossible in practice). The solid dot at 350 indicates the inclusion of 350.

Examples: $x > 9$



Note: Open dot at $x = 9$
Solid bar to the right of 9 since all numbers are the domain.

Graph $x \leq -3$



Note: Solid dot at -3 , solid bar to left of -3 , since all points are included in the domain.

Solve for x and graph the solution set:

$$\begin{array}{r} x + 9 > 4 \\ -9 \quad -9 \\ \hline x > -5 \end{array}$$



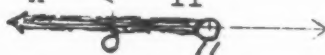
$$3x \geq 21$$

$$\frac{3x}{3} \geq \frac{21}{3}$$

$$x \geq 7$$



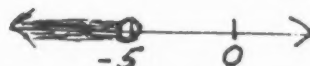
$$\begin{array}{r} x - 7 < 2 \\ +9 \quad +9 \\ \hline x < 11 \end{array}$$



$$-2x > 10$$

$$\frac{-2x}{-2} > \frac{10}{-2}$$

$$x < -5$$



Note change of direction of inequality symbol.

Applications. The payroll, minus withholding allowances, was mandated to be kept under \$30,000 per week. If the withholding allowances totalled 5400,

then

$P - 5400 < 30,000$ represents the relationship between payroll and withholding allowance of \$5400. Find the total payroll expenditure allowed.

$$\begin{array}{r} P - 5400 < 30,000 \\ + 5400 \quad + 5400 \\ \hline P < 35400 \end{array} \quad \text{or}$$



Broken line since negative payroll not possible.

The monthly production of electronic components was to be at least 892 units. Write an inequality which illustrates the weekly production expected. Let x represent weekly production expectation.

$$4x \geq 892$$

$$\frac{4x}{4} \geq \frac{892}{4}$$

$$x \geq 223 \text{ units}$$



Drill and Practice:
Rising, p. 112-115
Dressler, p. 280

29. CONCEPT: SOLVING INEQUALITIES INVOLVING ADDITION AND MULTIPLICATION

A transport rental agency offers a discount if a commercial renter contracts according to these conditions:

- 1) A total of at least 100 truck-rental days are contracted for.
- 2) 5 small delivery trucks are contracted for 4 days.
- 3) A sufficient number of trailers are contracted for 10 days.

Find the minimum number of trailers that must be rented to get the discount.

x = minimum no. of trailers

Therefore

$$\begin{array}{rclcl} 10x & + & (4)(5) & > & 100 \\ \text{Truck Rental} & + & \text{Truck Rental} & > & \text{100 Truck Rental} \\ \text{Days} & & \text{Days} & & \text{Days} \\ (\text{Delivery Trucks}) & & (\text{Trailers}) & & \end{array}$$

$$\begin{array}{rcl} \text{Solve for } x: & 10x + 20 & > 100 \\ & - 20 & - 20 \\ \hline & 10x & > 80 \end{array}$$

$$x > 8$$

$\therefore$ 9 trailers must be rented.

Examples:

Solve for x :

$$\begin{array}{rcl} -3x + 5 & < & 2 \\ - 5 & - 5 & \\ \hline -3x & < & -3 \end{array}$$

$$x > 1$$

Divide
by -3.

Solve for m :

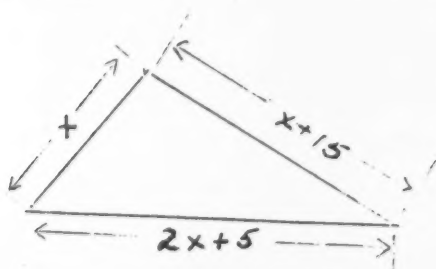
$$\begin{array}{rcl} \underline{m} - 3 & \geq & 4 \\ \underline{5} & & \\ + 3 & = & +3 \\ \hline \underline{m} & \geq & 7 \end{array}$$

$$\frac{5}{1} \left(\frac{m}{5} \right) \geq (7) \frac{5}{1}$$

$$m \geq 35$$

Applications.

The perimeter of a triangular plate must be at least 105cm.



If one side of the triangle is 5 more than twice the smaller and the third side is 15cm longer than the smaller, the length of the smaller must at least equal what value?

$$x + 2x + 5x + 15 \geq 105$$

$$\begin{array}{rcl} 4x + 20 & \geq & 105 \\ -20 & & -20 \\ \hline 4x & \geq & 85 \\ x & \geq & 21.25\text{cm.} \end{array}$$

The shortest side must be greater than or equal to 21.15cm.

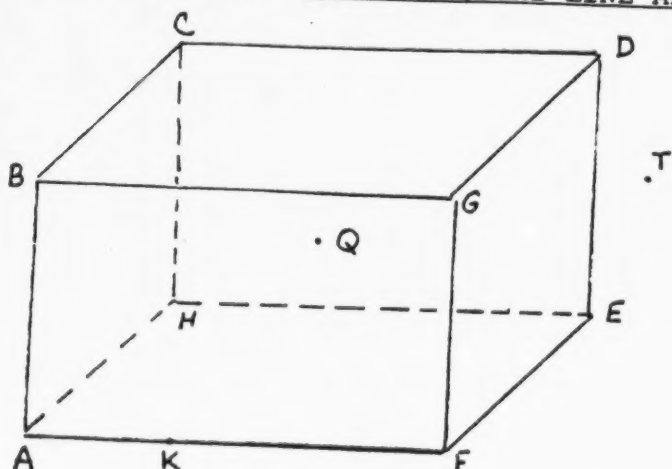
Drill and Practice:

Rising, p. 120

Dressler, p. 280-281

**AN INTRODUCTION
TO GEOMETRY**

1. CONCEPT: THE POINT, THE LINE AND THE PLANE



If the sketch to the left represents a room, most people can identify the floor, ceiling, walls and possible locations of doors and windows, etc.

However, there are other ways of viewing this room. Certain "points" are indicated, B, G, D, H, etc. These points seem to be corners of the room. It is possible to imagine other points, such as K which lies on the edge of AF, Q which lies within the room and T which lies outside the room.

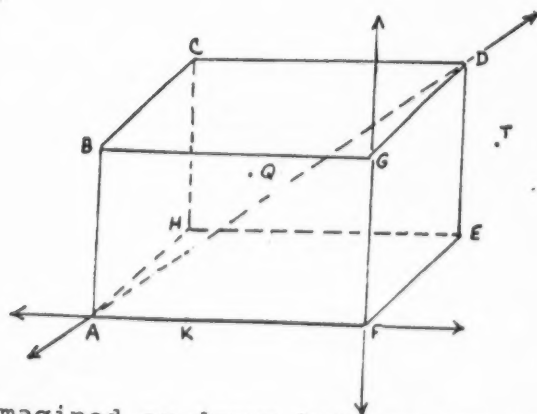
Each point is merely a location in space. There are an infinite number of these locations in space. A point is pictured as a dot and labeled with a single capital letter.

If we consider a set of points along a "straight edge" then we can understand the idea of a "straight line" or merely a "line".



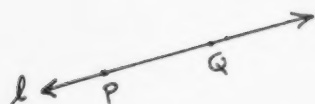
Lines can be named by using any two points on them.

$\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$, $\overleftrightarrow{BC}$, or $\overleftrightarrow{CA}$ or $\overleftrightarrow{CB}$



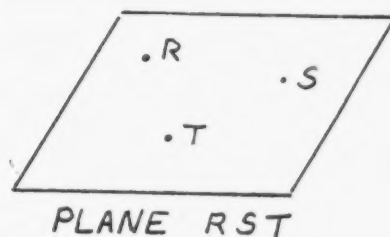
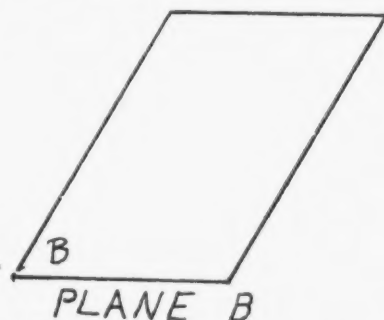
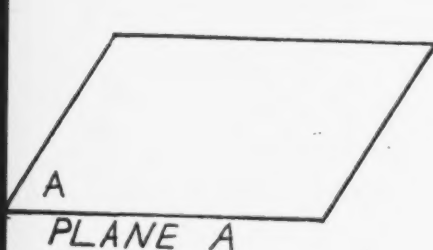
A line is determined by any two points in space. A line has no end points and is infinite in length.

$\overleftrightarrow{AF}$, $\overleftrightarrow{FG}$ and $\overleftrightarrow{AD}$ are examples. A line can be imagined or drawn between any two points in space without reference to a stylized rectangular solid as shown above.

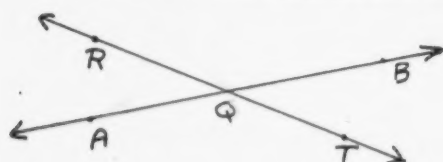


Ex: Line PQ or $\overleftrightarrow{PQ}$
 Sometimes a lower case letter such as l is used to name a line. l refers to no particular point: line l.

A plane is a set of all points on a flat surface. Pictorial representations of a plane are:



The intersection of two lines in space is a point.

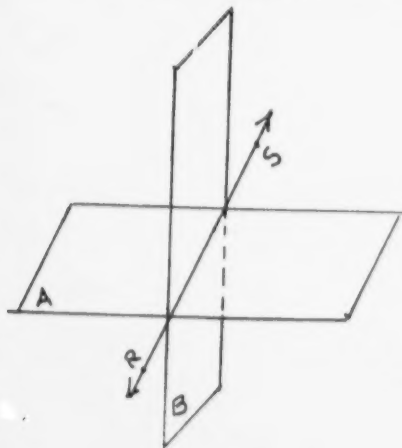


The intersection of $\overleftrightarrow{RT}$ and $\overleftrightarrow{AB}$ is point Q.

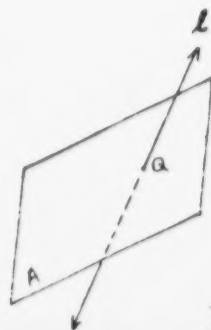
Points on the same line are said to be collinear. Three points not all on the same straight line are called non-collinear.

Ex: R, Q and T are collinear
 A, Q and B are collinear
 A, Q and T are non-collinear

The intersection of two planes is a line.

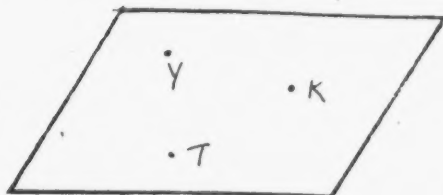


The intersection of plane A and plane B is $\overleftrightarrow{RS}$.

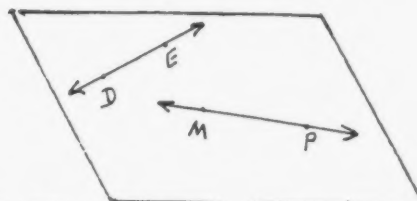


The intersection of a line and a plane is a point.
 The intersection of plane A and line l is point Q.

Points in the same plane are said to be coplanar points.
Lines in the same plane are said to be coplanar lines.



Points Y, T, and K are coplanar..



Lines DE and MP are coplanar.

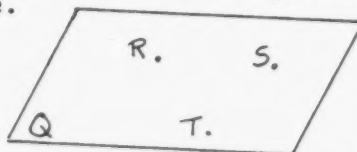
Some properties of planes, lines, and points.

1. Through any two points, there is exactly one line.
(Two points determine a line).



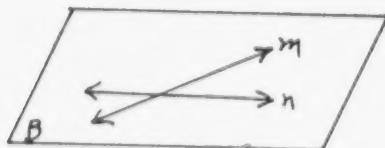
V and W determine $\overleftrightarrow{VW}$.

2. Through any three non-collinear points, there is exactly one plane.



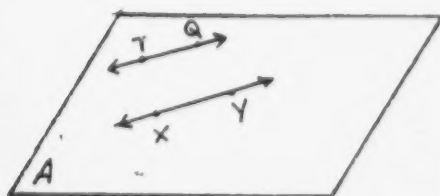
R, S, and T
determine plane Q.

3. Through two intersecting lines there is exactly one plane (Two intersecting lines determine one plane).



m and n determine plane B.

Two lines in the same plane which do not intersect are called parallel lines. (symbol $\parallel$)

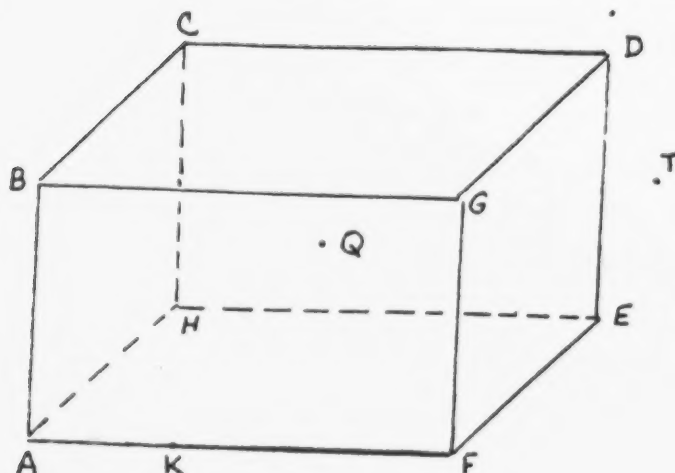


$\overleftrightarrow{TQ}$ and $\overleftrightarrow{XY}$ lie in plane A and $\overleftrightarrow{TQ} \parallel \overleftrightarrow{XY}$.

Application.

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Returning to a sketch of a work room.



- 1) Using 3 letters, name the plane of the floor.
Plane AFE or plane AHE or plane HEF.
- 2) Using 3 letters, name the planes which represent the walls. Plane BAF, plane GDE, plane BHA, plane CHE.
- 3) Using 3 letters, name the plane of the ceiling.
Plane CDG
- 4) Name the line which represents the intersection of the plane HAF (floor) and plane BGF (nearest wall). AF
- 5) Name two lines which are parallel
 $\longleftrightarrow$ CD and $\longleftrightarrow$ AF also $\longleftrightarrow$ GD and $\longleftrightarrow$ FE etc.

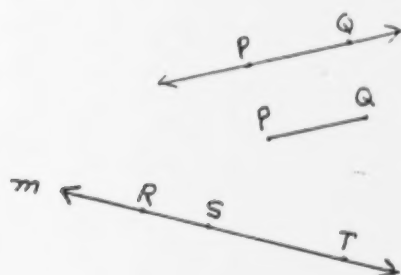
Drill and Practice:

Rising, p. 136

Dressler, p. 302

2. CONCEPT: AN UNDERSTANDING OF DISTANCE,
THE LINE SEGMENT AND RAY

A line segment is the set of all points between and including two points on a line.



The line PQ ($\overleftrightarrow{PQ}$) contains a segment PQ ($\overline{PQ}$) which contains P and Q and all points between them. A segment is named by its end points.

m contains $\overline{RS}$, $\overline{ST}$, $\overline{RT}$

(Note: $\overline{RS}$ can also be named $\overline{SR}$.)

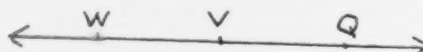
The length of a segment is the distance between the end points. PQ represents the length of $\overline{PQ}$.

RS is the length of $\overline{RS}$

ST is the length of $\overline{ST}$

RT is the length of $\overline{RT}$

Length (distance) is always positive. Two segments are said to be congruent ($\cong$) if their lengths are equal.



If WV equals VQ, then $\overline{WV} \cong \overline{VQ}$, ($\overline{WV}$ is congruent to $\overline{VQ}$).

A midpoint of a line segment is a point equally distant from the end points.

Ex: If $WV = VQ$, then V is the midpoint of WQ.

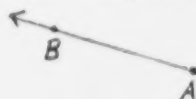
or If V is the midpoint of WQ, then $WV = VQ$.

A ray is the set of points which begins at a point in a line and extends infinitely in one direction.

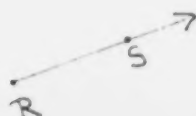


Ray KQ or $\overrightarrow{KQ}$
(Note: to name a ray write the end point K first followed by one other point.)

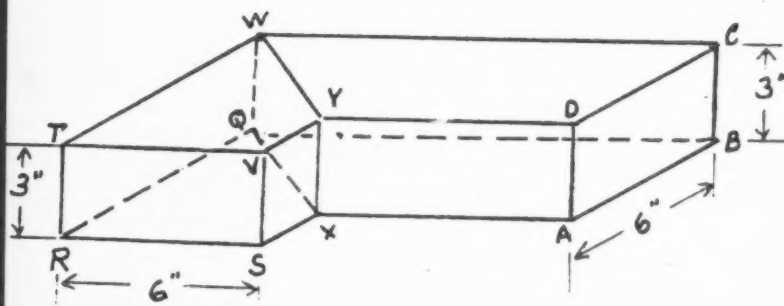
K is the endpoint of $\overrightarrow{KQ}$.



Ray AB



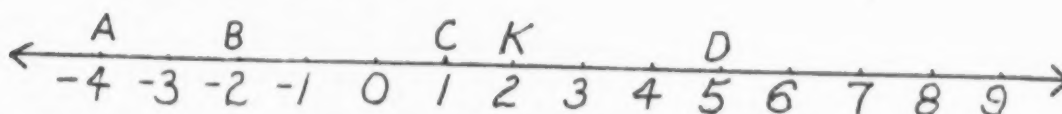
Ray RS

Application 1.

Name two pairs of congruent segments.
 $\overline{RS}$, $\overline{AB}$ and $\overline{TR}$, $\overline{CB}$

Using 4 letters name the plane of the weld.

Plane WYXQ

Application 2.

Find the distance $AB = 2$ (count divisions)

$$\text{or } AB = |-4 - (-2)|$$

$$AB = |-4 + 2| = |-2| = 2$$

Find the absolute value of the difference of the coordinates

Find $AC = 5$ (counting divisions)
 or $AC = |-4 - (+1)| = |-4 - 1| = |-5| = 5$

Find $BD = 7$ (try either method)

Find the coordinate of the midpoint of $\overline{AR}$: -1

Two pieces of iron are welded together. From this sketch name the line segments which you can identify.

$\overline{TW}$, $\overline{WC}$, $\overline{CB}$, $\overline{AB}$,
 $\overline{XA}$, $\overline{SX}$, $\overline{RS}$, $\overline{RV}$,
 $\overline{TR}$, $\overline{TV}$, $\overline{VY}$, $\overline{YD}$,
 $\overline{DC}$, $\overline{DA}$, $\overline{YX}$, $\overline{VS}$,
 $\overline{WQ}$, $\overline{QB}$, $\overline{WY}$, $\overline{QX}$

Name 4 segments shared by both pieces when welding was complete

$\overline{WQ}$, $\overline{WY}$, $\overline{YX}$ and $\overline{QX}$

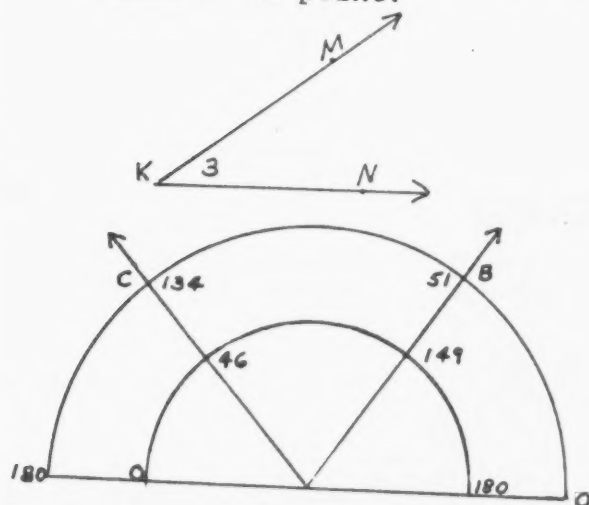
Drill and Practice:

Rising, p. 138-139

Dressler, p. 302

3. CONCEPT: THE MEASUREMENT OF AN ANGLE AND NAMES OF SPECIAL ANGLES

An angle is the set of points formed by two rays with a common end point.



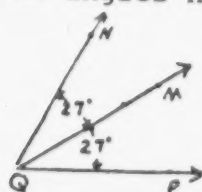
$\overset{\curvearrowright}{K}$ is called the vertex.
KM and KN are the sides
of the angle.

To name the angle we say $\angle K$, or $\angle MKN$ or $\angle NKM$ (if 3 letters are used, the vertex K is listed between the other two letters). Sometimes a number written between the sides is used, such as $\angle 3$ or a lower case letter such as x in place of 3.

The unit of measure of an angle is often the degree. Using a protractor there are 180 degrees or 180° on the outer scale. By reading the numbers on the scale on the protractor we can read the measure of $\angle COB$. The measure (m) of $\angle COB = m \angle COB = 134 - 51 = 83^\circ$

Most protractors also have an inner scale which allows us to read $m \angle COB = 149 - 46 = 83^\circ$.

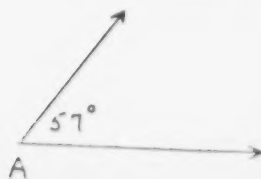
Congruent angles have the same measure.



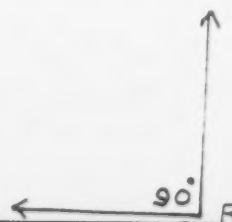
Since $m \angle NQM = 27^\circ$
and $m \angle MQP = 27^\circ$
We say $\angle NQM \cong \angle MQP$
" $\angle NQM$ is congruent
to $\angle MQP$ "

$\overrightarrow{QM}$ divides $\angle NQP$ into two congruent angles, and is called an angle bisector. An angle bisector is a ray (or a line or a line segment which divides an angle into two congruent angles.

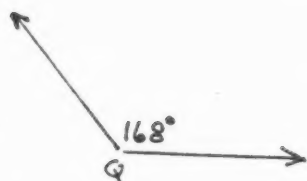
- 1) An acute angle has a measure less than 90°



- 2) A right angle has a measure $= 90^\circ$

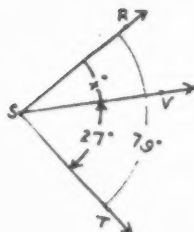
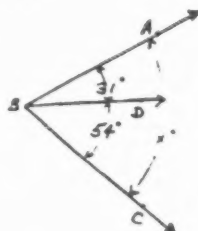


3)

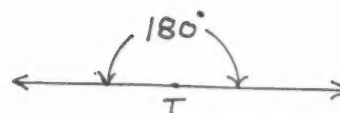


An obtuse angle has a measure greater than 90° and less than 180°

Applications.



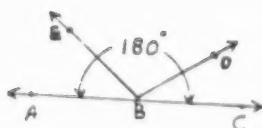
4)



A straight angle is an angle whose measure = 180°

Find m/x if
 $m\angle DBC = 54^\circ$
 Solution: m/x
 $= m\angle ABD + m\angle DBC$
 $m/x = 31 + 54 = 85^\circ$

Find m/x if
 $m\angle RST = 79^\circ$
 and $m\angle VST = 27^\circ$
 Solution:
 $m/x = m\angle RST - m\angle VST$
 $m/x = 79^\circ - 27^\circ = 52^\circ$



If $m\angle DBC = 24^\circ$
 $m\angle EBD = (2x - 18)^\circ$
 and $m\angle ABE = x^\circ$
 Find $m\angle ABE$

Solution: $x + 2x - 18 + 24 = 180$
 $3x + 6 = 180$
 $\frac{-6}{3x} = \frac{-6}{174}$

$$\frac{1}{3} (3x) = \frac{174}{3}$$

$$x = 58^\circ = m\angle ABE$$

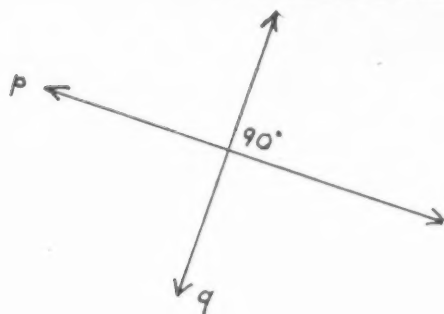
Drill and Practice:

Rising, p. 141-145

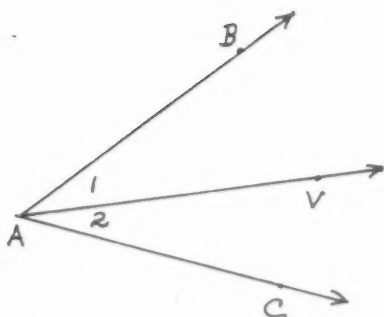
Dressler, p. 304-306, 308

Smith, p. 195 (4,8), 196, (1,27,33), 197

4. CONCEPT: UNDERSTANDING COMPLEMENTARY ANGLES, AND VERTICAL ANGLES

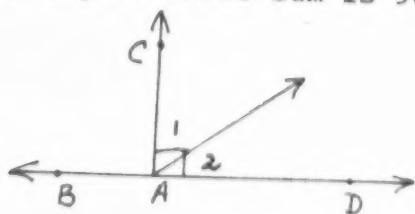


If lines p and q intersect so as to form a 90° angle, p and q are said to be perpendicular. ($\perp$) (Perpendicular lines form right angles). Therefore $p \perp q$.



If $\angle 1$ and $\angle 2$ share a common vertex A and a common side AV between them, then $\angle 1$ and $\angle 2$ are called adjacent angles. (Note $\angle BAC$ and $\angle 2$ have a common vertex A , and a common side AC ; however, since AC is not between the two angles, the angles are not adjacent).

Two angles whose sum is 90° are called complementary.



If $CA \perp AB$, then $\angle CAB = 90^\circ$ therefore $\angle 1$ and $\angle 2$ are complementary angles or

$\angle 1$ is the complement of $\angle 2$.
 $\angle 2$ is the complement of $\angle 1$.

Two angles whose sum is 180° are called supplementary.

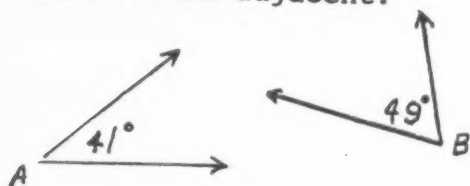


If $m\angle ACB = 180^\circ$, then $\angle 1$ and $\angle 2$ are supplementary. or

$\angle 1$ is the supplement of $\angle 2$.
 $\angle 2$ is the supplement of $\angle 1$.

Note: Complementary angles and supplementary angles do not have to be adjacent.

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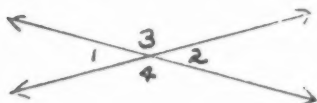


Since $41 + 49 = 90$
then $\angle A$ and $\angle B$ are
complements.



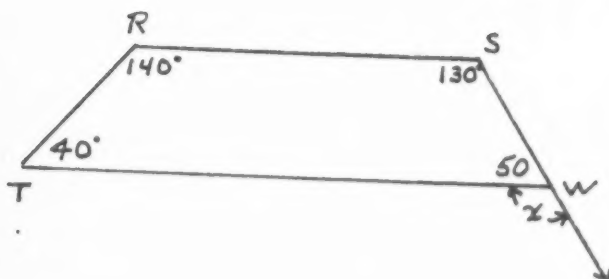
Since $38 + 142 = 180$
then $\angle R$ and $\angle S$ are
supplements.

If two lines intersect, two pairs of vertical angles are formed.

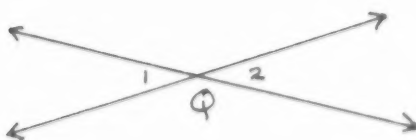


$\angle 1$ and $\angle 2$ are vertical
angles.
 $\angle 3$ and $\angle 4$ are vertical
angles.

If $m\angle 1 = 40$, then the $m\angle 2 = 40$
since they both are supplements of $\angle 3$
Vertical angles are congruent.



Line segments can also
form angles. RS and SW
intersect to form $\angle S$.
Name a pair of
complementary angles.
 $\angle T$ and $\angle SWT$.
Name a pair of
supplementary angles.
 $\angle R$ and $\angle T$
or $\angle S$ and $\angle SWT$
If $m\angle SWT = 50$, find $m\angle x$.
 $m\angle x = 180 - 50 = 130^\circ$



If the lines p and q
intersect at Q and
 $m\angle 1 = 5x - 10$
and $m\angle 2 = 2x + 8$
Find the value of x .
Since vertical angles
are congruent,
then $5x - 10 = 2x + 8$
$$\begin{array}{r} 5x - 10 = 2x + 8 \\ +10 \quad +10 \\ \hline 5x = 2x + 18 \\ -2x \quad -2x \\ \hline 3x = 18 \\ x = 6 \end{array}$$

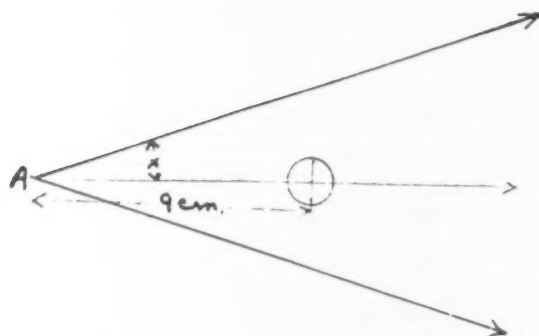
Find the $m\angle 1$ and $m\angle 2$

$$\begin{aligned} m\angle 1 &= 5x - 10 = 5(6) - 10 \\ &= 30 - 10 = 20^\circ \end{aligned}$$

$m\angle 2 = m\angle 1$ (Vertical angles are congruent)
 $m\angle 2 = 20^\circ$

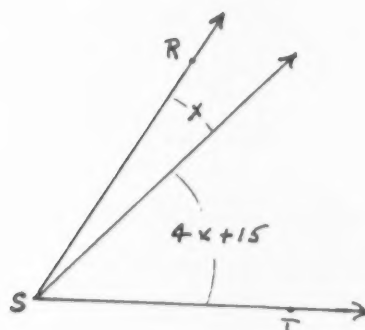
Applications.

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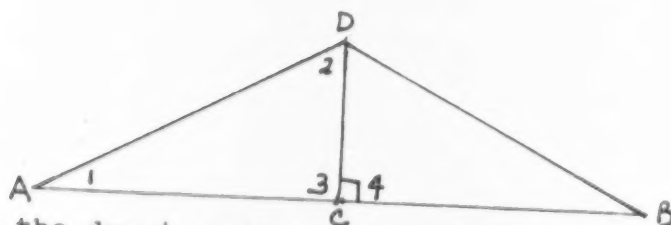
$$m\angle x = 26/2 = 13^\circ$$

The corner of a decorative piece of sheet metal must be drilled at a point 9cm. from A, and on the bisector of $\angle A$. If $\angle A = 26^\circ$, find the measure of $\angle x$



If the measure of $\angle RST = 75^\circ$, find $m\angle x$

$$\begin{aligned} x+4x + 15 &= 75 \\ -15 &-15 \\ \hline x+4x &= 60 \\ 5x &= 60 \\ \frac{1}{5} (5x) &= (60) \frac{1}{5} \\ x &= 12 \end{aligned}$$



In the drawing showing the support beams for a roof, $\overline{DC} \perp \overline{AB}$. Name a right angle.

$\angle 3$ or $\angle 4$

If $\angle 1$ and $\angle 2$ are complements, and $\angle 1 = 37^\circ$,

What is the $m\angle 2$? $m\angle 2 = 90 - 37 = 53^\circ$.

If $\angle 1$ is congruent with $\angle 5$, find $m\angle 5$. $m\angle 5 = 37^\circ$.

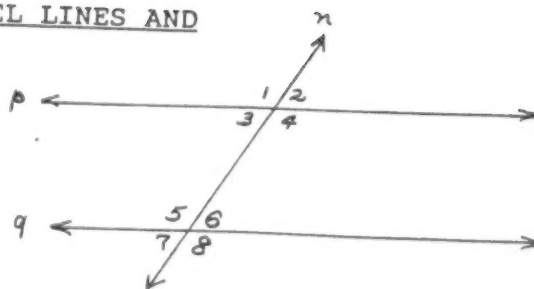
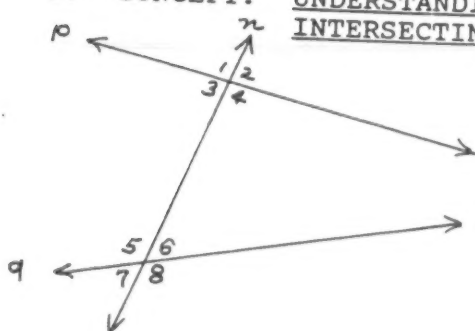
Drill and Practice:

Rising, p. 148-150

Dressler, p. 312-313

Smith, p. 207

5. CONCEPT: UNDERSTANDING PARALLEL LINES AND INTERSECTING LINES



If line n intersects 2 coplanar lines, 8 angles are formed. Line n is called a transversal.

$\angle 1$, $\angle 2$, $\angle 7$ and $\angle 8$ are called exterior angles.

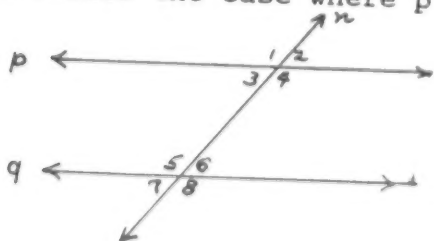
$\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$ are called interior angles.

There are other special names however:

Pairs of alternate interior angles: $\angle 3$ and $\angle 6$
 $\angle 4$ and $\angle 5$

Pairs of corresponding angles: $\angle 3$ and $\angle 7$
 $\angle 4$ and $\angle 8$
 $\angle 1$ and $\angle 5$
 $\angle 2$ and $\angle 6$.

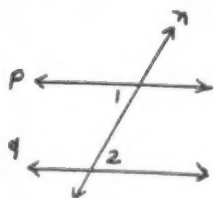
Consider the case where p and q are parallel.



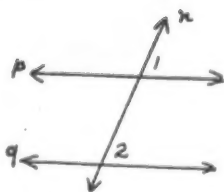
n is the transversal.
Remember that parallel lines are coplanar lines which do not intersect.

It is often useful to have some certainty that certain lines are indeed parallel in a drawing or print.

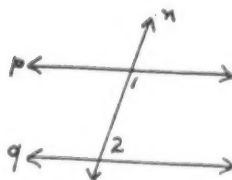
- Th. 1. If a pair of alternate interior angles are congruent, the lines are parallel.
- Th. 2. If a pair of corresponding angles are congruent, the lines are parallel.
- Th. 3. If a pair of interior angles on the same side of the transversal are supplementary, the lines are parallel.
- Th. 4. Two coplanar lines perpendicular to the same line are parallel.



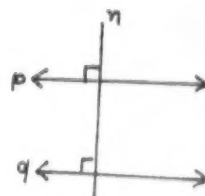
If $\angle 1 = \angle 2$
then $p \parallel q$.



If $\angle 1 = \angle 2$
then $p \parallel q$.



If $m\angle 1 + m\angle 2 = 180^\circ$
then $p \parallel q$.

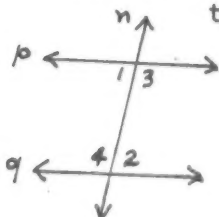


If $p \perp n$
and $q \perp n$
then $p \parallel q$.

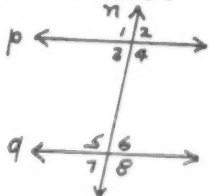
The proofs or demonstration of these theorems are available in any text on geometry.

Conversely,
If two lines are parallel,

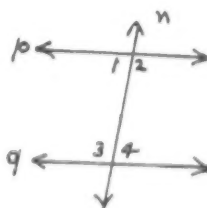
- Th. 5. The alternate interior angles are congruent.
Th. 6. The corresponding angles are congruent.
Th. 7. The interior angles on the same side of the transversal are supplementary.
Th. 8. In a plane, a line perpendicular to one parallel line, is perpendicular to the other one also.
Th. 9. Two lines parallel to a third line are parallel to each other.



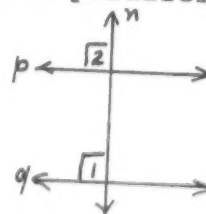
If $p \parallel q$
then
 $\angle 1 = \angle 2$
and
 $\angle 3 = \angle 4$



If $p \parallel q$
then
 $\angle 1 = \angle 5$
 $\angle 2 = \angle 6$
 $\angle 3 = \angle 7$
 $\angle 4 = \angle 8$



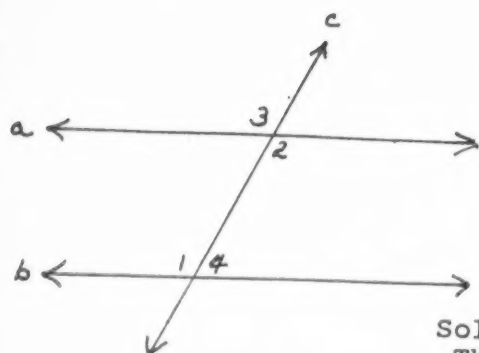
If $p \parallel q$
then
 $m\angle 1 + m\angle 3 = 180^\circ$
 $m\angle 2 + m\angle 4 = 180^\circ$



If $p \parallel q$
and $p \perp n$
then $q \perp n$
or
If $p \parallel q$
and $m\angle 1 = 90^\circ$
then
 $m\angle 2 = 90^\circ$.



If $p \parallel q$ and $p \parallel n$
then $q \parallel n$.



Given conditions: $a \parallel b$, c is a transversal.

If $m\angle 1 = 110^\circ$

Find the $m\angle 2$.

Solution: $m\angle 2 = m\angle 1$

Therefore
 $m\angle 2 = 110^\circ$

Since, if 2 lines are parallel, the alternate interior angles are congruent ($\cong$).

Find $m\angle 3$

Solution:

$m\angle 3 = m\angle 1 = 110^\circ$

Since, if 2 lines are parallel, the corresponding angles are congruent.

Find $m\angle 4$

Solution: $m\angle 2 + m\angle 4 = 180^\circ$

Therefore

$$m\angle 2 + m\angle 4 = 180$$

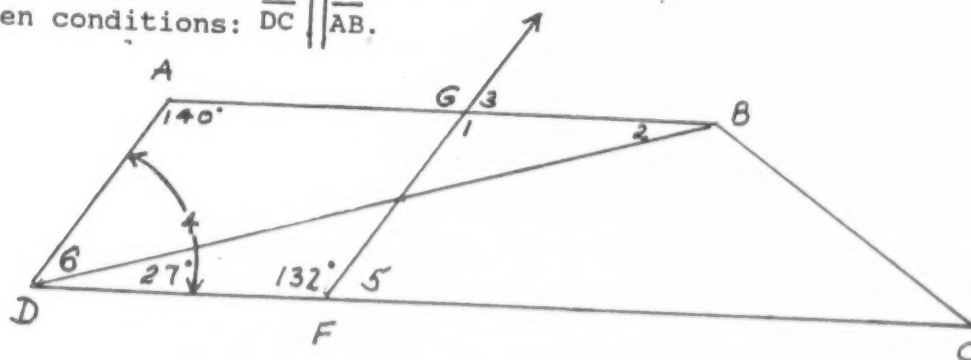
$$110 + m\angle 4 = 180$$

$$\begin{array}{r} -110 \quad -110 \\ \hline m\angle 4 = 70^\circ \end{array}$$

Since, if 2 lines are parallel, the interior angles on the same side of transversal are supplementary.

Application.

Given conditions: $\overline{DC} \parallel \overline{AB}$.



Given the information in this drawing, determine the measures of $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, and $\angle 5$.

1. $\angle 1$ and $\angle DFG$ are alternate interior angles:

$$\angle 1 \cong \angle DFG$$

and since

$$\angle DFG = 132^\circ$$

then, $\angle 1 = 132^\circ$

2. $\angle 2$ and $\angle BDC$ are alternate interior $\angle$'s

Therefore $\angle 2$ and $\angle BDC$ are $\cong$. If $m\angle BDC = 27^\circ$

then $m\angle 2 = 27^\circ$.

3. $\angle 3$ and $\angle 5$ are corresponding angles.
 $\angle DFG$ and $\angle 5$ are supplementary. Why?
 Therefore since $m\angle DFG = 132^\circ$, $m\angle 5 = 180 - 132 = 48^\circ$
 If $m\angle 5 = 48$, then $m\angle 3 = 48^\circ$.
4. $\angle 4$ and $\angle A$ are supplementary (interior angles on the same side of transversal)
 (If $m\angle A = 140$, then $m\angle 4 = 180 - 140 = 40^\circ$).
5. $m\angle 6 = m\angle 4 - m\angle BDC$
 $m\angle 6 = 40^\circ - 27^\circ = 13^\circ$.

Drill and Practice:

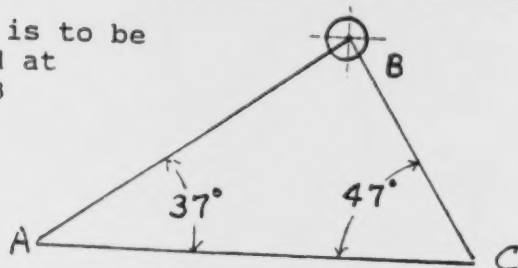
Rising, p. 153-155

Dressler, p. 319-320

Smith, p. 208-209 (5, 8, 9, 10-12)

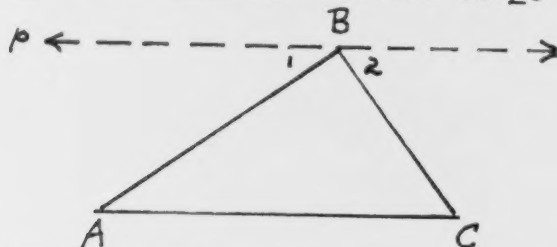
6. CONCEPT: THE SUM OF THE MEASURES OF THE ANGLES OF A TRIANGLE IS 180 DEGREES

A hole is to be drilled at point B



The figure formed by segments $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ (where A, B, and C are noncollinear) is a triangle.

If $m\angle A = 37^\circ$ and the measure of $\angle C$ is 47° , find the $m\angle ABC$.



Method of naming a triangle is $\triangle ABC$

Let's draw a line p through B $\parallel$ AC. This produces $\angle 1$ and $\angle 2$.

$m\angle 1 + m\angle B + m\angle 2 = 180^\circ$ (the measure of a straight angle is 180° .)

$\angle 1 = \angle A$, $\angle 2 = \angle C$ (if 2 lines are parallel, the alternate interior angles are congruent.)

Therefore,

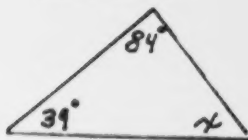
$m\angle A + m\angle B + m\angle C = 180$ (Substitution)

We have proven that: The sum of the measures of the angles of a triangle is 180° .

Therefore: If $m\angle A + m\angle B + m\angle C = 180$

$$\begin{array}{rcl} 37 + m\angle B + 47 & = & 180 \\ m\angle B + 84 & = & 180 \\ -84 & - & 84 \\ \hline m\angle B & = & 96^\circ \end{array}$$

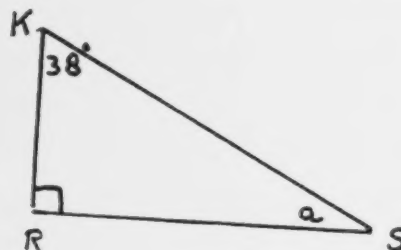
Examples:



Find m/x

Solution: $39 + 84 + x = 180$

$$\begin{array}{rcl} 123 + x & = & 180 \\ -123 & - & 123 \\ \hline x & = & 57^\circ \end{array}$$

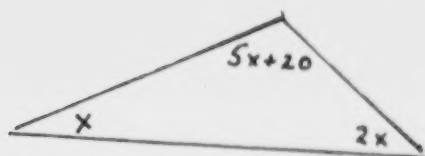


Find m/a

$$\begin{array}{rcl} 38 + 90 + m/a & = & 180 \\ 128 + m/a & = & 180 \\ -128 & - & 128 \\ \hline m/a & = & 52^\circ \end{array}$$

194 Note:

A triangle which contains a right angle such as $\triangle KRS$ above, is called a right triangle.



Find $m\angle x$.

$$x + 2x + 5x + 20 = 180$$

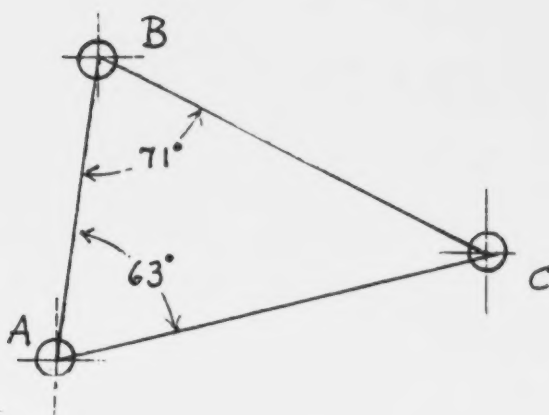
$$\frac{8x + 20}{8} = \frac{180}{8}$$

$$8x = 160$$

$$\frac{1}{8} \cdot (8x) = \left(\frac{160}{8}\right) \cdot \frac{1}{8}$$

$$x = 20^\circ$$

Application.



Hole centerlines
are at the vertices
of the triangle ABC
If $m\angle A = 63^\circ$
 $m\angle B = 71^\circ$,
Find the
measure of $\angle C$.

$$m\angle C = 180 - (63^\circ + 71^\circ)$$

$$m\angle C = 180 - 134 = 46^\circ$$

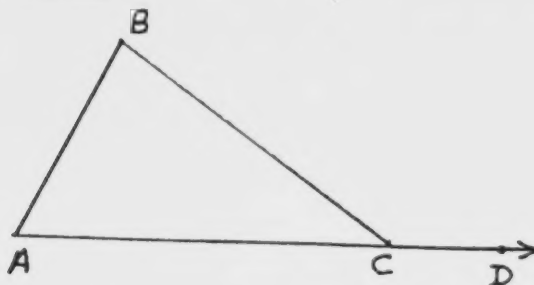
Drill and Practice:

Rising, p. 157-158

Dressler, p. 326-327

Smith, p. 213-214

7. CONCEPT: UNDERSTANDING THE RELATIONSHIPS BETWEEN THE INTERIOR AND EXTERIOR ANGLES OF A TRIANGLE



$\angle A$, $\angle B$, $\angle BCA$ are called interior angles of $\triangle ABC$. If we extend AC through C and through the point D , we produce exterior angle BCD .

Relative to $\angle BCD$, $\angle A$ and $\angle B$ are called the remote interior angles.

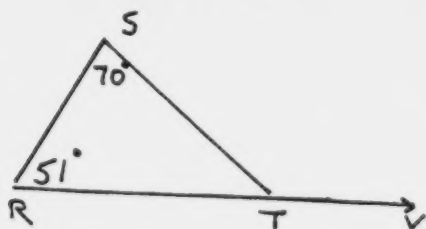
Now, $m\angle A + m\angle B + m\angle BCA = 180^\circ$ (the sum of the interior angles of a $\triangle = 180$)

And, $m\angle BCA + m\angle BCD = 180^\circ$ (the measure of a straight angle is 180)

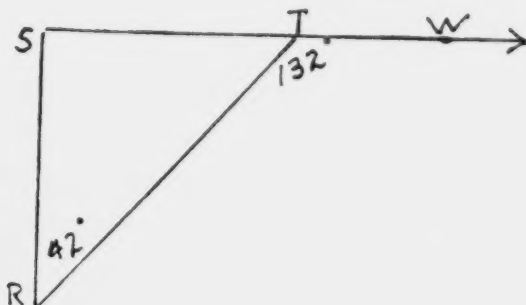
Therefore, $m\angle A + m\angle B + m\angle BCA = m\angle BCA + m\angle BCD$ (Substitution).

$$m\angle A + m\angle B = m\angle BCD \text{ (Subtraction property of equality).}$$

or The measure of an exterior angle equals the sum of the the measures of the two remote interior angles.
($m\angle BCD = m\angle A + m\angle B$).

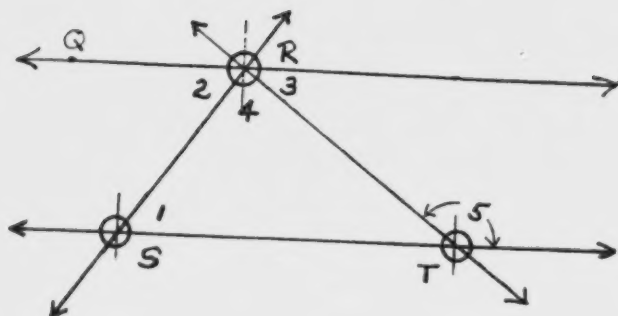


If $m\angle R = 51^\circ$, $m\angle S = 70^\circ$
Find $m\angle STV$
 $m\angle STV = 51 + 70 = \underline{121}^\circ$



If $m\angle WTR = 132^\circ$
and $m\angle R = 42^\circ$
Find $m\angle S$.

$$\begin{aligned} m\angle WTR &= m\angle S + m\angle R \\ 132 &= m\angle S + 42 \\ -42 &\quad -42 \\ \hline 90^\circ &= m\angle S \\ m\angle S &= 90^\circ \end{aligned}$$

Application.

Centerlines for holes
to be drilled lie on
parallel lines $\overleftrightarrow{QR}$ and $\overleftrightarrow{ST}$
If $m\angle 1 = 44^\circ$, $m\angle 3 = 39^\circ$
Find $m\angle 2$
 $m\angle 4$
 $m\angle 5$

Solution: Since $\overleftrightarrow{QR} \parallel \overleftrightarrow{ST}$, $\angle 1 \cong \angle 2$,
therefore if $m\angle 1 = 44^\circ$, $m\angle 2 = 44^\circ$.
(alt. int. $\angle$ s of $\parallel$ lines are $\cong$.)
and since $m\angle 2 + m\angle 4 + m\angle 3 = 180$
then, $44 + m\angle 4 + 39 = 180$
 $m\angle 4 + 83 = 180$
 $\begin{array}{r} -83 \\ m\angle 4 = 97^\circ \end{array}$

Finally, in $\triangle RST$,
 $m\angle 5 = m\angle 4 + m\angle 1$
 $m\angle 5 = 97 + 44$
 $m\angle 5 = 141^\circ$

Drill and Practice:

Rising, p. 157-159

Dressler, p. 329

Smith, p. 213 (17, 19) 214

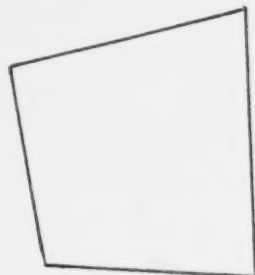
8. CONCEPT: POLYGONS AND THE PERIMETER OF POLYGONS

Drawings and prints are overflowing with polygons, many of which have important properties. Triangles, rectangles, squares, and hexagons are the most familiar.

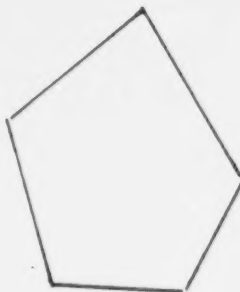
Polygons are difficult to define but, in general, the ones that we are concerned with are made up of a series of line segments, connected only at their end points, which enclose a portion of a plane.



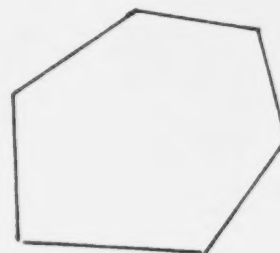
Triangle



Quadrilateral

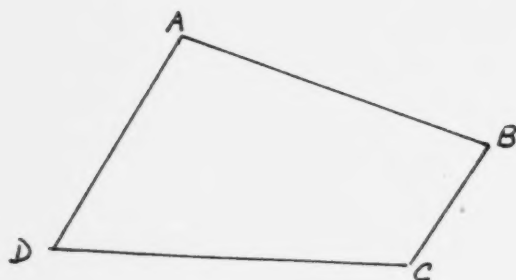


Pentagon



Hexagon

<u>Name</u>	<u>No. of Sides</u>
Triangle	3
Quadrilateral	4
Pentagon	5
Hexagon	6
Octagon	8
Decagon	10



Name: Polygon ABCD
or Quadrilateral
ABCD

Interior angles:

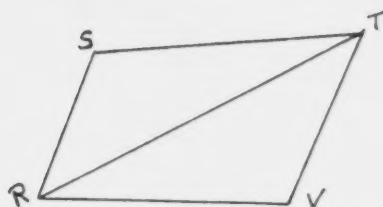
Sides: $\angle A, \angle B, \angle C, \angle D$
 $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA}$

Consecutive vertices:
A, B, C, D

The perimeter of a polygon is the sum of the lengths of the sides.

$$\text{Perimeter} = \overline{AB} + \overline{BC} + \overline{CD} + \overline{DA}$$

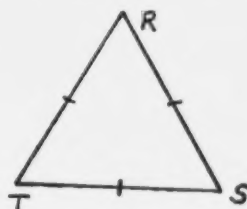
A diagonal is a segment drawn between two non-consecutive vertices.



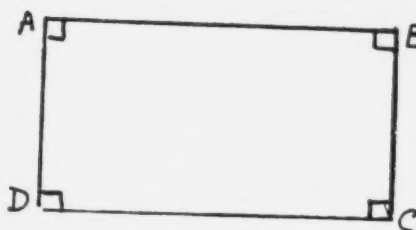
$\overline{RT}$ is a diagonal.

An equilateral polygon is a polygon with congruent sides.
 An equiangular polygon is a polygon with congruent angles.
 A regular polygon is a polygon with both congruent sides and congruent angles.

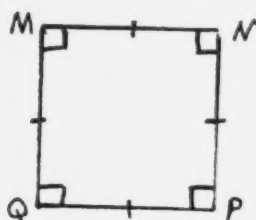
Examples:



Equilateral $\triangle$ ($RS=ST=RT$)

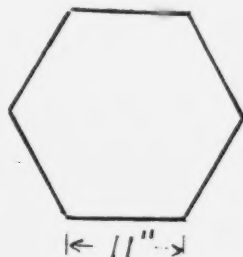


Rectangle ($\angle A \cong \angle B \cong \angle C \cong \angle D$)
 (Equilateral)



Square $MN = NP = PQ = QM$
 (Regular) $\angle M \cong \angle N \cong \angle P \cong \angle Q$

Application 1.

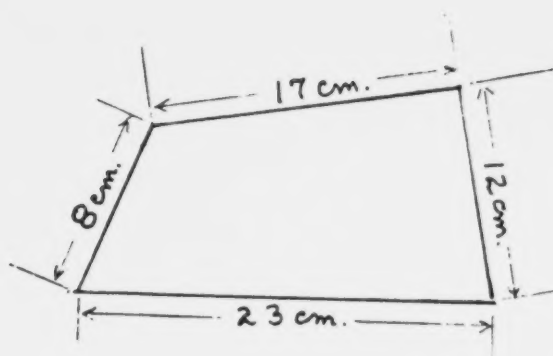


A piece of sheet metal was cut in the shape of a regular hexagon. Find the perimeter of the hexagon, if one side = 11 inches.

$$P = 11 + 11 + 11 + 11 + 11 + 11$$

$$\text{or } P = 6 \cdot 11$$

$$P = 66 \text{ inches}$$

Application 2.

A flat piece of iron is to be welded in place. What is the total length of the weld?

$$\text{Perimeter} = 8 + 17 + 12 + 23$$

$$\text{Perimeter} = 60\text{cm}$$

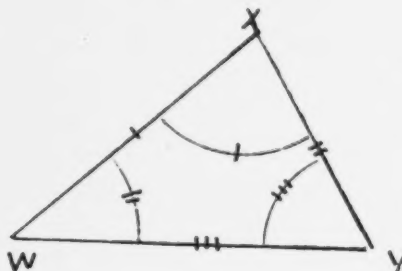
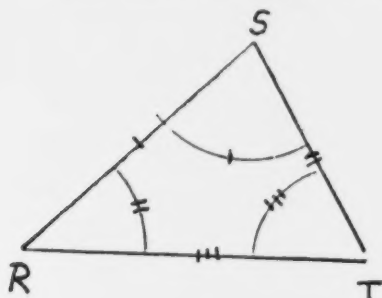
Drill and Practice:

Rising, p. 162-163

Dressler, p. 322-323

9. CONCEPT: CONGRUENT TRIANGLES

Congruent polygons is the name given to two or more polygons which have the same size and shape. (Symbol: $\cong$)



If $\triangle RST \cong \triangle WXY$

then the corresponding sides are congruent.

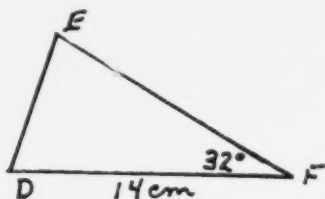
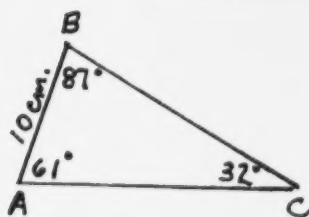
$$\overline{RS} \cong \overline{WX}, \overline{ST} \cong \overline{XY}, \overline{RT} \cong \overline{WY}$$

and the corresponding angles are congruent

$$m\angle R = m\angle W, m\angle S = m\angle X, m\angle T = m\angle Y$$

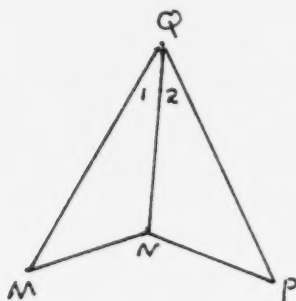
Note the method used to mark corresponding parts which are congruent.

Examples:



If $\triangle ABC \cong \triangle DEF$
using data from $\triangle ABC$,
find $DE, m\angle D, m\angle E$,
 AC .

Solution: $DE = AB = 10\text{cm}$.
 $m\angle D = m\angle A = 61^\circ$
 $m\angle E = m\angle B = 87^\circ$
 $AC = DF = 14\text{cm}$

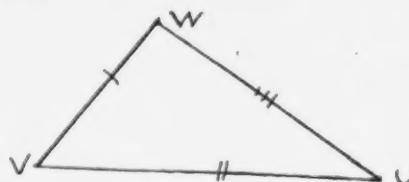
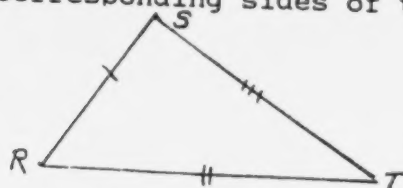


If $\triangle MNG \cong \triangle PNQ$
and $MN = 4$ feet,
then $NP = 4$ feet
If $m\angle 1 = 27^\circ$
then $m\angle 2 = 27^\circ$.
If $MQ = 9$ feet,
 $QP = 9$ feet.

Recognizing Congruent Triangles (Short Cuts)

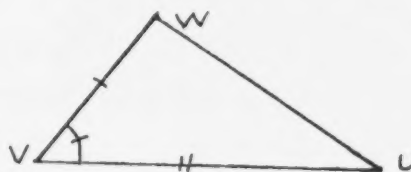
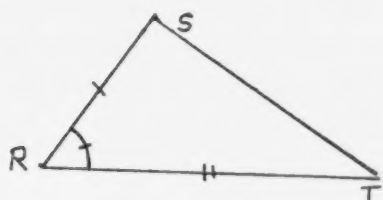
Two triangles are congruent if:

- 1) The corresponding sides of one are congruent to the corresponding sides of the other.



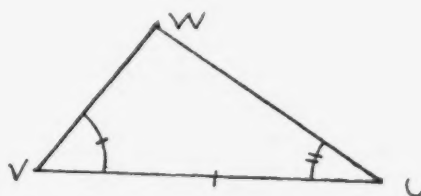
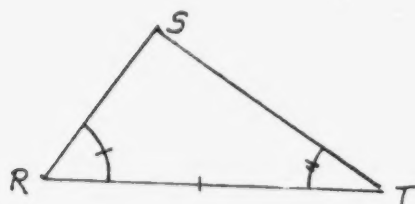
If $\overline{RS} \cong \overline{WV}$, $\overline{ST} \cong \overline{WU}$, $\overline{RT} \cong \overline{VU}$ then $\triangle RST \cong \triangle VWU$.
Abbreviation: SSS

- 2) If 2 sides and the included angle of one triangle are congruent to two sides and the included angle of the other.



If $\overline{RS} \cong \overline{WV}$, $\overline{RT} \cong \overline{VU}$, and $m\angle R = m\angle V$, then $\triangle RST \cong \triangle VWU$.
Abbreviation: SAS

- 3) If 2 angles and the included sides of one triangle are congruent to two angles and the included side of the other.



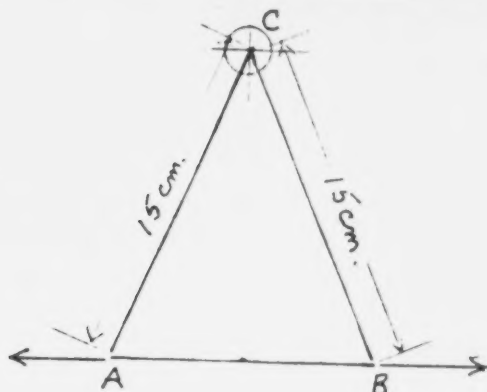
If $m\angle R = m\angle V$, $m\angle T = m\angle U$, $\overline{RT} \cong \overline{VU}$, then $\triangle RST \cong \triangle VWU$.
Abbreviation: ASA

Drill and Practice:

Rising, p. 185

Dressler, p. 362-364

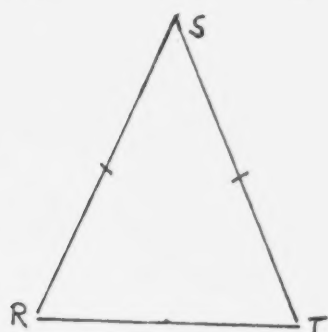
10. CONCEPT: THE ISOSCELES TRIANGLE AND ITS PROPERTIES



A hole is to be drilled, so that its center (C) will be 15cm from point A and 15cm from point B.

This creates a triangle ACB in which $AC = CB$

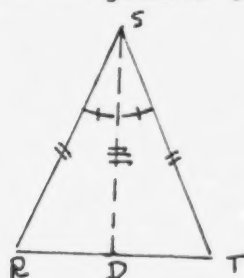
A triangle in which two sides are congruent is an isosceles triangle.



In an isosceles triangle such as $\triangle RST$, $\overline{RS}$ and $\overline{ST}$ the two congruent sides are called legs; $\overline{RT}$, the unequal side is called the base.

$\angle S$ is the vertex angle and $\angle R$ and $\angle T$ are the base angles.

If we draw a line segment through S so that $\angle 1 \cong \angle 2$, we produce two congruent triangles.



$$\triangle RSD \cong \triangle DST$$

Why are they congruent?

Mark all of the pairs of congruent parts.

(Note that $SD = SD$, since it is shared by both .

The triangles are congruent because of S.A.S.

Since $\angle R$ corresponds to $\angle T$. Then $m\angle R = m\angle T$.
(The corresponding parts of congruent triangles are congruent).

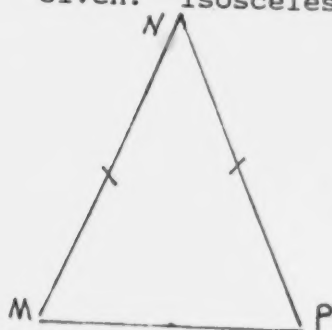
So The base angles of an isosceles triangle are congruent.

Example:

Given: Isosceles triangle MNP, where $MN = NP$
if $m\angle M = 41^\circ$

Find $m\angle P$, $m\angle N$

Solution: $m\angle P = m\angle M = 41^\circ$ (Base angles of isosceles triangles are congruent.)

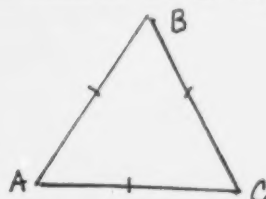


$$m\angle N = 180 - (41 + 41)$$

$$m\angle N = 180 - 82 = 98^\circ$$

An equilateral triangle is a triangle with three congruent sides.

Example:



Find the measure of $\angle A$, $\angle B$, $\angle C$

If $AB = BC$, then $\angle A \cong \angle C$

If $BC = AC$, then $\angle A \cong \angle B$

Therefore $\angle A \cong \angle B \cong \angle C$.

An equilateral triangle is also equiangular.

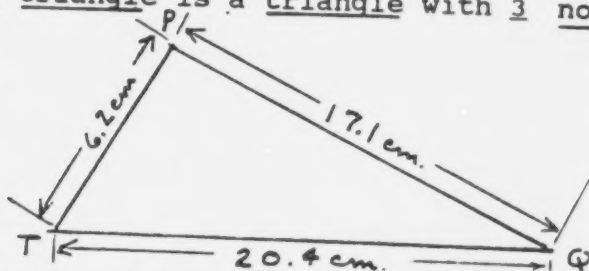
$$\text{Since } m\angle A + m\angle B + m\angle C = 180$$

$$m\angle A = 180 \div 3 = 60$$

$$m\angle A = m\angle B = m\angle C = 60^\circ$$

A third triangle to be considered is the scalene triangle.

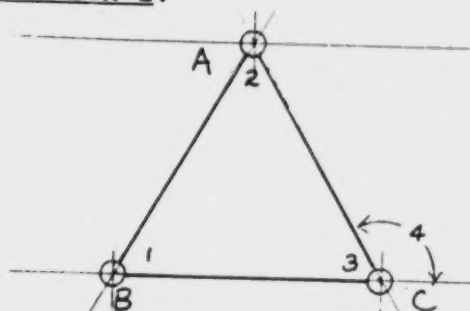
A scalene triangle is a triangle with 3 non-congruent sides.



$\triangle PTQ$ is scalene since $\overline{PT}$, $\overline{PQ}$, and $\overline{TQ}$ are not congruent.

In summary, the classification of triangles according to the length of the sides:

<u>Name</u>	<u>Relationship</u>
Scalene	3 non-congruent sides
Isosceles	2 congruent sides
Equilateral	3 congruent sides

Application 1.

Three spot welds are to be programmed for points A, B, and C.
 Given: $BC=AB=AC=17.1\text{cm}$.
 Find $m\angle 1$, $m\angle 2$, $m\angle 3$, $m\angle 4$.

Solution:

$\triangle ABC$ is equilateral, therefore

$$m\angle 1 = m\angle 2 = m\angle 3$$

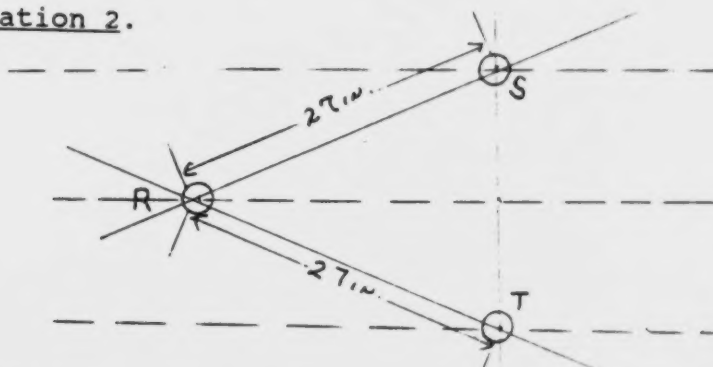
$$m\angle 1 + m\angle 2 + m\angle 3 = 180$$

$$m\angle 1 = 180 \div 3 = 60$$

$$m\angle 2 = m\angle 3 = 60$$

$$m\angle 4 = 180 - m\angle 3 = 180 - 60 = 120^\circ \text{ (Sum } \angle\text{s Theorem).}$$

or $m\angle 4 = m\angle 2 + m\angle 1 = 60 + 60 = 120^\circ \text{ (Ext. Angle Theorem).}$

Application 2.

In the drawing above $RS = ST$.

If $m\angle R = 44^\circ$, find $m\angle S$, $m\angle T$.

Solution: $\angle S$ and $\angle T$ are base angles of isosceles $\triangle TRS$.
 Therefore $\angle S \cong \angle T$.

$$\begin{aligned} m\angle S + m\angle T &= 180 - m\angle R \\ &= 180 - 44 = 136 \end{aligned}$$

$$m\angle S = 136/2 = 68^\circ$$

$$m\angle T = m\angle S = 68^\circ$$

Drill and Practice:

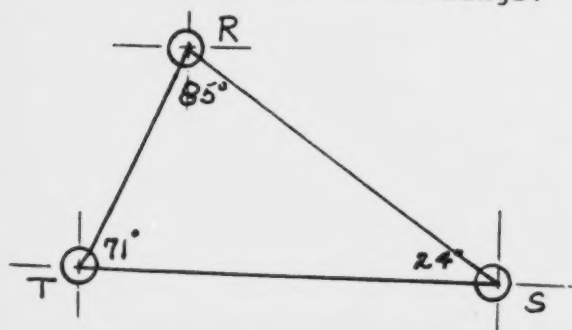
Rising, p. 187

Dressler, p. 331-333

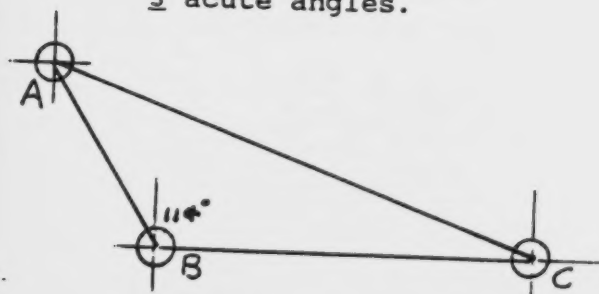
Smith, p. 220 (10-14)

11. CONCEPT: TRIANGLES CLASSIFIED ACCORDING TO ANGLES

Consider three drawings:



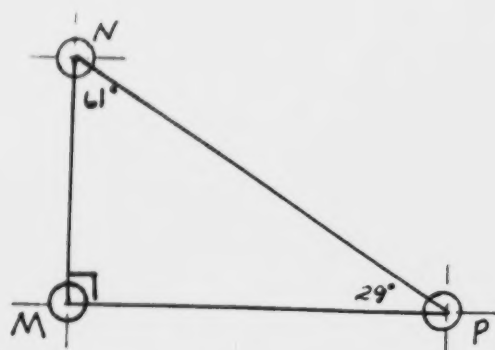
Note that all three angles are acute.
An acute triangle is a triangle with 3 acute angles.



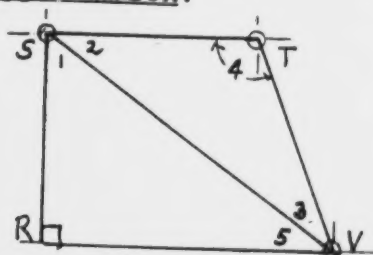
In ABC , $m\angle B = 114^\circ$.
Therefore $\angle B$ is obtuse.
An obtuse triangle is a triangle with one obtuse angle.

Note:

The equilateral triangle can also be classified as an equiangular triangle since all three angles are congruent.



Note that $m\angle M = 90^\circ$.
(The square at the vertex indicates a right triangle.)
A right triangle is a triangle with one right angle.
 $\angle N$ and $\angle P$ are the acute angles of the right triangle.
Note that $\angle N$ and $\angle P$ are complementary ($m\angle N + m\angle P = 90^\circ$)
The acute angles of a right triangle are complementary. The longest side NP is called the hypotenuse.
 NM and MP , the two shorter sides are called legs.

Application.

Three bolt holes are to be drilled through a piece of metal at points S, T, and V.

Given: $\overline{ST} \parallel \overline{RV}$ and

$$m\angle SRV = 90^\circ$$

$$m\angle 1 = 53^\circ, m\angle 3 = 40^\circ$$

Find $m\angle 2$, $m\angle 4$, $m\angle 5$

Solution: $m\angle TSR = 90^\circ$, since $\angle SRV = 90^\circ$ and $\overline{ST} \parallel \overline{RV}$.

Therefore, $m\angle 2 = 90 - m\angle 1$

$$m\angle 2 = 90 - 53 = 37^\circ$$

In $\triangle STV$, $m\angle 4 = 180 - (37 + 40)$

$$m\angle 4 = 180 - 77 = 103^\circ$$

In $\triangle SRV$, $m\angle 5 = 90 - m\angle 1 = 90 - 53 = 37^\circ$

What kind of a triangle is $\triangle SRV$? Right.

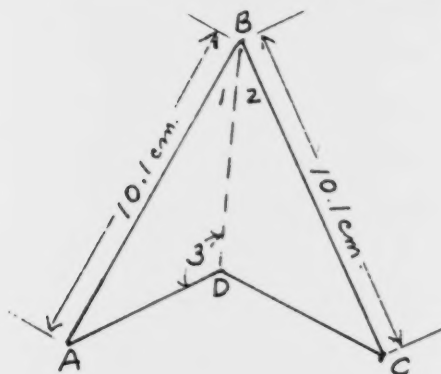
What kind of a triangle is $\triangle STV$? Obtuse.

Drill and Practice:

Rising, p. 157-158

Smith, p. 220-221 (10-14)

12. CONCEPT: THE USE OF THE STATEMENT: CORRESPONDING PARTS OF CONGRUENT TRIANGLES ARE CONGRUENT



A print often gives a minimum of information to a technician or operator. Consider this drawing.

Given:

$$AB = 10.1\text{cm}$$

$$BC = 10.1\text{cm}$$

BD bisects $\angle CBD$

$$m\angle 1 = 17^\circ \quad m\angle 3 = 141^\circ$$

Find $m\angle C$.

Solution: We may assume that $m\angle A = m\angle C$. But are we certain? We can be certain if we can identify congruent triangles by one of three methods: SSS, SAS or ASA.

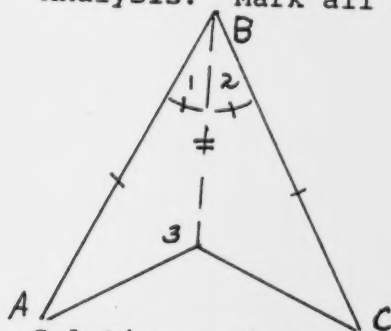
Analysis: Mark all known pairs of $\cong$ parts on the diagram.

$$AB = BC \text{ (Both measure } 10.1\text{cm)}$$

$$BD = BD \text{ (Shared by both } \triangle)$$

$$m\angle 1 = m\angle 2 \text{ (Definition of angle bisector)}$$

Therefore $\triangle ABD \cong \triangle CBD$ (S.A.S.)
and $\angle A \cong \angle C$ (Corresponding parts of $\cong \triangle$ are $\cong$.)



Solution: Since the two triangles are congruent

$$m\angle A = m\angle C$$

$$\text{so if } m\angle A = 180 - (m\angle 1 + m\angle 3)$$

$$= 180 - (17 + 141)$$

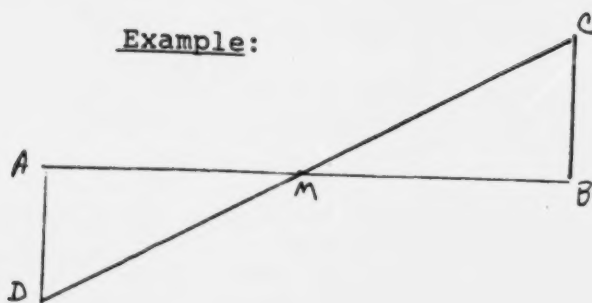
$$m\angle A = 180 - 158$$

$$m\angle A = 22^\circ$$

$$\text{then } \angle C = 22^\circ$$

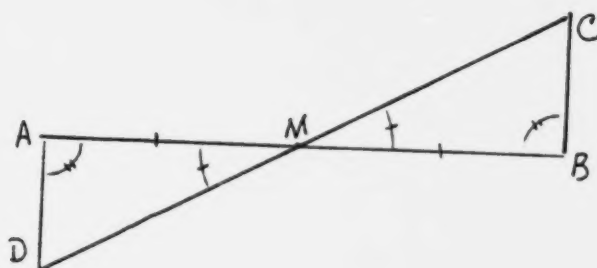
Rule: The corresponding parts of congruent triangles are congruent.

Example:



Given: $\overline{CD}$ and $\overline{AB}$
 intersect
 at M. M is
 the midpoint
 of $\overline{AB}$.
 $\angle A \cong \angle B$. If
 $m\angle D = 37^\circ$
 Find $m\angle C$.

Analysis: Are the triangles congruent?
 Mark congruent parts.



Yes: ASA since $\angle 1 \cong \angle 2$, (Vertical angles
 are congruent)

$\overline{AM} \cong \overline{MB}$ (Definition of
 midpoint)

$\angle A \cong \angle B$ (Given)

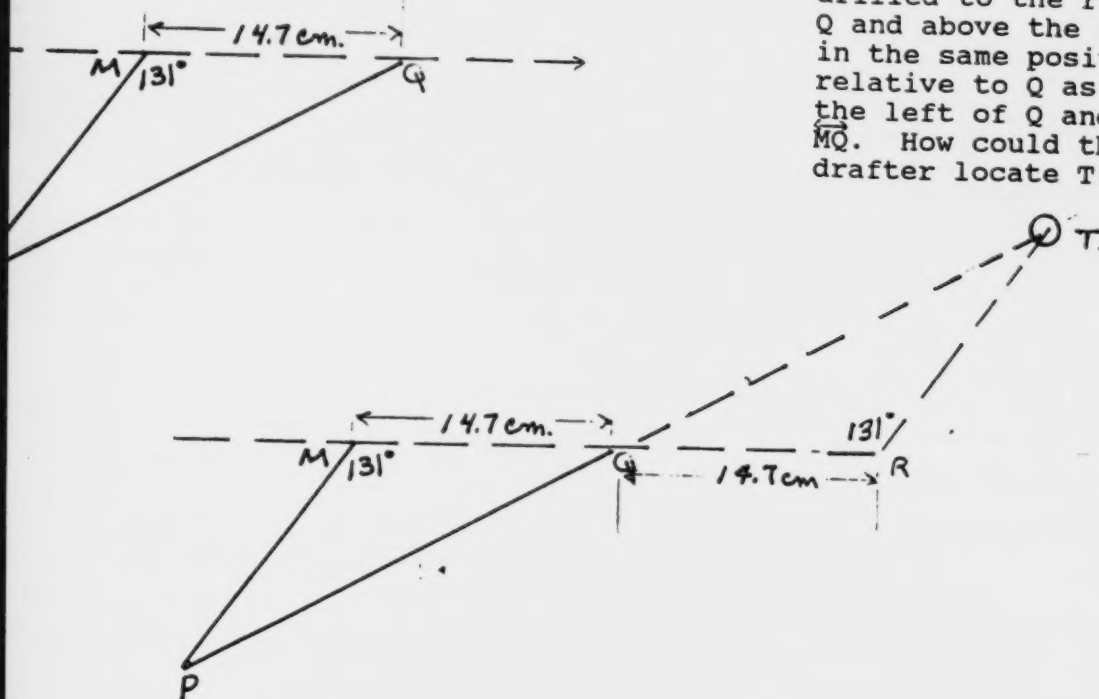
Solution: Since $\triangle DMA \cong \triangle CMB$
 Then $\angle D \cong \angle C$ (Corresponding parts of
 congruent triangles)

If $m\angle D = 37^\circ$
 Then $m\angle C = 37^\circ$

Application.

T (?)

A hole at T is to be drilled to the right of Q and above the line $\overline{MQ}$ in the same position relative to Q as P is to the left of Q and below $\overline{MQ}$. How could the drafter locate T?

Solution:

1. On $\overline{MQ}$ mark off $\overline{QR} = 14.7\text{cm}$.
2. Copy $\angle M = 131^\circ$ at R.
3. Extend $\overline{PQ}$ until it intersects the side of $\angle R$ at T.

The drafter has actually used the method ASA to produce two congruent triangles.

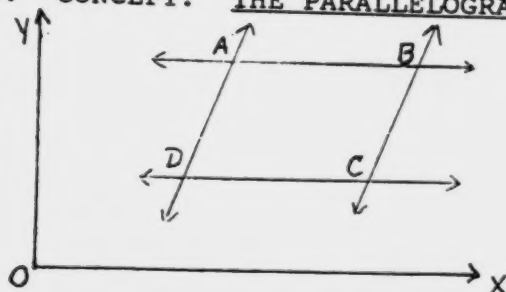
[$\overline{MQ} = \overline{QR}$, $\angle MQP = \angle TQR$ (Vertical Angles), and $\angle M = \angle R$]
 Now $\overline{PQ} \cong \overline{QT}$; corresponding parts of congruent triangles are congruent and T has the same position relative to Q as does P.

Drill and Practice:

Rising, p. 181-183

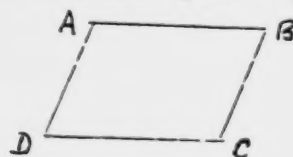
Smith, p. 220-221

13. CONCEPT: THE PARALLELOGRAM AND ITS PROPERTIES



Two sets of parallel lines are drawn on a page of coordinate paper to develop instructions for an automated drill. That is, $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$.

The figure formed by segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ is called a parallelogram. Symbol ($\square$ ABCD).



$\overline{AB}$ and $\overline{CD}$ are called opposite sides.

$\overline{BC}$ and $\overline{DA}$ are also opposite sides.

Pairs of opposite angles are $\angle A$ and $\angle C$, $\angle B$ and $\angle D$.

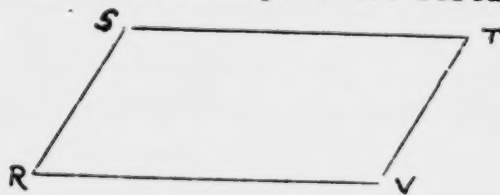
Pairs of consecutive angles are $\angle A$, $\angle B$
 $\angle B$, $\angle C$
 $\angle C$, $\angle D$
 $\angle D$, $\angle A$

Pairs of consecutive sides are $\overline{AB}$, $\overline{BC}$
 $\overline{BC}$, $\overline{CD}$
 $\overline{CD}$, $\overline{AD}$
 $\overline{AD}$, $\overline{AB}$

Definition:

A parallelogram is a quadrilateral with two pairs of opposite sides parallel (symbol $\square$).

Every parallelogram has certain properties:

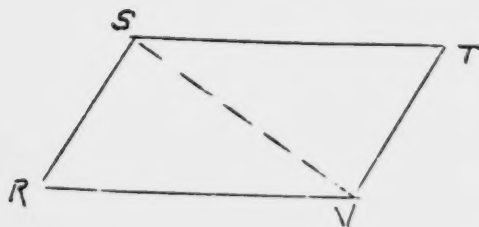


Given: $\square$ RSTV

From the definition

$$\overline{RS} \parallel \overline{TV}, \quad \overline{ST} \parallel \overline{RV}$$

The opposite sides of a parallelogram are parallel.



Now draw a diagonal $\overline{SV}$ (broken line). This produces 2 congruent triangles.

Why? $\angle 1 = \angle 2$ (Alt. Int. $\angle$ 2 of lines)
 $\angle 3 = \angle 4$ (" " " ")

$SV = SV$ (Identity)

$$\triangle SVR \cong \triangle STV$$

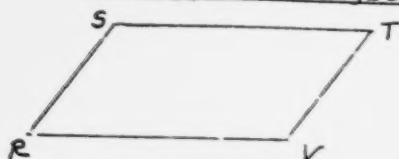
We have shown that a diagonal divides a parallelogram into two congruent triangles.

Therefore, $SR = TV$, $ST = RV$.

The opposite sides of a parallelogram are congruent.

Also, $\angle R \cong \angle T$ and $\angle S \cong \angle V$ (Draw RT instead of SV).

The opposite angles of a parallelogram are congruent.

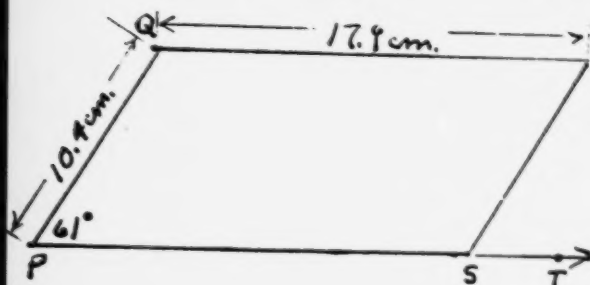


Since $\overline{ST} \parallel \overline{RV}$, $\angle R$ and $\angle S$ are supplementary. This is true for $\angle S$ and $\angle T$, etc.

The consecutive angles of a parallelogram are supplementary.

Example:

Given: $\square PQRS$, $m\angle P = 61^\circ$, $PQ = 10.4\text{cm}$
 $QR = 17.9\text{cm}$. Exterior angle RST .



Given this information, what else do we know?

$$\angle Q = 180 - m\angle P = 180 - 61 = 119^\circ$$

$$m\angle R = m\angle P = 61^\circ$$

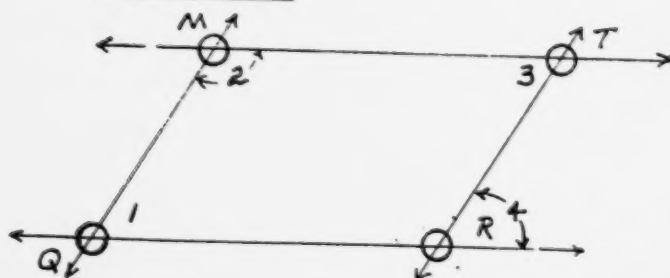
$$m\angle PSR = m\angle Q = 119^\circ$$

$$PS = QR = 17.9\text{cm}$$

$$RS = PQ = 10.4\text{cm}$$

$$m\angle RST = 180 - \angle PSR = 180 - 119 = 61^\circ$$

Application.



Given: Centers for holes M , T , Q and R fall on $\overline{MT}$, $\overline{MQ}$, $\overline{RT}$ and $\overline{QR}$.

$\overline{MT} \parallel \overline{QR}$, $\overline{TR} \parallel \overline{MQ}$

$MT = 17.3$ inches

$m\angle 1 = 71^\circ$

Find: $m\angle 3$
 $m\angle 4$
 $m\angle 2$

Solution: $QMTR$ is a $\square$

Therefore $m\angle 1 = m\angle 3$

$$m\angle 3 = m\angle 1 = 71^\circ$$

$m\angle 4 = m\angle 3$ (Alt. Int. $\angle$ s of $\parallel$ lines)

$$m\angle 4 = 71^\circ$$

$$m\angle 2 + m\angle 1 = 180$$

$$m\angle 2 = 180 - 71 = 109^\circ$$

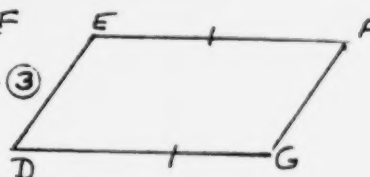
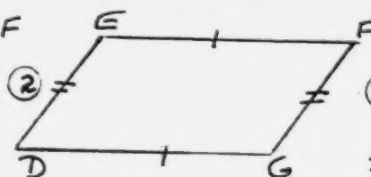
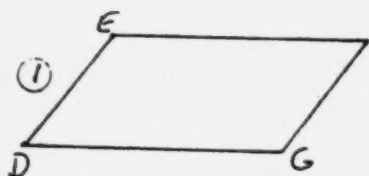
Drill and Practice:

Rising, p. 190, (1, 2, 3, 9-14), 191

14. CONCEPT: METHODS OF RECOGNIZING WHEN A QUADRILATERAL IS A PARALLELOGRAM

A quadrilateral is a parallelogram:

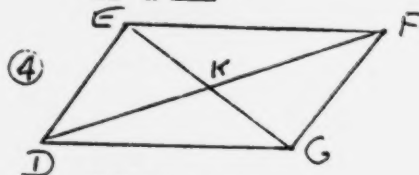
1. when its opposite sides are parallel.
2. when its opposite sides are congruent.
3. when one pair of sides are parallel and congruent.
- or 4. when its diagonals bisect each other.



If $\overline{EF} \parallel \overline{DG}$
and $\overline{ED} \parallel \overline{FG}$
then DEFG
is a

If $EF = DG$
and $DE = FG$
then DEFG
is a

If $\overline{EF} \parallel \overline{DG}$
and $EF = DG$
then DEFG
is a



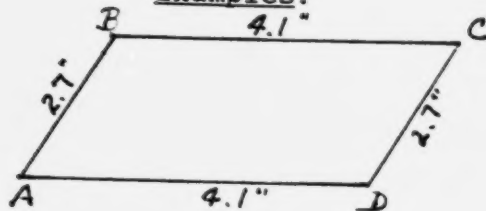
If $EK = KG$
and $DK = KF$
then DEFG is a
parallelogram.

#1 depends upon the definition
of a

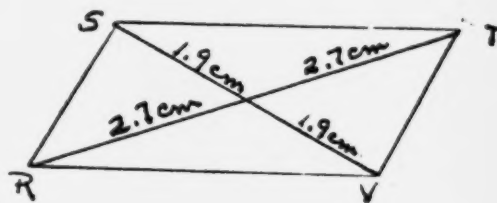
#2, #3, and #4 can be proven by
proofs provided in any text
on geometry.

We will not attempt the proofs
here.

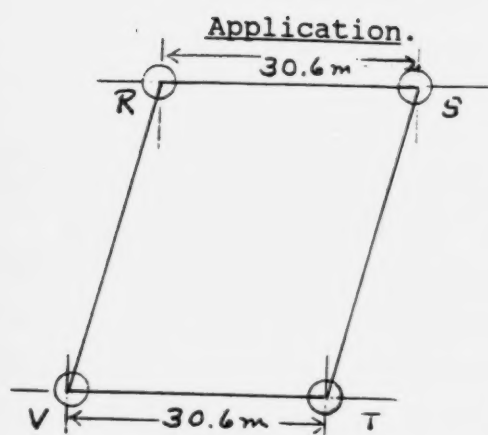
Examples:



Is ABCD a
parallelogram?
Yes. (Opposite sides
are $\cong$).



Is RSTV a
parallelogram?
Yes. (Diagonals
bisect each other.)



If $\overline{RS} \parallel \overline{TV}$,
 is RSVT a ?
 Yes. Since $RS = TV$
 and $\overline{RS} \parallel \overline{TV}$.

Drill and Practice:

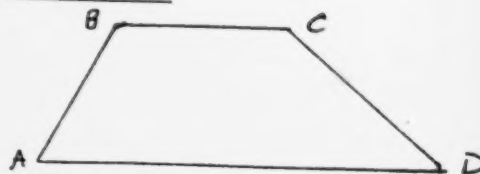
Rising, p. 190 (4-9), 191 (1-6)

15. CONCEPT: THE TRAPEZOID AND ITS PROPERTIES

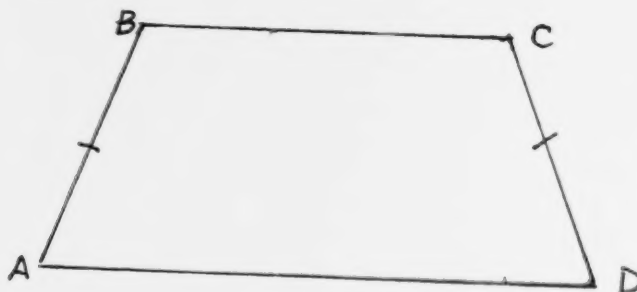
A piece of sheet metal was cut in the shape of a trapezoid.

A trapezoid is a quadrilateral with one and only one pair of parallel sides. Therefore, $\overline{BC} \parallel \overline{AD}$.

$\overline{BC}$ and $\overline{AD}$ are called the bases; $\overline{AB}$ and $\overline{CD}$, the two non-parallel sides are called the legs. $\angle A$ and $\angle D$ are called the base angles. Since $\overline{BC} \parallel \overline{AD}$, then $\angle A$ and $\angle B$ are supplementary. The same relationship is true for $\angle C$ and $\angle D$.

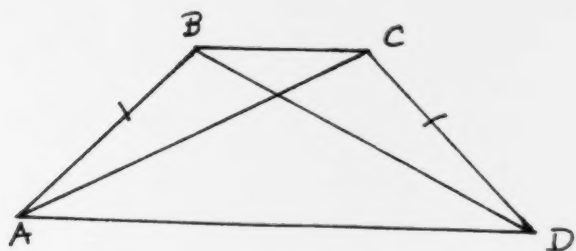


If the legs $\overline{AB}$ and $\overline{CD}$ are congruent, the trapezoid is an isosceles trapezoid.

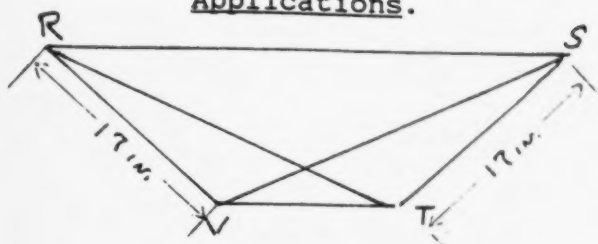


It can be shown that in an isosceles trapezoid, the base angles A and D are congruent.

Two diagonals can be drawn. In the isosceles trapezoid, the diagonals are congruent.

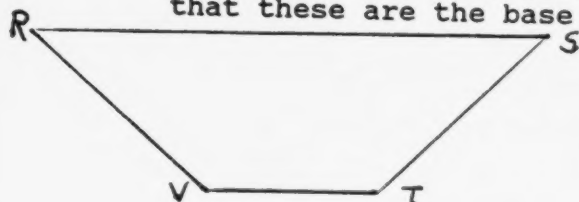


If trapezoid ABCD is isosceles, then $AC = BD$.

Applications.

Trapezoid RSTV represents the cross section of a hopper which is to be fabricated from sheet metal.

Since $RV = ST = 17$ in., RSTV is an isosceles trapezoid and therefore we can conclude that $RS \parallel VT$, $\angle RVT = \angle STV$, (Note that the trapezoid can "sit" on its shorter base), $RT = VS$ (Diagonals are congruent) and $\angle VRS$ is congruent to $\angle RST$ (Assume that these are the base angles).



Therefore, given:
 trapezoid RSTV, $RV = ST = 17$ in.
 $RS \parallel VT$, $m\angle V = 130^\circ$.
 Find $m\angle T$, $m\angle R$, $m\angle S$.

Solution: If $RV = ST = 17$ in., RSTV is isosceles
 $m\angle T = m\angle V = 130^\circ$.

$$\begin{array}{rcl} \text{Since } RS \parallel VT, m\angle V + m\angle R & = & 180 \\ 130 + m\angle R & = & 180 \\ -130 & & -130 \\ \hline m\angle R & = & 50^\circ \end{array}$$

$$\begin{array}{l} m\angle R = m\angle S \text{ (Base } \angle\text{s)} \\ \angle m\angle S = m\angle R = \underline{50^\circ} \end{array}$$

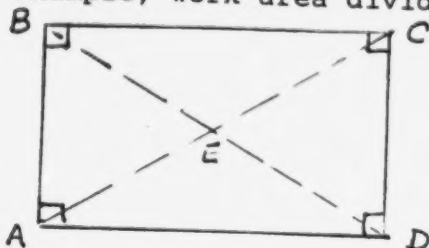
Drill and Practice:

Rising, p. 195 (19), 196 (2)

Dressler, p. 339

16. CONCEPT: THE RECTANGLE, THE RHOMBUS AND THE SQUARE, AND THEIR PROPERTIES

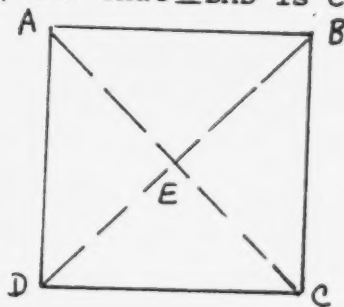
A rectangle is a commonly experienced quadrilateral; for example, work area dividers, wall charts, desk tops.



A rectangle is a parallelogram with four right angles.

Properties of a rectangle (Remember a rectangle is a parallelogram)

1. The opposite sides of a rectangle are parallel.
 $\overline{BC} \parallel \overline{AD}$ and $\overline{BA} \parallel \overline{CD}$
2. The opposite sides of a rectangle are congruent.
 $AB = CD$, $BC = AD$
3. The angles of a rectangle = 90°
4. The diagonals of a rectangle bisect each other.
 $BE = ED$, $EC = AE$
5. The diagonals of a rectangle are congruent.
(Show that $\triangle BAD$ is congruent to $\triangle CDA$.)



A square is a rectangle with four congruent sides.

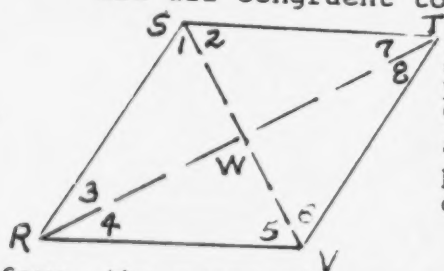
This means that a square has all the properties of a rectangle plus a few of its own.

Therefore:

The properties of a square (Remember a square is a rectangle)

1. The opposite sides of a square are parallel.
 $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$
2. The opposite sides of a square are congruent.
 $AB = DC$, $AD = BC$
3. A square has four right angles.
 $\angle A = \angle B = \angle C = \angle D = 90^\circ$

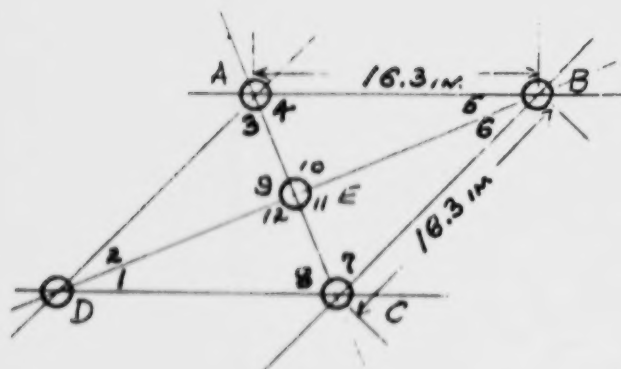
4. The diagonals of a square are congruent.
 $AC = DB$
5. The diagonals of a square bisect each other.
 $AE = EC, DE = EB$
- and 6. The diagonals of a square are perpendicular to each other.
 $AC \perp DB$
(Show that $\triangle DEA, \triangle AEB, \triangle BEC, \triangle DEC$ are all congruent to each other.)



A rhombus is a parallelogram with four congruent sides.
This means that a rhombus has all the properties of a parallelogram plus a few of its own.

Therefore, the properties of a rhombus (Remember a rhombus is a parallelogram)

1. The opposite sides of a rhombus are parallel.
 $\overline{ST} \parallel \overline{RV}, \overline{SR} \parallel \overline{TV}$
2. The opposite sides of a rhombus are congruent.
 $ST = RV, SR = TV$
3. The opposite angles of a rhombus are congruent.
 $m\angle RST = m\angle RVT, m\angle SRV = m\angle STV$
4. Diagonals of a rhombus bisect each other.
 $SW = WV, RW = WT$
5. Diagonals of a rhombus are perpendicular to each other.
 $\overline{SV} \perp \overline{RT}$
6. The diagonals of a rhombus bisect the angles of the rhombus.
 $m\angle 1 = m\angle 2, m\angle 3 = m\angle 4, m\angle 5 = m\angle 6, m\angle 7 = m\angle 8$

Application.

The instructions to an automated drill press indicate that holes are to be drilled at A, B, C, D and E. $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$. $m\angle 1 = 23^\circ$

From this information what more can we conclude from this drawing?

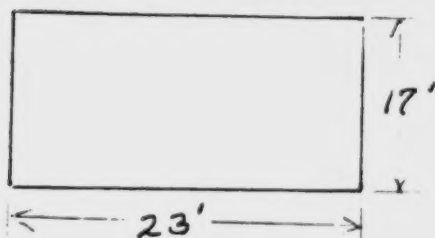
- Conclusions:
- 1) ABCD is a parallelogram
 - 2) $AD = BC = 16.3$, $AB = DC = 16.3$
 - 3) ABCD is a rhombus
 - 4) Since $\angle 1 = 23^\circ$, then $m\angle 2 = 23^\circ$ and $m\angle ADC = 2 \cdot (23) = 46^\circ$
 - 5) $m\angle DAB = 180 - 46^\circ = 134^\circ$
 - 6) $m\angle 3 = (1/2) \cdot (134) = 67^\circ$, and $\angle 4 = 67^\circ$
 - 7) $\overline{AC} \perp \overline{BC}$, then $\angle$ s 9, 10, 11, 12 are rt. $\angle$.

Drill and Practice:

Rising, p. 195-196

Dressler, p. 338 (11-18)

17. CONCEPT: THE AREA OF A RECTANGLE AND THE AREA OF A SQUARE



In the shape of a rectangle, a work area with the dimensions shown in the diagram is to be covered with a special tile. Find the area of this rectangle.

Most of us are familiar with the formula for the area of a rectangle.

$$A = lw$$

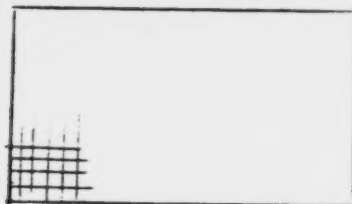
Where A = area
(square units)
 l = length
 w = width

Therefore

$$A = 23 \times 17$$

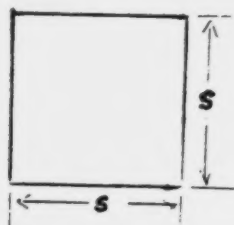
$$A = 391 \text{ square feet}$$

Actually what we are doing is counting the number of square units making up the area of the rectangle.



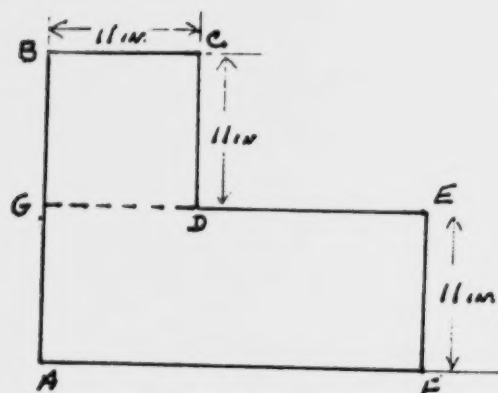
The area of a rectangle equals the product of its length and width.
 $A = lw.$

A square is a rectangle, so its area can also be calculated from $A = lw.$



However, all sides of a square are congruent, therefore $A = s \cdot s$
or $A = s^2$ where
 s = side of the square.

The area of a square equals the square of a side.

Application.

The piece of metal is shown here with the dimensions as indicated. Find the area of this fixture.
Note that $\overline{GD}$ has been drawn.

Solution: The total area is made up of Square GBCD and Rectangle GEFA.

Total Area = Area GBCD + Area GEFA

Total Area = $s^2 + lw$

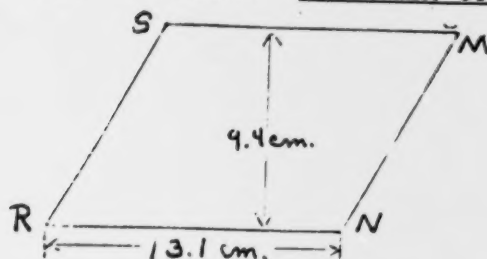
Total Area = $11^2 + 24 \cdot 11$

Total Area = $121 + 264 = 385$ square inches.

Drill and Practice:

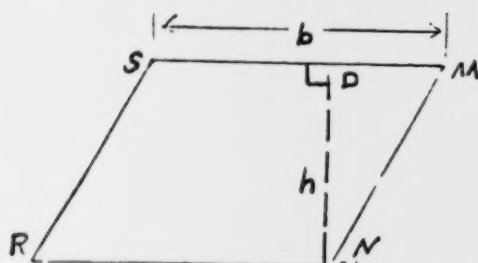
Rising, p. 199 (1, 2, 3), 200 (1-8),
201 (23, 29, 33, 34), 202 (36)

18. CONCEPT: THE AREA OF A PARALLELOGRAM



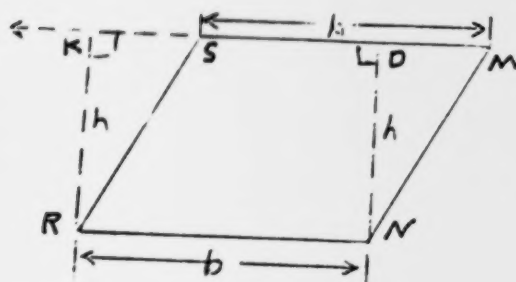
Given: Parallelogram SMNR.

Find: The area of the parallelogram.



Any side of the parallelogram can be considered the base (b). If we consider $\overline{SM}$ the base, we draw a line segment $\overline{ND}$ through N perpendicular to $\overline{SM}$, $\overline{ND}$ is called an altitude of the parallelogram.

An altitude of a parallelogram is a line segment drawn perpendicular to a line containing any base from a point on the opposite side. (Sometimes the base must be extended. The altitude is often labeled h , its height.)



If we extend $\overline{SM}$ and draw $\overline{RK} \perp \overline{SM}$ at K , we produce another altitude $\overline{RK}$ congruent to altitude $\overline{DN}$.

Now: $\triangle KSR \cong \triangle DMN$

Then: Area $\triangle KSR$ = Area $\triangle DMN$

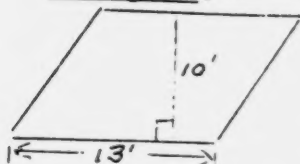
$KDNR$ is a rectangle with base = b and altitude = h .

Area $\triangle KSR$ + Area $SDNR$ = Area $SDNR$ + Area $\triangle DMN$

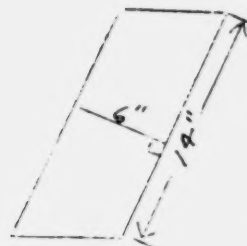
Then area rectangle $KDNR$ = area parallelogram $SMNR$.

So if area rectangle = $b \times h$

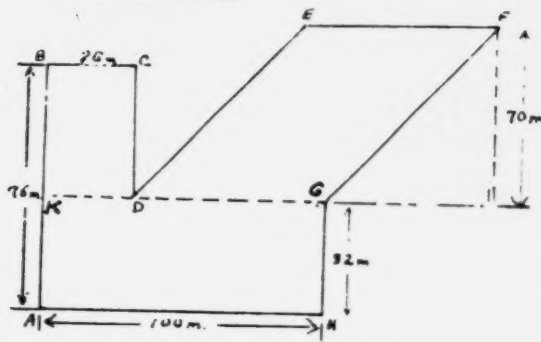
Then area parallelogram = $b \times h$.

Examples:

Given: parallelogram
 altitude = 10 feet
 base = 13 feet
 Find area.
 $A = bh$
 $A = 13 \cdot 10$
 $A = 130$ square feet



Given: parallelogram
 altitude = 6"
 base = 14"
 Find area.
 $\text{area} = b \cdot h$
 $\text{area} = 14 \cdot 6$
 $\text{area} = 84$ square inches

Application.

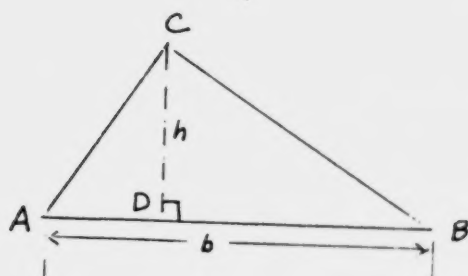
Find area of this
 irregular floor
 space.

Total area = Area BCDK + Area KGHA + Area EFGD
 = Rectangle + Rectangle + Parallelogram
 $= 26 \cdot (75 - 32) + 32 \times 100 + DG \cdot 70 *$
 $= 1118 + 3200 + 74 \cdot 70$
 $= 1118 + 3200 + 5180$
 Total area = 9498 square meters.

$$* DG = 100 - 26 = 74 \text{ m.}$$

Drill and Practice:

Rising, p. 199 (5-10), 200 (9-14) (19-22,
 24, 25), 202 (35)

19. CONCEPT: THE AREA OF A TRIANGLE

The area of a triangle equals one-half the product of the length of its base (b) and the length of its altitude (h) drawn to the line containing the base.

$$A = \left(\frac{1}{2}\right)bh$$

where A = Area

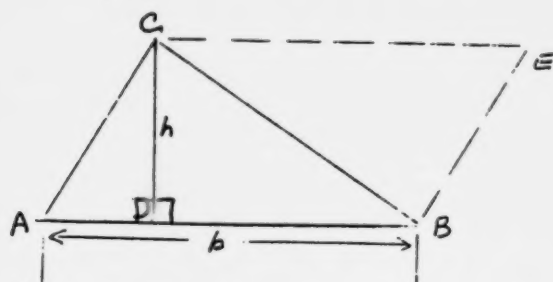
b = Length of Base

h = Length of Altitude

Any side of a triangle can be considered a base. In this diagram $\overline{AB}$ is the base.

An altitude of a triangle is a line segment drawn from a vertex perpendicular to the opposite side. $\overline{CD} \perp \overline{AB}$.

Any triangle can be expanded to be part of a parallelogram.



Through C draw $\overline{CE} \parallel \overline{AB}$.

Through B draw $\overline{BE} \parallel \overline{AC}$.

Then $ACEB$ is a parallelogram.

The area of $ACEB = bh$.

Area $\triangle ACB = \left(\frac{1}{2}\right) \cdot \text{Area } ACEB$.

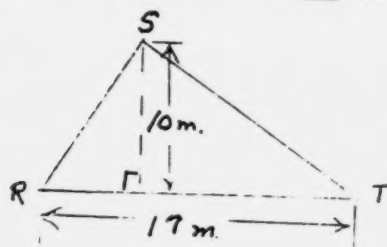
(Congruent triangles)

Therefore,

$$\text{Area } \triangle ACB = \left(\frac{1}{2}\right)bh.$$

Examples:

Find the area of $\triangle RST$.



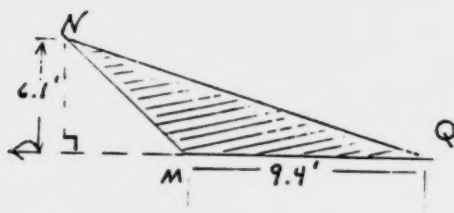
Find the area of MNQ .

$$A = \left(\frac{1}{2}\right)bh$$

$$A = \left(\frac{1}{2}\right) \cdot 17 \cdot 10$$

$$A = 170/2 = 85 \text{ square meters.}$$

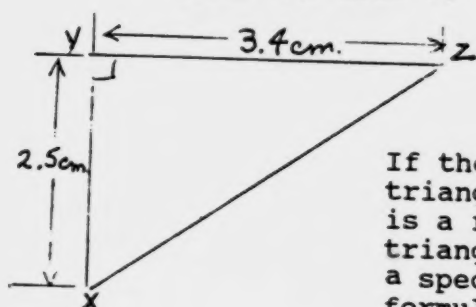
Find the area of $\triangle MNQ$.



Note: If $\overline{MQ}$ is considered the base (obtuse triangle), side $\overline{MQ}$ must be extended in order to draw the altitude perpendicular to the line of the base.

$$\begin{aligned}\text{Area } \triangle MNQ &= (1/2) \cdot b \cdot h \\ &= (1/2)(9.4)(6.1) \\ &= (1/2)(57.34) \\ &= 28.67 \text{ square feet}\end{aligned}$$

Find the area of $\text{Rt. } \triangle XYZ$, where $\angle Y$ is a right angle. Consider $\overline{YZ}$ as the base.



If the triangle is a right triangle, a special formula can be used.

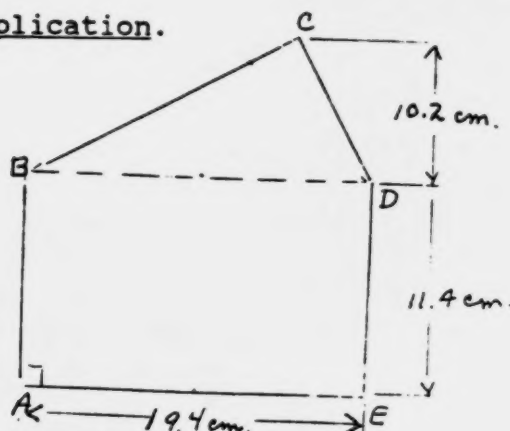
This makes $\overline{YX}$ the altitude.

Therefore,

$$\begin{aligned}\text{Area } \triangle XYZ &= (1/2) \cdot b \cdot h \\ &= (1/2)(3.4)(2.5) \\ &= (1/2)(8.50) \\ &= 4.25 \text{ square cm.}\end{aligned}$$

$$\text{Area right triangle} = (1/2) \cdot (\text{LEG}) \cdot (\text{LEG})$$

Application.



Find the total area if ABDE is a rectangle.

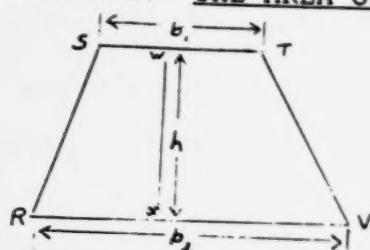
$$\begin{aligned}\text{Total Area} &= \text{Area Rect.} + \text{Area Triangle.} \\ &= \text{Area ABDE} + \text{Area } \triangle BCD \\ &= bh + (1/2) \cdot bh \\ &= (19.4)(11.4) + (1/2)(19.4)(10.2)\end{aligned}$$

Note: $\triangle$ has same base as rectangle.

$$\begin{aligned}&= 221.16 + 98.94 \\ \text{Total area} &= 320.1 \text{ square centimeters}\end{aligned}$$

Drill and Practice:

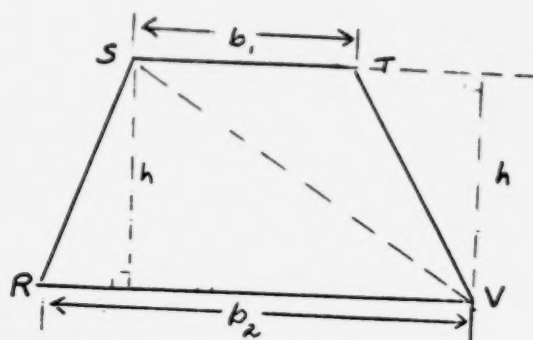
Rising, p. 204-205 (oral 1-7),
205 (1-6, written), 206 (13-16)
Dressler, p. 70 (5)

20. CONCEPT: THE AREA OF A TRAPEZOID

If $\overline{RV} \parallel \overline{ST}$, then $\overline{RV}$ and $\overline{ST}$ are the bases of the trapezoid. h is the length of the altitude WX which is a line segment drawn from a point on one base perpendicular to the other base.

Therefore, $\overline{WX} \perp \overline{RV}$

By drawing a diagonal $\overline{SV}$



It can be seen that

Area of trapezoid $RSTV = \text{Area } \triangle RSV + \text{Area } \triangle STV$.

The length of base of $\triangle RSV$ is b_2 , and the length of the base of $\triangle STV$ is b_1 . The altitude of $\triangle RSV$ can be drawn from S and equals h . The altitude of $\triangle STV$ can be drawn from V to $\overline{ST}$ extended and also equals h .

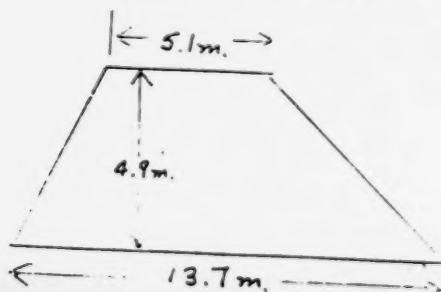
Therefore,

$$\begin{aligned} \text{The area trapezoid } RSTV &= (1/2) \cdot h \cdot b_1 + (1/2) \cdot h \cdot b_2 \\ \text{or area trapezoid} &= (1/2) \cdot h \cdot (b_1 + b_2) \end{aligned}$$

(Distributive Law)

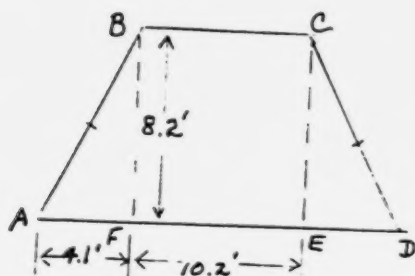
Examples:

Find the area trapezoid.



$$\begin{aligned} \text{Area} &= (1/2) \cdot h (b_1 + b_2) \\ &= (1/2) \cdot (4.9) (5.1 + 13.7) \\ &= (1/2) \cdot (4.9) (18.8) \\ &= 46.06 \text{ square meters} \end{aligned}$$

Find the area of isosceles trapezoid.



It sometimes helps to draw two altitudes which cut off two congruent triangles on either end.

Therefore,

$$AD = AF + FE + ED$$

$$AF = ED = 4.1 \text{ ft.}$$

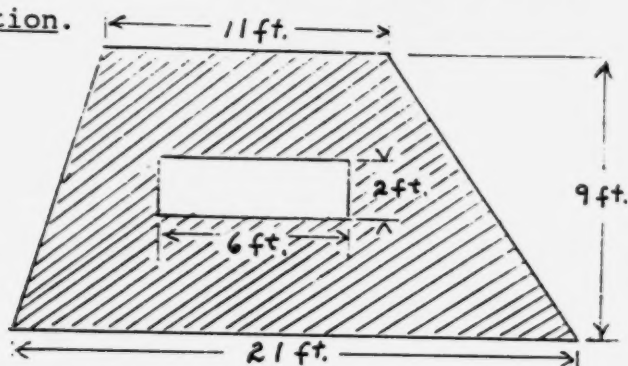
$$AD = 4.1 + 10.2 + 4.1$$

$$AD = 18.4 \text{ ft.} = b_2$$

$$BC = FE = 10.2 = b_1$$

$$\begin{aligned} \text{Then, Area trapezoid} &= (1/2) \cdot h \cdot (b_1 + b_2) \\ &= (1/2) \cdot (8.2) \cdot (10.2 + 18.4) \\ &= (1/2) \cdot (8.2) \cdot (28.6) \\ &= 117.26 \text{ square feet.} \end{aligned}$$

Application.



Several pieces of sheet metal are to be cut into the form in the diagram (with the rectangle removed) and then sprayed with an insulating material. Find the area of the shaded part.

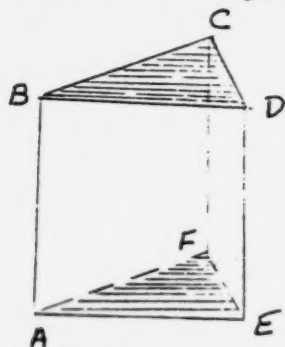
$$\begin{aligned} \text{Shaded area} &= \text{Area trapezoid} - \text{Area rectangle} \\ &= (1/2) \cdot (b_1 + b_2) \cdot h - b \cdot h \\ &= (1/2) \cdot 9 \cdot (11 + 21) - 6 \cdot 2 \\ &= (1/2) \cdot 9 \cdot 32 - 12 \\ &= 144 - 12 = 132 \text{ square feet} \end{aligned}$$

Drill and Practice:

Rising, p. 205 (oral) (8, 9),
205 (written) (7-10), 206 (11, 12,
17-20)

21. CONCEPT: THE VOLUMES OF A PRISM AND A PYRAMID

A prism is a space (or solid) enclosed by two congruent polygons (called bases) in parallel planes and several lateral faces which are parallelograms and which connect the bases. In a right prism each lateral face is a rectangle.



Triangular
Right Prism

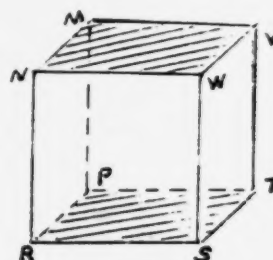
Bases: $\triangle BCD$, $\triangle AFE$

Lateral

Faces: Rectangles
BCFA, CDEF,
BDEA

$\overline{BA}$, $\overline{CF}$, $\overline{DE}$ are called
lateral edges.

In a right prism
 $\overline{BA}$ = altitude.



Rectangular
Right Prism

Bases: Rectangles
NMVW, RPTS

Lateral

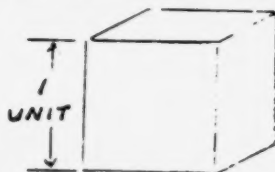
Faces: Rectangles
NMPR, MVTP,
WVTS, NWSR.

$\overline{NR}$, $\overline{MP}$, $\overline{VT}$, and $\overline{WS}$
are lateral edges.

In this right prism
 $\overline{NR}$ is an altitude.

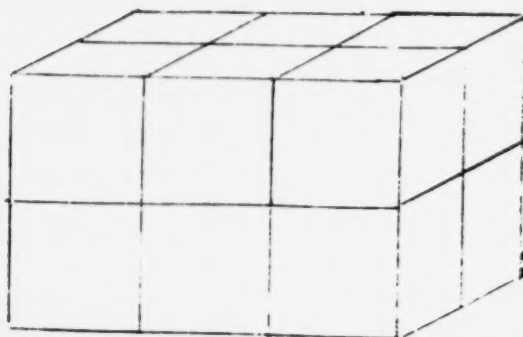
The altitude of a prism is a segment drawn from a point in one base perpendicular to the plane of the other base.

A cube is a prism in which all six faces are congruent squares. The intersection of two faces is an edge.

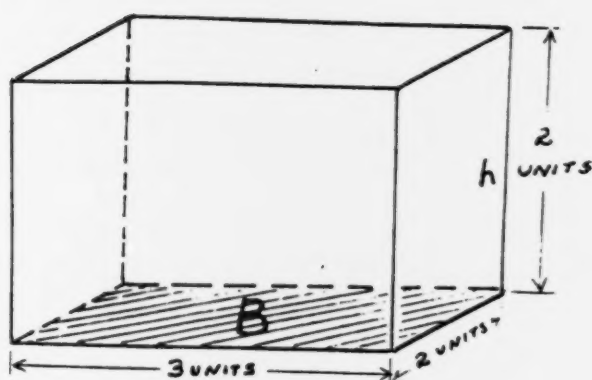


A unit cube is a
cube with length of
an edge equal to
1 unit.

The volume of a unit
cube is 1 cubic unit.



A rectangular prism
with a volume of
12 unit: cubes is
said to have a
volume of 12 cubic
units, or
Volume = $3 \times 2 \times 2 = 12$
cubic units.

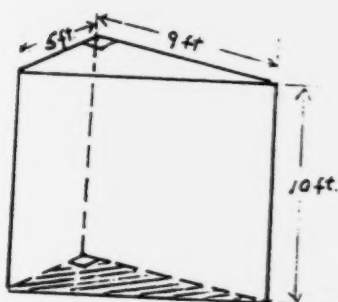


The base of this solid is a rectangle with an area $B = 3 \cdot 2 = 6$. The altitude is the lateral edge with a length of $h = 2$.

Therefore the volume $= 6 \times 2 = 12$.

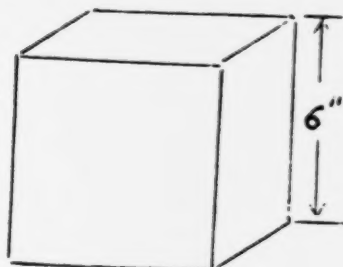
or Volume $= B \cdot h$
Prism where $B =$ Area of Base
and $h =$ altitude

Example:



In the right triangle prism, the bases are right triangles with legs equal to 5 ft. and 9 ft. The altitude is 10 ft.
 $B = \text{area} = \frac{1}{2} \cdot (5)(9) = 22.5$ sq. ft.
Volume $= Bh = (22.5)(10) = 225$ cubic ft.

The volume of a cube $= s^3$ where s is the length of an edge.



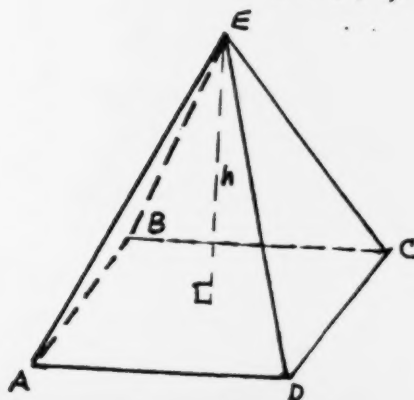
Volume of cube $= s^3$
 $V = 6^3 = \underline{216}$ cubic inches

The volume of a pyramid equals one-third the product of the length of the altitude and the area of the base.

$$V = \frac{1}{3} \cdot Bh$$

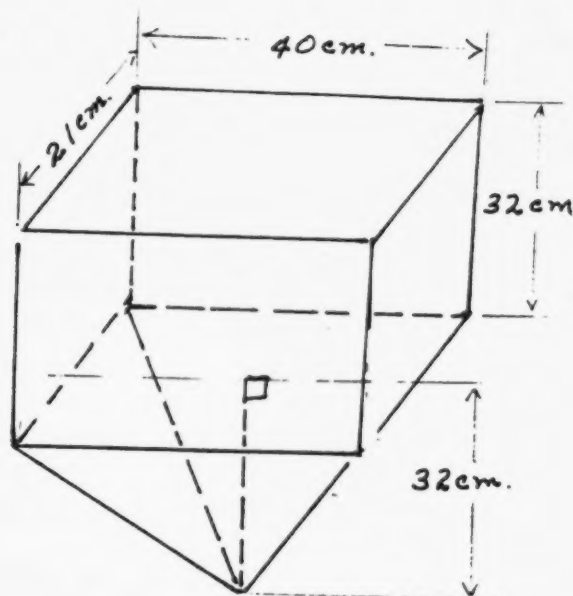
where $B =$ area of base
Find the volume of a square if $h = 32.7$ cm., and the area of the base is 200 sq. cm.

Volume $= \frac{1}{3} \cdot Bh$
 $= \frac{1}{3} \cdot 200 (32.7)$
Volume $= 21,800$ cubic cm.



Application.

119



A hopper is to be fabricated out of sheet metal to the indicated dimensions. Find the volume of the hopper. The hopper is composed of a rectangular prism and a right pyramid.

$$\begin{aligned}\text{Total volume} &= \text{volume prism} + \text{volume pyramid.} \\ &= B \cdot h + \left(\frac{1}{3}\right) \cdot B \cdot h\end{aligned}$$

$$B \text{ for prism and pyramid} = 21 \cdot (40) = 840 \text{ sq. cm.}$$

$$\begin{aligned}\text{Total volume} &= 840 \cdot 32 + \left(\frac{1}{3}\right) \cdot 840 \cdot 32 \\ &= 26,880 + 8960 \\ &= 35,840 \text{ cubic centimeters}\end{aligned}$$

Drill and Practice:

Rising, p. 213-214 (1-20)

**A FURTHER
DEVELOPMENT
OF ALGEBRA**

1. CONCEPT: FACTORS, POWERS, EXPONENTS AND BASES

A work area is to be designed so as to have 36 square yards available for the installation of equipment. Some of the ways in which the area could be laid out are:

$$9 \text{ yd.} \times 4 \text{ yd.}$$

$$6 \text{ yd.} \times 6 \text{ yd.}$$

or 12×3 , or 18×2 or 36×1 or $1/2 \times 72$.

There really isn't any limit to the numbers of sets of factors of 36.

When all or some of the factors of a number are equal we can use exponents.

$$81 = 9 \times 9 = 9^2$$

9 is called the base

2 is called the exponent

81 is called the power

or $27 = 3 \times 3 \times 3 = 3^3$

$$a a b = a^2 b.$$

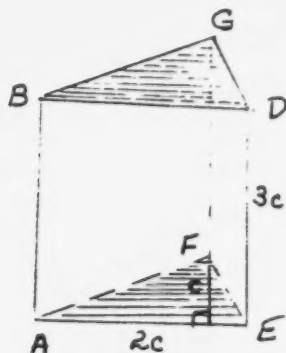
$$7 y y y z z = 7y^3 z^2$$

Evaluate:

$$\left(\frac{1}{3}\right)^4 = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{81}$$

$$\left(5 \frac{1}{2}\right)^2 = \left(\frac{11}{2}\right)^2 = \frac{11}{2} \cdot \frac{11}{2} = \frac{121}{4} = 30 \frac{1}{4}$$

Application.



Write a formula for the volume of this triangular right prism in terms of the letter c .

$$B = \left(\frac{1}{2}\right) b \cdot h$$

$$B = \left(\frac{1}{2}\right) (2c) (c)$$

$$B = \left(\frac{1}{2}\right) \cdot 2c \cdot c$$

$$B = \left(\frac{1}{2}\right) 2 \cdot c \cdot c$$

$$B = \frac{1}{2} \cdot 2 \cdot (c \cdot c)$$

$$B = c^2$$

$$V = Bh$$

$$V = c^2 \cdot 3c$$

$$V = 3 \cdot c^2 \cdot c$$

$$V = 3 \cdot (c^2 \cdot c)$$

$$V = 3c^3$$

Drill and Practice:

Rising, p. 226-228

Similar terms are terms with identical literal coefficients.
 Example: $-14x^2y$ and $9x^2y$.

$32a$ and $-17a$

To simplify a polynomial, combine its similar terms.

$$\begin{aligned} &17a^2 - b + 23a^2 + 4b \\ &= 17a^2 + 23a^2 - b + 4b \end{aligned}$$

$$= \underline{40a^2 + 3b}$$

no terms are similar.
 - stop!!!

The degree of a polynomial is equal to the highest sum of the exponents of the variables in any term.

Example: $x^3 - 6x^2 + 7x - 2$
 $x^2y^3 - 16x^3y^3$

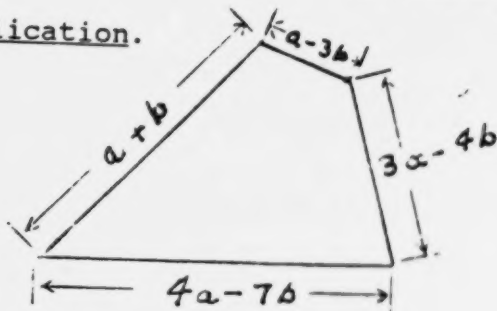
Degree 3.

Degree 6.

A polynomial written with a single variable can be written in standard form, that is, written with the terms in descending order of the exponents.

Example: $2x^2 + 5x^3 - 2 - 7x$ becomes $5x^3 + 2x^2 - 7x - 2$ in standard form.

Application.



Write a formula for the perimeter of this lot in terms of a and b .

$$\text{Perimeter} = a - 3b + a + b + 4a - 7b + 3a - 4b$$

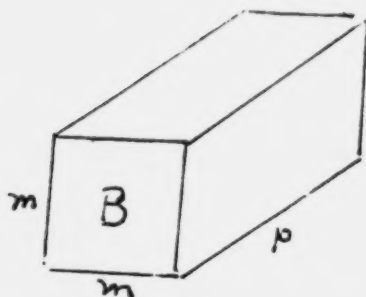
$$\begin{aligned} \text{Perimeter} &= a + a + 4a + 3a - 3b + b + 7b - 4b \\ &= \underline{9a - 13b} \end{aligned}$$

Drill and Practice:

Rising, p. 230-231

Dressler, p. 241

2. CONCEPT: AN UNDERSTANDING OF POLYNOMIALS



$$V = Bh$$

The volume of this rectangular prism can be written using the formula

where
B is the area
of the base
and h = length of
altitude.

$$\text{Therefore } V = m^2 \cdot p$$

$$V = m^2 p$$

Now $m^2 p$ is called a monomial since it is a product of m^2 and p . A monomial is a product of numbers and/or variables.

Examples

Factors

$$7a^2b$$

$$-4a$$

$$2$$

$$0$$

$$\frac{-15x^3y^2z}{4}$$

$$7 \cdot a \cdot a \cdot b$$

$$-4 \cdot a \text{ or } (-2) \cdot 2 \cdot a$$

$$2 \cdot 1$$

$$0 \cdot 1$$

$$\frac{-15 \cdot x^3 \cdot y^2 \cdot z}{4} \text{ or } \frac{-15 \cdot x^3 \cdot y^2 \cdot z}{4} \text{ etc.}$$

Note: $\frac{m}{4}$ is the same as $m \div 4$ or $m \cdot \frac{1}{4}$ or $\frac{1}{4} \cdot m$

In a monomial such as $-5ab$, -5 is called the numerical coefficient or sometimes just the coefficient. A polynomial is the sum of monomials or merely a single monomial. The monomials which are part of a polynomial are called terms.

Examples:

Polynomial

No. of Terms

Terms

$$-4mn^2p$$

$$a + b$$

$$a^2 - 3a + 4$$

$$1$$

$$2$$

$$3$$

$$-4mn^2p$$

$$a, + b$$

$$a^2, -3a, +4$$

Polynomials with one term are called monomials:

Example: $-28r^2s$

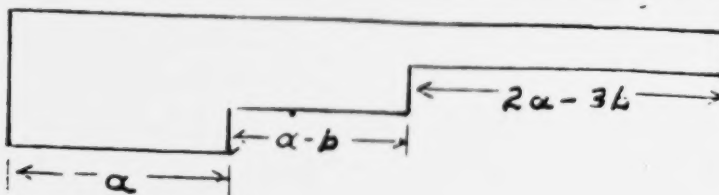
Polynomials with two terms are called binomials:

Example: $5 - 3r^2$

Polynomials with three terms are called trinomials:

Example: $x^2 - 7x + b$

3. CONCEPT: ADDITION AND SUBTRACTION OF POLYNOMIALS



Write a formula for the length of this metal bracket in terms of a and b.

$$\text{Length} = a + a - b + 2a - 3b$$

$$\text{Length} = a + a + 2a - b - 3b$$

$$\text{Length} = 4a - 4b$$

To find the sum, combine similar terms.

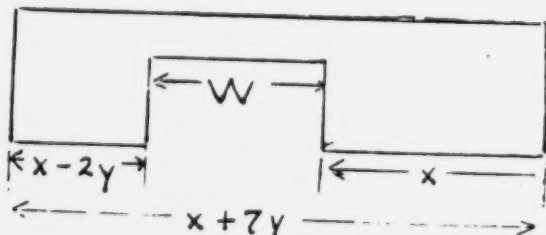
To subtract two polynomials: add the opposite of the subtraend to the minuend.

$$\begin{aligned} \text{Example: } & 9x^2 - 3x + 2 - (4x^2 - 6x + 12) \\ & 9x^2 - 3x + 2 + 4x^2 + 6x - 12 \\ & 9x^2 + 4x^2 - 3x + 6x + 2 - 12 \\ & 13x^2 + 3x - 10 \end{aligned}$$

$$\begin{aligned} \text{Examples: To } 9a + 6b - 3c \text{ add } 13a - 7b - 4c \\ & 9a + 6b - 3c + (13a - 7b - 4c) = \\ & 9a + 13a + 6b - 7b - 3c - 4c = 22a - b - 7c \end{aligned}$$

$$\begin{aligned} \text{From } 5x^2 - 6x + 9 \text{ subtract } 2x^2 - 7x + 1 \\ & 5x^2 - 6x + 9 - (2x^2 - 7x + 1) \\ & 5x^2 - 6x + 9 - 2x^2 + 7x - 1 \\ & 3x^2 + x + 8 \end{aligned}$$

Application.



In this drawing find w in terms of x and y.

$$w = x + 7y - (x - 2y) - x$$

$$= x + 7y - x + 2y - x$$

$$w = -x + 9y$$

Drill and Practice:

Rising, p. 232-233

Dressler, p. 241-242, 244-246

Smith, p. 127-128

4. CONCEPT: MULTIPLICATION INVOLVING MONOMIALS WHICH CONTAIN EXPONENTS

To multiply two powers with the same base:

$$\begin{aligned} 4^2 \cdot 4^3 &= (4 \cdot 4) (4 \cdot 4 \cdot 4) = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^5 \\ x^3 \cdot x^4 &= (x \cdot x \cdot x) (x \cdot x \cdot x \cdot x) = x^7 \end{aligned}$$

or

$$x^a x^b = x^{a+b}$$

Note: to multiply two powers with the same base, add the exponents.

Examples:

$$\begin{aligned} m^5 \cdot m^2 &= m^7 \\ q^4 \cdot q_2 &= q_5^5 \\ 2^3 \cdot 2^2 &= 2^5 \end{aligned}$$

$$\begin{aligned} s^7 \cdot s \cdot s^6 &= s^{14} \\ \text{Note: } s &= s^1 \end{aligned}$$

To find the power of a product:

$$(xyz)^2 = (xyz) \cdot (xyz) = x \cdot x \cdot y \cdot y \cdot z \cdot z = x^2 y^2 z^2$$

$$\begin{aligned} (-2m)^3 &= (-2m) (-2m) (-2m) = (-2) (-2) (-2) m \cdot m \cdot m \\ &= (-2)^3 \cdot m^3 = -8m^3 \end{aligned}$$

or

$$(xy)^a = x^a \cdot y^a$$

Examples:

$$\begin{aligned} (-4x)^4 &= (-4)^4 m^4 = 64 m^4 \\ (-6m)^2 &= (-6)^2 \cdot m^2 = 36 m^2 \\ (+2mnp)^3 &= 2^3 \cdot m^3 \cdot n^3 \cdot p^3 = 8 \cdot m^3 \cdot n^3 \cdot p^3 \end{aligned}$$

To find the power of a power:

$$\begin{aligned} (2^3)^2 &= 2^3 \cdot 2^3 = 2^6 \\ (p^5)^3 &= p^5 \cdot p^5 \cdot p^5 = p^{15} \\ (m^4)^5 &= m^4 \cdot 5 = m^{20} \end{aligned}$$

or

$$(x^a)^b = x^{ab}$$

Examples:

$$(9^2)^3 = 9^2 \cdot 3 = 9^6 = 531,441$$

$$(-1)^{3 \cdot 4} = (-1)^{12} = +1$$

$$(m^6)^7 = m^{42}$$

$$(3 n^2 p^4)^4 = 3^4 (n^2)^4 (p^4)^4 = 81 n^8 p^{16}$$

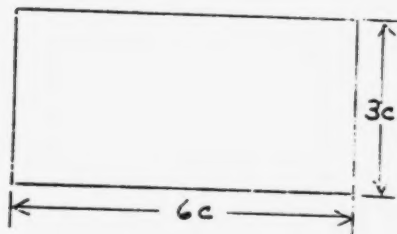
To multiply monomials:

$$\begin{aligned} & (5a^2b)(-7a^3b^2c) \\ & 5 \cdot a^2 \cdot b \cdot (-7) \cdot a^3 \cdot b^2 \cdot c \\ & 5 \cdot (-7) \cdot (a^2a) \cdot (b \cdot b^2) \cdot c \\ & -35a^5b^3c \end{aligned}$$

Use the commutative and associative properties of multiplication.

$$\begin{aligned} (8m^2y)^3 &= 8^3 \cdot m^6 \cdot y^3 = 512 m^6 y^3 \\ (7a^2)(4r^2)(-2a^3) &= 7 \cdot 4 \cdot (-2) \cdot a^2 \cdot a^3 \cdot r^2 \\ &= -56a^5r^2 \end{aligned}$$

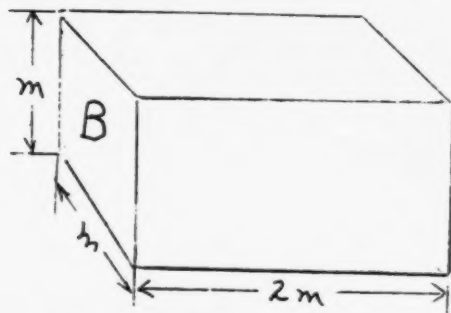
Application 1.



Find the area of a space divider in terms of c as indicated in this diagram.

$$\begin{aligned} A &= L \cdot W & L &= 6c \\ A &= 6c \cdot 3c & W &= 3c \\ A &= 6 \cdot 3 \cdot c \cdot c \\ A &= \underline{18c^2} \text{ square units.} \end{aligned}$$

Application 2.



Find the volume of this container if $2m$ represents the height.

$$\begin{aligned} V &= B \cdot h \\ & \quad h = 2m \\ B &= m \cdot m = m^2 \end{aligned}$$

$$\begin{aligned} V &= m^2 \cdot 2m \\ V &= 2 \cdot m^2 \cdot m \\ V &= \underline{2m^3} \text{ cubic units} \end{aligned}$$

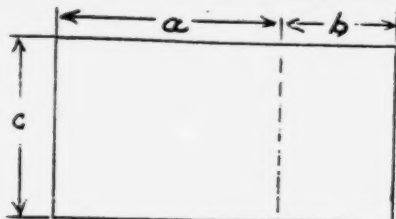
Drill and Practice:

Rising, p. 236

Dressler, p. 223, 226

Smith, p. 129 (62-82), p. 133 (33-52)

5. CONCEPT: MULTIPLICATION OF A POLYNOMIAL BY A MONOMIAL



The area of a display board in a conference room can be determined by the formula:

$$A = LW$$

where L = length

W = width

find the area in terms of a , b , and c .

Solution:

$$A = L W$$

$$\text{Now } L = a + b$$

$$W = c$$

Therefore,

$$A = (a + b) \cdot c$$

$$A = c \cdot (a + b)$$

$$A = ca + cb.$$

This is an application of the distributive property which we can use to multiply other monomials and polynomials.

Example:

$$5(a^2 + a + 3) = 5 \cdot a^2 + 5 \cdot a + 5 \cdot 3$$

$$= 5a^2 + 5a + 15$$

$$m(m^3 - 5m + 6) = m \cdot m^3 - m \cdot 5m + m \cdot 6.$$

$$= m^4 - 5m^2 + 6m$$

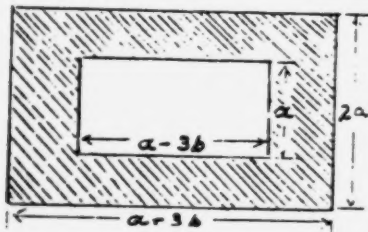
$$y(y^2 + yx + 2) = y^3z + y^2x + 2y$$

$$-6x(x^2 + 7x + 1) = (-6x) \cdot x^2 + (-6x) \cdot 7x + (-6x) \cdot 1$$

$$= -6x^3 - 42x^2 - 6x$$

$$14x^2y(7x^2z - 2xy + 20xy^2) = 98x^4yz - 28x^3y^2 + 280x^3y^3$$

Application.



Find the area of the shaded part if the inner rectangle is cut from the area of the outer rectangle.

$$\text{Shaded Area} = \text{Area outer rectangle} - \text{Area inner rectangle}$$

$$= 2a(a+3b) - a(a-3b)$$

Simplify.

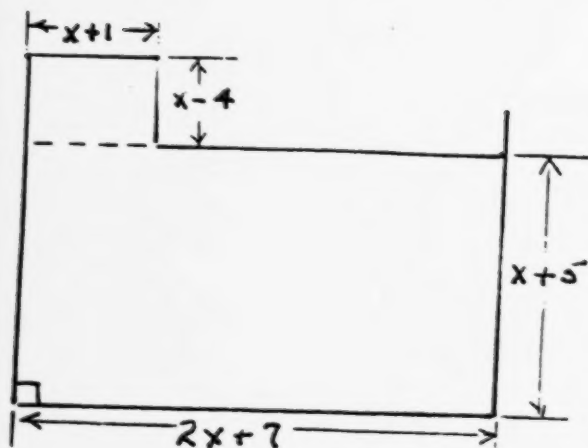
$$= 2a^2 + 6ab - a^2 + 3ab$$

Combine similar terms.

$$\text{Shaded Area} = a^2 + 9ab. \text{ Square units (in terms of } a \text{ and } b).$$

Drill and Practice:

Rising, p. 238 (Oral) (1-9), 239 (Written) (1-10), (31-34)
Dressler, p. 247 (1-18), 248, (1-31)
Smith, p. 129 (83-88)



Find the area (in terms of x) of this metal bracket.

$$\text{Area} = (x+1)(x-4) + (x+5)(2x+7)$$

$$\text{Area} = x^2 - 3x - 4 + 2x^2 + 17x + 35$$

$$\text{Area} = 3x^2 + 14x + 31 \text{ square units.}$$

Drill and Practice:

Rising, p. 239-240 (11-30, 35-46)

Dressler, p. 250-252

Smith, p. 129 (88-92)

6. CONCEPT: MULTIPLICATION OF TWO POLYNOMIALS

Rule

To multiply a polynomial by another polynomial, multiply each term in the first polynomial by each term in the second polynomial. Then combine all similar terms in the product.

Example: Find the product $(x + 3)(2x + 6)$
The work can be arranged two ways.

Method 1.

$$\begin{aligned}(x+3)(2x+6) &= 2x(x+3)+6(x+3) \\ &= 6x^2+6x+6x+3 \\ &= 6x^2+12x+3\end{aligned}$$

In this method we multiplied each term in $(2x+6)$ by $(x+3)$. This gave $2x(x+3)+6(x+3)$. We then cleared parentheses and combined similar terms.

Method 2.

$$\begin{array}{r} 2x+6 \\ \times x+3 \\ \hline 2x^2+6x \\ +6x+18 \\ \hline 2x^2+12x+18 \end{array}$$

In this method we first multiplied $(2x+6)$ by x , giving $2x^2+6x$ in the first row of the product. We then multiplied $(2x+6)$ by 3 giving $6x+18$ in the second row of the product, lining up similar terms in columns. Then we combined similar terms.

Examples:

$$\begin{aligned}(3x-5)(x+4) &= (3x-5)x+(3x-5)4 \\ &= 3x^2-5x+12x-20 \\ &= 3x^2+7x-20\end{aligned}$$

$$\begin{aligned}(x-3)(x+2)(4x-1) &= \\ (x-3)(4x^2+7x-2) &= (x-3)4x^2+(x-3) \cdot 7x-(x-3)2 \\ &= 4x^3-12x^2+7x^2-21x-2x+6 \\ &= 4x^3-5x^2-23x+6\end{aligned}$$

7. CONCEPT: DIVISION OF MONOMIALS WHICH CONTAIN EXPONENTS

To divide two powers with the same base:

$$5^3 \div 5^2 = \frac{5 \cdot 5 \cdot 5}{5 \cdot 5} = \frac{5 \cdot 1 \cdot 1}{1 \cdot 1} = \frac{5}{1} = 5 \text{ or } 5^1$$

$$a^7 \div a^3 = \frac{\overset{1}{\cancel{a}} \cdot \overset{1}{\cancel{a}} \cdot \overset{1}{\cancel{a}} \cdot a \cdot a \cdot a \cdot a}{\underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{a}}} = \frac{a}{1}^4 = a^4$$

or $\boxed{x^a \div x^b = x^{a-b}}$ where $a > b$

Example: $m^5 \div m^2 = m^{5-2} = m^3$

$$q^8 \div q = \frac{q^8}{q^1} = q^7$$

$$8^4 \div 8^2 = 8^{4-2} = 8^2 = 64$$

Divide: $10^3 \div 10^3 = \frac{\overset{1}{\cancel{10}} \cdot \overset{1}{\cancel{10}} \cdot \overset{1}{\cancel{10}}}{\underset{1}{\cancel{10}} \cdot \underset{1}{\cancel{10}} \cdot \underset{1}{\cancel{10}}} = \frac{1}{1} = 1$

$$x^7 \div x^7 = 1$$

$$p \div p = 1$$

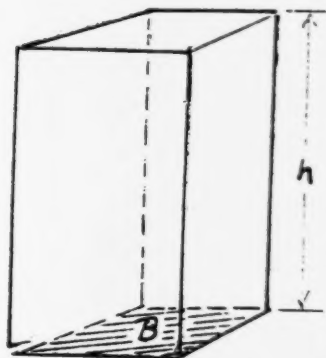
Divide: $6^3 \div 6^5 = \frac{\overset{1}{\cancel{6}} \cdot \overset{1}{\cancel{6}} \cdot \overset{1}{\cancel{6}}}{\underset{1}{\cancel{6}} \cdot \underset{1}{\cancel{6}} \cdot \underset{1}{\cancel{6}} \cdot 6 \cdot 6} = \frac{1}{6^2} = \frac{1}{36}$

$$t^5 \div t^9 = \frac{t^5}{t^9} = \frac{1}{t^4}$$

or $\boxed{x^a \div x^b = \frac{1}{x^{b-a}}}$ if $b > a$.

Divide: $(-10m^3t^2p) \div (5mt^5p)$

$$\begin{aligned} \frac{-10m^3t^2p}{5mt^5p} &= \frac{(-10)}{5} \cdot \frac{(m^3)}{m} \cdot \frac{(t^2)}{t^5} \cdot \frac{p}{p} \\ &= -2m^2 \cdot \frac{1}{t^3} \cdot 1 = -\frac{2m^2}{t^3} \end{aligned}$$

Application.

The volume of rectangular prism is $14a^3$. If the area of the base is $2a^2$ find the length of the altitude in terms of a .

$$\begin{aligned}\text{Volume} &= Bh \\ 14a^3 &= 2a^2h\end{aligned}$$

$$\frac{14a^3}{2a^2} = \frac{2a^2}{2a^2} \cdot h$$

$$7a = h \quad \text{or} \quad h = 7a$$

Drill and Practice:

Rising, p. 241 (1-12), 242 (1-16)

Dressler, p. 228 (1-33), 229-230 (1-22)

Smith, p. 132 (1-20)

8. CONCEPT: DIVISION OF POLYNOMIAL BY A MONOMIAL

$$\begin{aligned}\text{Divide: } (ax + bx) \div c \\ = (ax + bx) \cdot \frac{1}{c}\end{aligned}$$

$$(ax + bx) \div c = \frac{ax}{c} + \frac{bx}{c} \quad \text{Distributive Law.}$$

Examples:

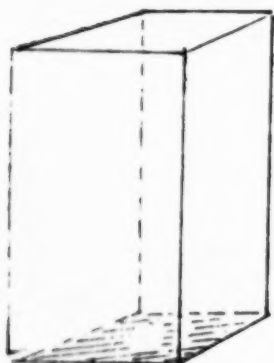
$$1) \quad (3m^3 - 6m^2 - 3m) \div 3m$$

$$\frac{3m^3}{3m} - \frac{6m^2}{3m} - \frac{3m}{3m} = m^2 - 2m - 1$$

$$2) \quad (-25m^7n^2 + 5m^5m^2) \div (-5m^2n^2)$$

$$= \frac{-25m^7n^2}{-5m^2n^2} + \frac{5m^5m^2}{-5m^2m^2} = 5m^5 - 1m^3$$

Application.



If the volume of this prism is $3v^3 - 21v^2 + 9$, divide the volume by 3.

$$\begin{aligned}(3v^3 - 21v^2 + 9) \div 3 &= \frac{3v^3 - 21v^2 + 9}{3} \\ &= v^3 - 7v^2 + 3\end{aligned}$$

Drill and Practice:

Rising, p. 242 (17-30)

Dressler, p. 253 (1-31)

Smith, p. 132 (21-32)

9. CONCEPT: PRIME FACTORS, COMMON FACTORS, GREATEST COMMON FACTORS AND COMMON MONOMIAL FACTORS

A prime number is an integer greater than 1 which has as factors only itself and 1.

The prime factors of:

12 are $2 \cdot 2 \cdot 3$
 20 are $2 \cdot 2 \cdot 5$
 25 are $5 \cdot 5$
 7 are $7 \cdot 1$ (this is a prime number)

The set of prime numbers:

$= \{2, 3, 5, 7, 11, 13, 17, \dots\}$

If $45 = 5 \cdot 3 \cdot 3$
 and $39 = 3 \cdot 13$ } $\longrightarrow$ Then 3 is a common factor of 45 and 49.

If $30 = 2 \cdot 3 \cdot 5$
 and $42 = 2 \cdot 3 \cdot 7$ } $\longrightarrow$ Then 2 and 3 are common factors. But 3 is the greatest common factor.

Example:

Consider 16 and 32. What is the greatest common factor?

$16 = 2 \cdot 2 \cdot 2 \cdot 2$
 $32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$2 \cdot 2 \cdot 2 \cdot 2$ or 16 is the greatest common factor (G.C.F.)

Consider

$4ab$ and $12a$.
 $4ab = 2 \cdot 2 \cdot a \cdot b$
 $12a = 2 \cdot 2 \cdot 3 \cdot a$ } $4a$ is the G.C.F.

$-12m^3n^2$ and $8m^2n^2$ } G.C.F. = $4m^2n^2$

Common Monomial Factor

Consider $ax + bx = x \cdot (a+b)$ [distributive property]

$3m-9n = 3 \cdot (m-3n)$

3 is the G.C.F.
 of $3m$ and $-9n$.
 Remove the 3 and
 divide it into $3m-9n$

We have now written a $3m-9n$ in the factored form

$3 \cdot (m-3n)$ by removing the G.C.F.

Examples:

Factor: $5x^2 - 25x = 5x \cdot (x-5)$
 Note: GCF = $5x$

Factor: $36m^2 - 6mn = 6m \cdot (6m-n)$
 Note: GCF = $6m$

Factor: $7x^2 - 14x^2y = 7x^2 \cdot (1-2y)$

Application. The formula for converting temperature in Fahrenheit to temperature in Celsius is

$$C = \frac{5}{9} F - \frac{160}{9}$$

Write this in its more familiar factored form:

$$C = \frac{1}{9} (5F - 160)$$

$$\text{Factor } \frac{1}{9}$$

$$C = \frac{1}{9} \cdot 5 \cdot (F - 32)$$

$$\text{Factor } 5$$

$$C = \frac{5}{9} (F - 32)$$

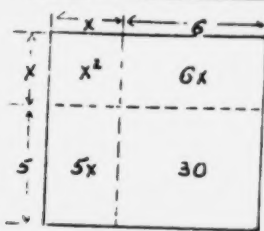
$$\text{GCF} = \frac{5}{9}$$

Drill and Practice:

Rising, p. 244-245

Dressler, p. 399

10. CONCEPT: A SHORT METHOD OF MULTIPLYING TWO BINOMIALS (FOIL)



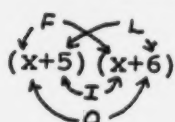
A floor area is divided in the manner indicated by the diagram. The individual areas are x^2 , $5x$, $6x$, and 30

This means that the total area which is indicated by $(x+5)(x+6) = x^2 + 5x + 6x + 30$.

Notice that $5x$ and $6x$ are similar.

Therefore the total area equals $x^2 + 11x + 30$

This result could have been obtained quickly by use of the FOIL method.



F = First product or $x \cdot x = x^2$
 O = Outside product or $x \cdot 6 = 6x$
 I = Inside product or $5 \cdot x = 5x$
 L = Last product or $5 \cdot 6 = 30$

Therefore: Product = $x^2 + 11x + 30$ (Add similar terms mentally)

Example:

$$\begin{aligned}(x-11)(x+4) &= x^2 - 7x - 44 \text{ (All work done mentally)} \\ (3x-4)(x+9) &= 3x^2 + 23x - 36 \text{ (Combine } 27x \text{ and } -4x) \\ (4x-5)(2x-3) &= 8x^2 - 22x + 15\end{aligned}$$

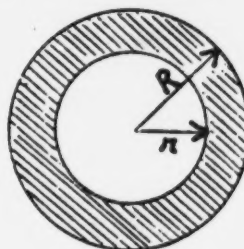
Application:

The formula for the area (shaded) of this washer is
 Area shaded = $\pi (R-r) \cdot (R+r)$

Perform the indicated multiplications in the formula

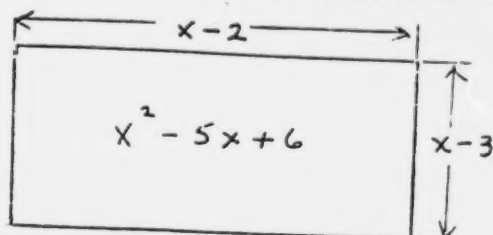
$$\begin{aligned}\text{Area shaded} &= \pi (R^2 - r^2) \\ \text{Area shaded} &= \pi R^2 - \pi r^2\end{aligned}$$

Note: Middle terms Rr and $-Rr$ have a sum = 0.



Drill and Practice:
 Rising, p. 246, 247
 Dressler, p. 410

11. CONCEPT: FACTORING A POLYNOMIAL OF THE FORM Ax^2+Bx+C



If x^2-5x+6 represents area of the rectangle, then the length and the width can be represented by $(x-2)$ and $(x-3)$.

To factor x^2-5x+6 write $(x-2)(x-3)$

x^2-5x+6 is a polynomial of the form Ax^2+Bx+C where $A=1$, $B=-5$ and $C=6$.

This is a polynomial of the quadratic form. One condition is essential, A cannot equal 0.

Example: Factor $x^2-9x+20$

$$x^2-9x+20 = (x \quad)(x \quad)$$

x and x are the possible factors of x^2 .

The possible factors of 20 are: 20 and 1, 9 and 2, or 4 and 5.

Try 20 and 1.

Write $(x-20)(x-1)$. To tell whether this choice is correct, find the inside product (FOIL), the outside product (FOIL) and add.

$$(-21x) + (-1x) = -21x \quad \text{This sum should equal the middle term } -9x.$$

Now $-21x$ does not equal $-9x$.

Therefore 20 and 1 are eliminated.

Now try 10 and 2.

$$(x-10)(x-2)$$

$$\begin{aligned} \text{Inside Product} &= -10x \\ \text{Outside Product} &= -2x \\ \text{Sum} &= -12x \end{aligned}$$

$-12x$ does not equal middle term $-9x$.

Therefore 10 and 1 are eliminated.

Try 4 and 5.

$$(x-4)(x-5)$$

$$\begin{aligned} \text{Inside Product} &= -4x \\ \text{Outside Product} &= -5x \\ \text{Sum} &= -9x \end{aligned}$$

$-9x$ does equal the middle term.

Therefore

$$x^2-9x+20 = (x-4)(x-5)$$

Examples: Factor

$$x^2 - 15x + 50 = (x-10)(x-5)$$

$$x^2 - x - 2 = (x-2)(x+1)$$

(signs different)
since $C = -2$

$$\text{Inside product} = -2x$$

$$\text{Outside product} = +1x$$

$$\text{Sum} = -1x$$

Note: If the sign of C is positive both signs in the factored form are the same. If the sign of C is negative both signs in the factored form are different.

$$3x^2 - 8x + 5 = (3x-5)(x-1)$$

$$\begin{array}{c} \downarrow \quad \downarrow \\ -5x \quad \downarrow \\ \rightarrow -3x \leftarrow \\ \text{Sum} \quad -8x \end{array}$$

$$\text{Factor : } 24x^2 - 2x - 15$$

possible factors

24, 1
12, 2 Try
8, 3 each
6, 4 pair.

possible factors

1, 15
3, 5 Try
 each
 pair.

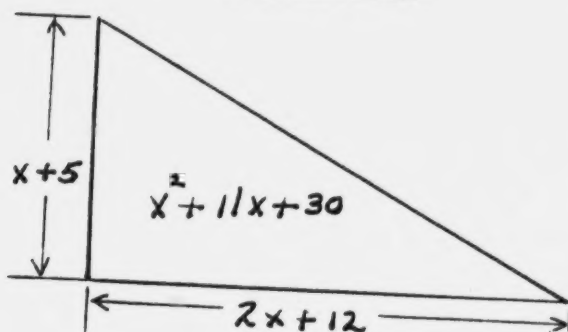
Until the sum of the inside product and the outside product equals $-2x$, this is trail and error. Use a pencil.

Solution:

$$24x^2 - 2x - 15 = (6x-5)(4x+3)$$

Note: signs (-5) and $(+3)$ are opposite, since $C = -15$.

Application.



The area of a triangular sheet of metal is $x^2 + 11x + 30$.

$$\text{Factored } x^2 + 11x + 30 = (x+6)(x+5)$$

$$\text{If area of } \triangle = \frac{1}{2} \cdot b \cdot h$$

$$\text{Then } (x+6)(x+5) = \frac{1}{2} \cdot b \cdot h$$

Multiplying both members by 2.

$$2(x+6)(x+5) = b \cdot h$$

$$\text{or } (2x+12)(x+5) = b \cdot h$$

$$\text{so } b \text{ could equal } 2x+12$$

$$\text{and } h \text{ could equal } x+5.$$

Drill and Practice:
Rising, p. 250-251
Dressler, p. 410

12. CONCEPT: SPECIAL PRODUCT AND FACTORING

Rewrite the formula for the volume of a hollow cylinder, if it is written in the form, $V = \pi \cdot h \cdot (R-r)(R+r)$
 $V = \pi \cdot h \cdot (R^2 - r^2)$ FOIL method.

Solution:

Note: $(R-r)(R+r) = R^2 + \overset{\uparrow}{Rr} - \overset{\uparrow}{Rr} - r^2$

zero
 $= R^2 - r^2$

This is a special product of the form:
 $(a-b)(a+b) = a^2 - b^2$ There is no middle term.

Rule: The product of a sum of two terms and the difference of two terms is the difference of the squares of the terms.

Example: $(x-9)(x+9) = x^2 - 81$
 $(a+4b)(a-4b) = a^2 - 16b^2$
 $(m-n)(m+n) = m^2 - n^2$
 $(3q+7r)(3q-7r) = 9q^2 - 49r^2$

To factor the difference of two squares, $a^2 - b^2$:
 $a^2 - b^2 = (a-b)(a+b)$

Examples:

$\frac{9x^2}{\uparrow} - \frac{16}{\nearrow} = (3x-4)(3x+4)$
 Must be a "perfect" square.

Let's review some commonly used integral perfect squares.

Square

1
4
9
16
25
36
49
64
81
100
121
144
225
400
625
900
10000

Square Root

1
2
3
4
5
6
7
8
9
10
11
12
15
20
25
30
100

These should be committed to memory.

Note: The square root
 of x^2 is x
 of x^6 is x^3
 of a^2b^2 is ab .
 Keep the base and divide the exponent by 2.

Examples:

Factor

$$\begin{aligned} 25m^2 - 1 &= (5m+1)(5m-1) \\ 36q^2 - 64 &= (6q-8)(6q+8) \\ 100r^2 - 49 &= (10r-7)(10r+7) \\ a^2b^2 - 81y^2 &= (ab+9y)(ab-9y) \end{aligned}$$

Find the square of a binomial

$$\begin{aligned} (a+b)^2 &= a^2 + 2ab + b^2 \\ \text{or } (a-b)^2 &= a^2 - 2ab + b^2 \end{aligned}$$

$$\begin{aligned} \text{Example: } (3x-5)^2 &= (3x)^2 - 2(3x)(+5) + (-5)^2 \\ &= 9x^2 - 30x + 25 \end{aligned}$$

$$\begin{aligned} (9x+4)^2 &= (9x)^2 + 2(9x)4 + 4^2 \\ &= 81x^2 + 72x + 16 \end{aligned}$$

If a polynomial can be recognized as a perfect trinomial square then it can be quickly factored.

$$\begin{aligned} a^2 + 2ab + b^2 &= (a+b)(a+b) = (a+b)^2 \\ a^2 - 2ab + b^2 &= (a-b)(a-b) = (a-b)^2 \end{aligned}$$

Is $x^2 + 12x + 36$ a perfect trinomial square?

perfect
square

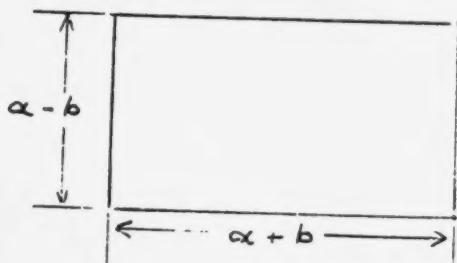
perfect
square

Two times
the product
of x and 6.

Yes

$$\text{Therefore } x^2 + 12x + 36 = (x + 6)^2$$

Application 1.

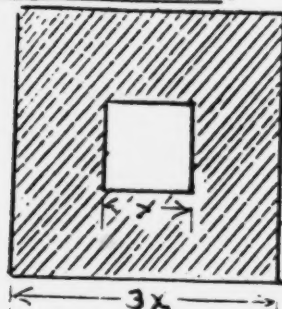


Write a formula for the area of this rectangle.

$$A = (a+b)(a-b)$$

Now rewrite the area without parentheses.

$$A = a^2 - b^2$$

Application 2.

$$\begin{aligned}\text{Shaded Area} &= (3x)^2 - y^2 \\ &= 9x^2 - y^2\end{aligned}$$

Rewrite this formula in factored form.

$$\text{Shaded Area} = (3x - y)(3x + y)$$

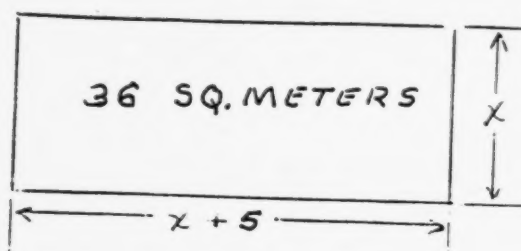
A square panel around a square opening in a furnace is to be insulated. Write the formula for the area of the panel if it is represented by the shaded area in the diagram.

Drill and Practice:

Rising, p. 253

Dressler, p. 401, 402, 403

13. CONCEPT: THE SOLUTION OF QUADRATIC EQUATIONS



The surface area of a patio to be paved is 36 square meters.

If the length of the patio is five meters longer than the width, find the width.

Solution: $A = lw$
Therefore $A = (x+5)x$

$$\text{or} \quad \begin{aligned} (x+5)x &= 36 \\ x^2+5x &= 36 \end{aligned}$$

Now x^2 and $5x$ are not similar terms and cannot be combined, so we must develop a new method of solution.

Consider the product $a \cdot b = 0$. If $a \cdot b = 0$ then either $a = 0$, $b = 0$, or a and b equal 0.

$$\text{Now rewrite } x^2+5x = 36$$

$$\begin{array}{r} x^2+5x = 36 \\ -36 \quad -36 \\ \hline \end{array}$$

$$\begin{aligned} x^2+5x-36 &= 0 \\ (x+9)(x-4) &= 0 \end{aligned}$$

Factor the left side

$$\begin{array}{l|l} \text{Either } x+9 = 0 & \text{or } x-4 = 0 \\ \hline \begin{array}{r} -9 = -9 \\ x = -9 \end{array} & \begin{array}{r} +4 = +4 \\ x = 4 \text{ m.} \end{array} \end{array}$$

Since, in a geometric problem $x = -9$ has no meaning we discard -9 , therefore $x = +4$.

Examples:

Solve for x

$$\begin{aligned} x^2-11x-26 &= 0 \\ (x-13)(x+2) &= 0 \end{aligned}$$

$$\text{Let } \begin{array}{l|l} x-13 = 0 & x+2 = 0 \\ x = 13 & x = -2 \end{array}$$

Since this is merely an algebraic example, do not discard any solution.

Solution (13, -2)

Solve for m

$$\begin{aligned} 3x^2-x-2 &= 0 \\ (3x+2)(x-1) &= 0 \end{aligned}$$

$$\text{Set } \begin{array}{l|l} 3x+2 = 0 & x-1 = 0 \\ \hline \begin{array}{r} -2 = -2 \\ 3x = -2 \\ x = -2/3 \end{array} & \begin{array}{r} x = 1 \end{array} \end{array}$$

Solution $\{-2/3, 1\}$

Solve for a:

$$\begin{array}{l} a^2 - 25 = 0 \\ (a-5)(a+5) = 0 \\ a-5 = 0 \quad | \quad a+5 = 0 \\ a = 5 \quad | \quad a = -5 \end{array}$$

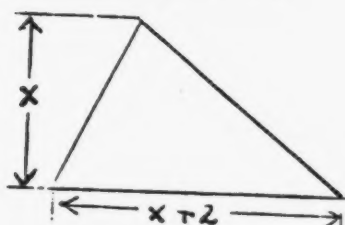
(5, -5)

Both solutions should check.

$$\begin{array}{l|l} a^2 - 25 = 0 & a^2 - 25 = 0 \\ (5)^2 - 25 = 0 & (-5)^2 - 25 = 0 \\ 25 - 25 = 0 & 25 - 25 = 0 \\ 0 = 0 & 0 = 0 \end{array}$$

Check

Application.



The area of a triangle is 24 square centimeters. If the base is two centimeters more than the altitude, find the altitude.

$$\begin{array}{l} \text{Solution: } A = (1/2) \cdot b \cdot h \\ 24 = (1/2) \cdot (x+2) \cdot x \\ 2 \cdot 24 = (2 \cdot 1/2) \cdot (x+2) \cdot x \\ 48 = (x+2) \cdot x \\ (x+2) \cdot x = 48 \\ x^2 + 2x = 48 \\ -48 \quad -48 \\ x^2 + 2x - 48 = 0 \\ (x-6)(x+8) = 0 \\ \text{Set } x-6 = 0 \quad | \quad x+8 = 0 \\ x = 6 \quad | \quad x = -8 \text{ discard} \end{array}$$

The altitude = 6 centimeters.

Drill and Practice:

Rising, p. 256-257 (1-48),
259 (9-12, par. 21), 260
Dressler, p. 654 (1-48)

14. CONCEPT: RATIONAL NUMBERS; CHANGING FRACTIONS TO DECIMALS

The set of real numbers is made up of two subsets, the rational numbers and the irrational numbers.

The rational numbers can be written in the form of p/q where p and q are integers and q cannot equal zero. The irrational numbers cannot be written in this form.

Examples of rational numbers are

$$\frac{2}{3}, -4, 0, 6\frac{1}{2}, .7, 1.05, \frac{67}{13}$$

since each of these numbers can be written as

$$\begin{array}{l} p : \frac{2}{3}, -4, 0, \frac{13}{2}, \frac{7}{10}, \frac{105}{100}, \frac{67}{13} \\ q : 3, 1, 1, 2, 10, 100, 13 \end{array}$$

To express a rational number as a decimal merely divide numerator by denominator.

$$\frac{2}{3} = 3 \overline{)2.000} = .\overline{6}$$

$\overline{.6}$ means .6666.....

A repeating decimal

$$\frac{2}{3} = .\overline{6}$$

$$\frac{7}{8} = 8 \overline{)7.000} \begin{array}{r} .875 \\ 64 \\ 60 \\ 56 \\ 40 \\ 40 \end{array}$$

A terminating decimal

$$\frac{7}{8} = .875$$

Every rational number can be expressed either as a repeating or terminating decimal.

Examples:

$$\frac{4}{5} = 5 \overline{)4.0} = .8 \quad \text{Terminating decimal}$$

$$\frac{1}{9} = 9 \overline{)1.000} = .\overline{1} \quad \text{Repeating decimal}$$

$$4 \frac{5}{8} = \frac{37}{8} = 8 \overline{) 37.000} = 4.625 \text{ Terminating decimal.}$$

$$\begin{array}{r} 4.625 \\ 8 \overline{) 37.000} \\ \underline{32} \\ 50 \\ \underline{48} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \end{array}$$

Application. $\frac{3}{8}$ of the employees of a small plant were hired within the past year. A notice on the bulletin board said that .4 of the employees were hired during the previous year. Which is the greater figure?

$$\frac{3}{8} = 8 \overline{) 3.000} = .375$$

$$.4 = .400 \leftarrow \text{Larger}$$

Therefore .4 is larger than $\frac{3}{8}$

Drill and Practice:

Rising, p. 273-274

Dressler, p. 588 (23-29)

15. CONCEPT: RATIONAL NUMBERS; CHANGING DECIMALS TO FRACTIONS

To change a terminating decimal to a fraction rewrite with an appropriate denominator of 10, 100 or 1000, etc. Then simplify.

Example:

$$.37 = \frac{37}{100}$$

$$.419 = \frac{419}{1000}$$

$$.48 = \frac{48}{100} = \frac{12}{25}$$

$$8.0006 = 8 \frac{6}{10000} = 8 \frac{3}{5000} \text{ or } \frac{40,003}{5000}$$

To change a repeating decimal to a fraction follow this procedure:

Example:

Convert $.\overline{3}$ to a decimal

- | | |
|---|---|
| 1. Let $N = .\overline{3}$ | Note: One repeating place.
Multiply both members by 10.
Subtract Step 1 from Step 2
Subtraction Property of Equality
Divide by 9. |
| 2. $10N = 3.\overline{3}$ | |
| 3. $10N - N = 3.\overline{3} - .\overline{3}$ | |
| 4. $9N = 3$ | |
| 5. $N = \frac{3}{9}$ | Reduce. |
| 6. $N = \frac{1}{3}$ | |

Example:

Convert $2.\overline{54}$ to a decimal.

- | | |
|-------------------------------|---|
| 1. Let $N = 2.\overline{54}$ | Note: 2 repeating places
Multiply both members by 100
Subtract (1) from (2)
Divide by 99 |
| 2. $100N = 254.\overline{54}$ | |
| 3. $99N = 252$ | |
| 4. $N = \frac{252}{99}$ | |

$$5. \quad N = \frac{252 \div 9}{99 \div 9} = \frac{28}{11} \quad \text{Reduce}$$

$$\text{or } 2 \frac{6}{11}$$

Application.

A calculator readout indicated

17.818181 or $17.\overline{81}$. Convert this number to a mixed number.

$$N = 17.\overline{81}$$

$$100N = 1781.\overline{81}$$

$$99N = 1781.\overline{81} - 17.\overline{81}$$

$$99N = 1764$$

$$N = \frac{1764}{99} = \frac{1764 \div 9}{99 \div 9} = \frac{196}{11} = 17 \frac{9}{11}$$

Drill and Practice:

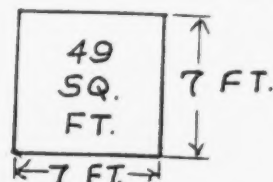
Rising, p. 275-276

Dressler, p. 588 (30-35)

16. CONCEPT: SQUARE ROOT

If 49 square feet is the area of a square then the length of one side is 7.

If $A = s^2$ and $49 = 7^2$
7 is said to be the
square root of 49.



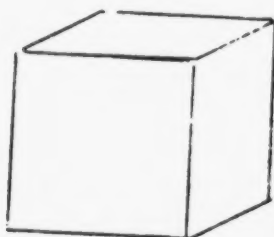
Numbers like 49, 64, 625 are called perfect squares. A rational number which has another rational number as its square root is called a perfect square.

A symbol $\sqrt{\quad}$ is used to indicate a root, this is called a radical sign. If no number or a 2 is written, as $\sqrt{\quad}$ or $^2\sqrt{\quad}$ then square root is indicated.

$$\sqrt{81} = 9 \quad ^2\sqrt{144} = 12, \quad \sqrt{\frac{25}{36}} = \frac{5}{6}$$

The number under the radical sign is called the radicand. The number in the position of the 2 is called the index. Only non negative numbers have rational square roots. Actually each rational number has two square roots. Since $(+8)(+8) = 64$ and $(-8)(-8)$ also equals 64. Therefore $\sqrt{64} = 8$ or $-\sqrt{64} = -8$ Note: A negative sign to the left of the radical sign indicates a negative square root.

The positive square root is the principal square root.



If the volume of this cube is 125 cubic centimeters, then one side equals 5 centimeters.

5 is said to be the cube root of 125. We express this as

$$^3\sqrt{125} = 5$$

Note that $^3\sqrt{-64} = -4$ since $(-4)(-4)(-4) = -64$

Examples:

$$^3\sqrt{1000} = 10, \quad ^3\sqrt{-8} = -2$$

$$^3\sqrt{\frac{27}{64}} = \frac{3}{4}$$

The above mentioned roots are all rational numbers. Now suppose that the area of a square is 47 square units. A side, then, is $\sqrt{47}$. $\sqrt{47}$ is an irrational number since it cannot be written in the form of p/q where p and q are integers and q is not 0.

Examples of irrational numbers, $\sqrt{5}$, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{7}$, π (3.14159.....)

Square roots can be calculated. This process, however, is tedious. Therefore calculators are the most efficient method of calculating square root. Tables of square roots are also useful.

To determine square root by calculator:

Example: Find $\sqrt{63.7}$

1. Read in 63.7 where 63.7 is radicand
2. Push $\sqrt{x}$ or similar key
3. Read in 2 where 2 is index
4. Push =
5. Read out 7.981228
6. Round off 7.98

Review of Perfect Squares

1, 4, 9, 16, 25, 36, 49, 64
81, 100, 121, 144, 225, 400, 625

Review of Perfect Cubes

1, 8, 27, 64, 125, 1000

Commit to Memory

Monomials are perfect squares when all exponents are divisible by 2 and all coefficients are perfect squares.

Example: $\sqrt{16x^4} = 4x^2$ $\sqrt{100m^6y^2} = 10m^3y$

Application .

The volume of of a cube is 117 cubic feet. Find the length of one side of the cube.

$$V = 117 \text{ cubic centimeters}$$

$$V = s^3$$

$$s = \sqrt[3]{117}$$

$$s = 4.8909733 \quad \text{Read out.}$$

Round off to correct to nearest tenth of a foot.

$$s = 4.9 \text{ Feet}$$

Drill and Practice:

Rising, p. 277-279

Dressler, p. 595-596, 598, 599, 603

Smith, p. 50-52

17. CONCEPT: THE MULTIPLICATION AND DIVISION OF RADICALS

Radicals can express rational numbers such as $\sqrt{9}$ or $\sqrt{25}$ or irrational numbers such as $\sqrt{7}$ or $\sqrt{80}$. Sometimes mathematical operations need to be performed on these numbers. (addition, subtraction, multiplication, division)

Example: $\sqrt{25} \cdot \sqrt{4} = 5 \cdot 2 = 10$

Now, if we write

$$\sqrt{25} \cdot \sqrt{4} = \sqrt{25 \cdot 4} = \sqrt{100} = 10$$

or $\boxed{\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}}$

Where a and b are non negative real numbers.

Examples:

$$\sqrt{3} \cdot \sqrt{5} = \sqrt{3 \cdot 5} = \sqrt{15} \quad \text{Irrational}$$

$$\sqrt{2} \cdot \sqrt{50} = \sqrt{2 \cdot 50} = \sqrt{100} = 10 \quad \text{Rational}$$

$$\sqrt{.09} \cdot \sqrt{9} = \sqrt{(.09)(9)} = \sqrt{.81} = .9 \text{ since } (.9)(.9) = .81$$

Ex: $\sqrt{16} \div \sqrt{25} = 4 \div 5 = \frac{4}{5}$

or $\frac{\sqrt{16}}{\sqrt{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

Therefore $\boxed{\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}}$

Where a and b are non-negative real numbers

Examples:

$$\sqrt{8} \cdot \sqrt{2} = \sqrt{16} = 4$$

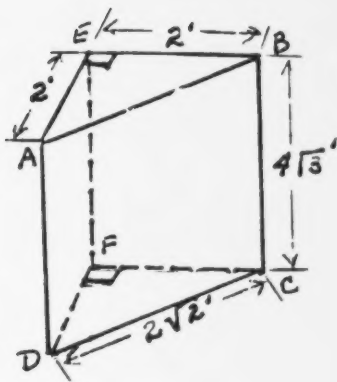
Now $5\sqrt{3}$ means 5 times $\sqrt{3}$.

$$5\sqrt{3} \cdot 2\sqrt{6} = 5\sqrt{3} \cdot 2\sqrt{6} = 5 \cdot 2 \sqrt{3\sqrt{6}} = 10\sqrt{18}$$

$$(2\sqrt{7})^2 = 2\sqrt{7} \cdot 2\sqrt{7} = 2 \cdot 2 \sqrt{7\sqrt{7}} = 4\sqrt{49} = 4 \cdot 7 = 28$$

$$\sqrt{98} \div \sqrt{2} = \frac{\sqrt{98}}{\sqrt{2}} = \sqrt{\frac{98}{2}} = \sqrt{49} = 7 \quad \text{Rational}$$

$$\frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}} \quad \text{Irrational}$$

Application.

The lateral faces of this triangular prism are AEFD, EBCF, and ABCD. Find the total area of the lateral faces.

$$\text{Area AEFD} = 4\sqrt{3} \cdot 2 = 4 \cdot 2\sqrt{3} = 8\sqrt{3}$$

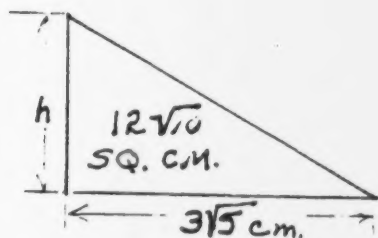
$$\text{Area EBCF} = 4\sqrt{3} \cdot 2 = 4 \cdot 2\sqrt{3} = 8\sqrt{3}$$

$$\text{Area ABCD} = 2\sqrt{2} \cdot 4\sqrt{3} = 2 \cdot 4\sqrt{2}\sqrt{3} = 8\sqrt{6}$$

$$\text{Total lateral area} = 8\sqrt{3} + 8\sqrt{3} + 8\sqrt{6}$$

These two terms have identical radicals, therefore can be combined like similar terms.

$$\text{Total lateral area} = 16\sqrt{3} + 8\sqrt{6} \text{ square units.}$$

Application.

Find the altitude of a right triangle whose area is $12\sqrt{10}$ square centimeters and whose base is $3\sqrt{5}$ centimeters.

$$A = \frac{1}{2} \cdot b \cdot h$$

$$12\sqrt{10} = \frac{1}{2} \cdot 3\sqrt{5} \cdot h$$

$$\frac{2}{1} \cdot (12\sqrt{10}) = \frac{2}{1} \cdot \left(\frac{1}{2} \cdot 3\sqrt{5} \cdot h \right)$$

$$24\sqrt{10} = 3\sqrt{5} \cdot h$$

$$\text{So } h = \frac{24\sqrt{10}}{3\sqrt{5}} = 8\sqrt{2} \text{ sq. cm}$$

Drill and Practice:
 Rising, p. 284-285
 Dressler, p. 609, 611

18. CONCEPT: EXPRESSING RADICALS IN SIMPLEST FORM

Rule: A radical expression is in "simplest" form when there is:

- 1) no perfect square (except 1) under the radical sign.
- 2) no radical in the denominator.
- 3) no fraction under the radical sign.

Two relationships are helpful.

$$(\sqrt[n]{a})^n = a \quad \text{Ex. } (\sqrt{5})^2 = 5$$

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \quad \text{Ex. } \sqrt{6} = \sqrt{3 \cdot 2} = \sqrt{3} \sqrt{2}$$

Examples: Simplify

$$\sqrt{200} = \sqrt{100 \cdot 2} = \sqrt{100} \sqrt{2} = 10\sqrt{2}$$

↑
perfect square

$$\sqrt{180} = \sqrt{36 \cdot 5} = \sqrt{36} \cdot \sqrt{5} = 6\sqrt{5}$$

$$\frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{4\sqrt{3}}{\sqrt{9}} = \frac{4\sqrt{3}}{3}$$

$$\sqrt{\frac{4}{7}} = \frac{\sqrt{4}}{\sqrt{7}} = \frac{2}{\sqrt{7}} = \frac{2\sqrt{7}}{\sqrt{7}\sqrt{7}} = \frac{2\sqrt{7}}{\sqrt{49}} = \frac{2\sqrt{7}}{7}$$

$$\sqrt{18x^3} = \sqrt{9 \cdot 2 \cdot x^2 \cdot x} = \sqrt{9x^2} \cdot \sqrt{2x} = 3x\sqrt{2x}$$

Write with a rational denominator.

$$\frac{3}{\sqrt{10}} = \frac{3 \cdot \sqrt{10}}{\sqrt{10} \cdot \sqrt{10}} = \frac{3 \cdot \sqrt{10}}{\sqrt{100}} = \frac{3\sqrt{10}}{10}$$

Rationalize the denominator.

$$\frac{\sqrt{6}}{\sqrt{5}} = \frac{\sqrt{6} \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{\sqrt{30}}{\sqrt{25}} = \frac{\sqrt{30}}{5}$$

Application 1.

The calculation of the length of a radius of a circle produced an answer of

$$6\sqrt{24} \text{ meters.}$$

Rewrite $6\sqrt{24}$ in simplest form.

$$6\sqrt{24} = 6\sqrt{4 \cdot 6} = 6\sqrt{4} \sqrt{6} = 6 \cdot 2\sqrt{6} = 12\sqrt{6}$$

↑
perfect square

Application 2.

The area of a triangular piece of
sheet metal was

154

$50\sqrt{27}$ square inches. Rewrite
this number in simplest form.

$$50\sqrt{27} = 50\sqrt{9 \cdot 3} = 50\sqrt{9}\sqrt{3} = 50 \cdot 3\sqrt{3} = 150\sqrt{3}$$

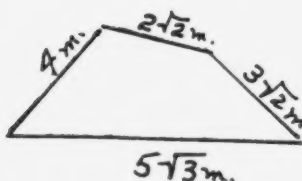
square
inches

Drill and Practice:

Rising, p. 285-286

Dressler, p. 604-605

19. CONCEPT: SIMPLIFYING RADICAL EXPRESSIONS INVOLVING ADDITION AND SUBTRACTION



Find the perimeter of this floor area.

$$\text{Perimeter} = 5\sqrt{3} + 4 + 2\sqrt{2} + 3\sqrt{2}$$

Can we simplify this expression?

Now, square root radicals

with the same radicand are similar or like radicals. Therefore we can combine

$$2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2} \text{ (using the distributive property)}$$

$$\text{Finally } P = 5\sqrt{3} + 4 + 2\sqrt{2} + 3\sqrt{2}$$

$$P = 5\sqrt{3} + 4 + 5\sqrt{2} \text{ Meters}$$

Note: This represents the exact value of the perimeter.

$$\begin{aligned} \text{In practice: } P &= 5 \cdot (1.721) + 4 + 5 \cdot (1.414) \\ P &= 8.667 + 4 + 7.070 \\ P &= 19.73 \text{ meters (Approximate value of perimeter)} \end{aligned}$$

Examples: Simplify

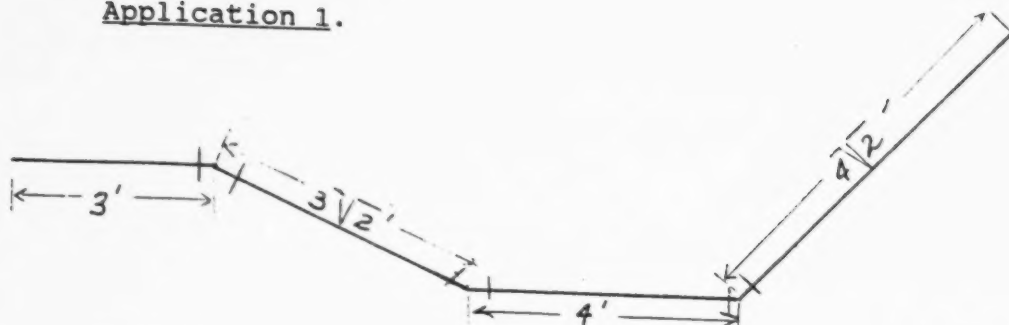
$$5\sqrt{7} - 2\sqrt{7} = 3\sqrt{7}$$

$$13\sqrt{11} + 2\sqrt{11} - 4\sqrt{11} = 11\sqrt{11}$$

Now consider

$$\begin{aligned} &\sqrt{200} + 5\sqrt{2} \\ &\sqrt{100 \cdot 2} + 5\sqrt{2} \\ &\sqrt{100} \sqrt{2} + 5\sqrt{2} \\ &10\sqrt{2} + 5\sqrt{2} = 15\sqrt{2} \end{aligned}$$

Sometimes, if a radical itself is simplified, the result allows addition or subtraction of similar radicals.

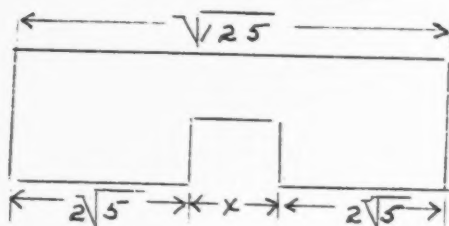
Application 1.

Find the total length of steam pipe.

$$\begin{aligned}\text{Total length} &= 3 + 3\sqrt{2} + 4 + 4\sqrt{2} \\ &= 3 + 4 + 3\sqrt{2} + 4\sqrt{2} \\ &= 7 + 7\sqrt{2} \text{ (exact length)}\end{aligned}$$

Combine
similar terms
and radicals.

$$\begin{aligned}\text{Total length} &= 7 + 7 \cdot (1.414) \\ &= 7 + 9.898 = 16.898 \text{ ft. (Approx.)}\end{aligned}$$

Application 2.

Find the exact measure of the dimension x.

$$\begin{aligned}x &= \sqrt{125} - 2\sqrt{5} - 2\sqrt{5} \\ x &= \sqrt{25 \cdot 5} - 2\sqrt{5} - 2\sqrt{5} \\ x &= 5\sqrt{5} - 4\sqrt{5} \\ x &= 1\sqrt{5} \text{ or } \sqrt{5} \text{ cm}\end{aligned}$$

Find the approximate value of x.

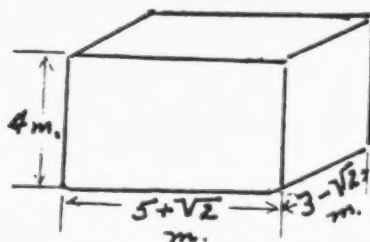
$$x = 5 = 2.236067977$$

Read out

$$x = 5 = 2.24 \text{ cm}$$

Drill and Practice:
Rising, p. 287-288
Dressler, p. 607

20. CONCEPT: MULTIPLICATION OF TWO BINOMIALS CONTAINING RADICALS



The volume of the container to the left is given by the formula

$$V = lwh$$

Where l = length

w = width

h = height

Using the dimensions as indicated in the diagram, calculate the volume.

$$V = (5 + \sqrt{2})(3 - \sqrt{2}) \cdot 4$$

Now $(5 + \sqrt{2})$ and $(3 - \sqrt{2})$ are binomials, so we can use the FOIL method.

$$V = (5 \cdot 3 - 5\sqrt{2} + 3\sqrt{2} - \sqrt{2}\sqrt{2}) \cdot 4$$

$$V = (15 - 2\sqrt{2} - \sqrt{4}) \cdot 4$$

$$V = (15 - 2\sqrt{2} - 2) \cdot 4$$

$$V = (13 - 2\sqrt{2}) \cdot 4 \quad \text{Now multiply by 4}$$

$$V = 52 - 8\sqrt{2} \quad \text{cubic meters.}$$

Examples:

$$(6 - \sqrt{3})(7 - \sqrt{3}) = 42 - 6\sqrt{3} - 7\sqrt{3} + \sqrt{9} = 42 - 13\sqrt{3} + 3 = 45 - 13\sqrt{3}$$

$$\begin{aligned} (5 + 2\sqrt{7})(8 - 5\sqrt{7}) &= 40 - 25\sqrt{7} + 16\sqrt{7} - 10\sqrt{49} = 40 - 9\sqrt{7} - 10 \cdot 7 \\ &= 40 - 9\sqrt{7} - 70 \\ &= -30 - 9\sqrt{7} \end{aligned}$$

$$\begin{aligned} (2 - \sqrt{10})^2 &= (2 - \sqrt{10})(2 - \sqrt{10}) = 4 - 2\sqrt{10} - 2\sqrt{10} + \sqrt{100} \\ &= 4 - 4\sqrt{10} + 10 = 14 - 4\sqrt{10} \end{aligned}$$

or remembering that $(a+b)^2 = a^2 + 2ab + b^2$

and $(a-b)^2 = a^2 - 2ab + b^2$

$$\begin{aligned} \text{Then } (2 - \sqrt{10})^2 &= 2^2 - 2(2\sqrt{10}) + (\sqrt{10})^2 \\ &= 4 - 4\sqrt{10} + 10 = 14 - 4\sqrt{10} \end{aligned}$$

Consider

$$\begin{aligned} (6 - \sqrt{15})(6 + \sqrt{15}) &= 36 + 6\sqrt{15} - 6\sqrt{15} - \sqrt{15} \cdot \sqrt{15} \\ &= 36 - \sqrt{225} = 36 - 15 = \underline{21} \end{aligned}$$

Notice that 21 is rational. Numbers like $6 - \sqrt{15}$ and $6 + \sqrt{15}$ (which are identical binomials except for the signs between the terms) are called conjugate pairs.

Example:

The conjugate of $9 - \sqrt{5}$ is $9 + \sqrt{5}$
 The conjugate of $8 + \sqrt{2}$ is $8 - \sqrt{2}$
 The conjugate of $6 - \sqrt{2}$ is $6 + \sqrt{2}$
 The conjugate of -7 is $+7$

Remember that $(a+b)(a-b) = a^2 - b^2$.

Therefore $(9 - \sqrt{17})(9 + \sqrt{17}) = 81 - (\sqrt{17})^2 = 81 - 17 = \underline{64}$

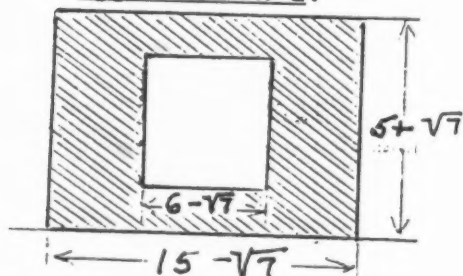
We can use this property of conjugate pairs to rationalize denominators which contain irrational binomials.

Example: Write $\frac{5}{1 + \sqrt{2}}$ with a rational denominator.

$$\begin{aligned} \frac{5}{1 + \sqrt{2}} &= \frac{5}{(1 + \sqrt{2}) \cdot (1 - \sqrt{2})} = \frac{5(1 - \sqrt{2})}{1^2 - (\sqrt{2})^2} = \frac{5(1 - \sqrt{2})}{1 - 2} = \frac{5(1 - \sqrt{2})}{-1} \\ &= -5(1 - \sqrt{2}) \end{aligned}$$

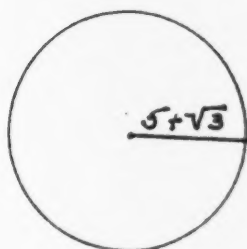
Note: Multiply numerator and denominator by the conjugate of the denominator.

Application 1.



If a square is cut from a rectangular piece of sheet metal with the indicated dimensions, find the area of the shaded part.

$$\begin{aligned} \text{Shaded Area} &= LW - s^2 \\ &= (15 - \sqrt{7})(5 + \sqrt{7}) - (6 - \sqrt{7})^2 \\ &= 75 + 15\sqrt{7} - 5\sqrt{7} - (\sqrt{7})^2 - [36 - 2 \cdot 6\sqrt{7} + (\sqrt{7})^2] \\ &= 75 + 10\sqrt{7} - 7 - (36 - 12\sqrt{7} + 7) \\ &= 75 + 10\sqrt{7} - 7 - 36 + 12\sqrt{7} - 7 \\ &= 25 + 22\sqrt{7} \text{ square inches} \end{aligned}$$

Application 2.

If the radius of a circle is $5 + \sqrt{3}$ cm find the area of the circle.

$$\begin{aligned}
 A &= \pi r^2 \\
 A &= \pi (5 + \sqrt{3})^2 \\
 &= \pi (25 + 2 \cdot 5\sqrt{3} + \sqrt{9}) \\
 &= \pi (25 + 10\sqrt{3} + 3) \\
 &= \pi (28 + 10\sqrt{3}) \text{ (exact)} \\
 &\quad \text{square} \\
 &\quad \text{centimeters.}
 \end{aligned}$$

$$\begin{aligned}
 \text{or} \\
 &= 3.14 [28 + 10(1.732)] \\
 &= 3.14 [28 + 17.32] \\
 &= 3.14 (45.32) = 142.3048 \\
 &= 142.3 \text{ sq. cm.}
 \end{aligned}$$

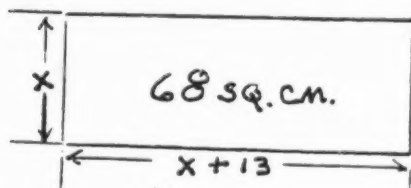
Drill and Practice:
 Rising, p. 290

21. CONCEPT: THE QUADRATIC FORMULA

In the design of a circuit board, the dimensions included an area of 68 square centimeters and a length which is 13 centimeters longer than the width. Find the dimensions.

Solution:

Let x = width
 $x+13$ = length



$$\begin{aligned} x(x+13) &= 68 \\ x^2+13x &= 68 \\ x^2+13x-68 &= 0 \leftarrow \text{(Subtract 68 from both members)} \\ (x+17)(x-4) &= 0 \\ x+17 &= 0 & x-4 &= 0 \\ x &= -17 & x &= 4 \text{ cm} \\ \text{discard} & & x+4 &= 17 \text{ cm} \end{aligned}$$

(Dimensions must be positive)

Now, there is a formula called the Quadratic Formula which will give the same result.

$$\text{Quadratic Formula: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

First write equation in standard form:

$$ax^2 + bx + c = 0$$

$$\begin{aligned} \text{So, } x^2 + 13x - 68 &= 0 \\ \text{where } a &= 1, b = +13, c = -68 \end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-13 \pm \sqrt{13^2 - 4 \cdot 1 \cdot (-68)}}{2(1)}$$

$$x = \frac{-13 \pm \sqrt{169+272}}{2}$$

$$x = \frac{-13 \pm \sqrt{441}}{2}$$

$$x = \frac{-13 \pm 21}{2}$$

$$\text{so } x = \frac{-13+21}{2} \quad \text{or} \quad x = \frac{-13-21}{2}$$

$$x = \frac{8}{2} = 4 \text{ cm} \quad \text{or} \quad x = \frac{-34}{2}$$

$$\text{and } x = -17 \text{ discard}$$

$$x+13 = 17 \text{ cm}$$

Dimensions (4 cm x 17 cm).

Example: If a quadratic trinomial in an equation is factorable you may prefer not to use the formula. But if the quadratic is not factorable the use of the formula is essential.

Solve: $x(x+10) = 16$
 $x^2+10x - 16 = 0.$
 $a = 1, b = 10, c = -16$

Therefore $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-10 \pm \sqrt{10^2 - 4 \cdot 1 \cdot -16}}{2(1)}$$

$$x = \frac{-10 \pm \sqrt{100 + 64}}{2}$$

$$x = \frac{-10 \pm \sqrt{164}}{2} \quad \sqrt{164} = 12.8$$

$$x = \frac{-10 \pm 12.8}{2}$$

$$x = \frac{-10 + 12.8}{2} = \frac{2.8}{2} = 1.4$$

or $x = \frac{-10 - 12.8}{2} = \frac{-22.8}{2} = -11.4$ } Roots

Applications. Find the outer radius R if the

$$h = 5 \text{ cm}$$

inner radius (r) = 2.4 cm. and the total volume is 207 cu. cm.

Use formula Volume = $1.05h(R^2 + Rr + 5.76)$

Therefore, substituting



$$207 = 1.05(5)(R^2 + 2.4R + 5.76)$$

$$207 = 5.25(R^2 + 2.4R + 5.76)$$

$$\frac{207}{5.25} = R^2 + 2.4R + 5.76$$

$$39.4 = R^2 + 2.4R + 5.76$$

$$0 = R^2 + 2.4R + 5.76 - 39.4$$

$$0 = R^2 + 2.4R - 33.64$$

This is a quadratic equation in standard form.

$$a = 1, b = 2.4, c = -33.64$$

Therefore

$$R = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$R = \frac{-2.4 \pm \sqrt{(2.4)^2 - 4(1)(-33.64)}}{2(1)}$$

$$R = \frac{-2.4 \pm \sqrt{5.76 + 134.56}}{2}$$

$$R = \frac{-2.4 \pm \sqrt{140.32}}{2}$$

$$R = \frac{-2.4 \pm 11.85}{2}$$

Therefore

$$R = \frac{-2.4 + 11.85}{2} \quad \text{or} \quad R = \frac{-2.4 - 11.85}{2}$$

$$R = \frac{9.45}{2} = 4.725 \quad R = \frac{-14.25}{2}$$

$$R = \underline{\underline{4.7 \text{ cm}}} \quad R = -7.125 \text{ discard}$$

Drill and Practice:
Rising, p. 292-293

RATIO AND PROPORTION

1. CONCEPT: AN UNDERSTANDING OF RATIO

If 977 parts out of 1000 parts examined pass inspection, we can compare these numbers by writing this fraction.

$$\frac{977}{1000}$$

or

$$977:1000$$

When two numbers are compared in this manner, it is called a ratio, and it is read, "nine hundred seventy-seven is to one thousand".

To compare the number of parts passing to that number which did not pass we would write $\frac{977}{33}$ or 977:33. This is read "nine hundred seventy-seven is to thirty-three".

Examples: Compare 5 and 4

$$"5 \text{ is to } 4" = \frac{5}{4} \text{ or } 5:4$$

Compare 20 and 30

$$"20 \text{ is to } 30" = \frac{20}{30} \text{ or } 20:30$$

However, like many fractions $\frac{20}{30}$ can be reduced. $\frac{20}{30} = \frac{2}{3}$ or 2:3

To sum up "20 is to 30" as "2 is to 3"

Equal Ratios are in the form of equal fractions.

$$\text{That is } \frac{20}{30} = \frac{2 \times 10}{3 \times 10} = \frac{2}{3}$$

Application.

72 out of 981 employees were absent on a work day.

Write a comparison which indicates the ratio of employees absent to the total number employees.

$$\frac{72}{981} = \frac{8}{109} \quad \text{or } 8:109$$

Drill and Practice:

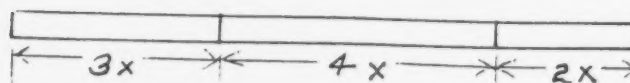
Rising, p. 302 (1-17), 303 (1-12)

Dressler, p. 286-287

Smith, p. 164

Sullivan, p. 109

2. CONCEPT: THE USE OF RATIO IN AN ALGEBRAIC OR GEOMETRIC SETTING



A sprinkler system included a pipe 18 ft. long. Three sections of the pipe were in the ratio of 3:4:2. Find the length of each section.

Let $3x$ = length of first section
 $4x$ = length of second section
 $2x$ = length of third section

$$\begin{aligned}\text{Therefore } 3x + 4x + 2x &= 18 \\ 9x &= 18 \\ \frac{9x}{9} &= \frac{18}{9} \\ x &= 2\end{aligned}$$

$$\begin{aligned}\text{So } 3x &= 3(2) = 6 \text{ feet} \\ 4x &= 4(2) = 8 \text{ feet} \\ 2x &= 2(2) = 4 \text{ feet} \\ \text{Total} &= 18 \text{ feet}\end{aligned}$$

Example:

The ratio of two numbers is 5:2. Their difference is 24. Find the numbers

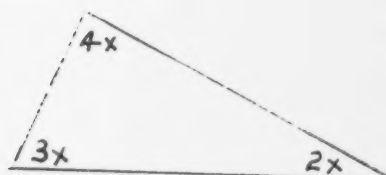
Let $5x$ = 1st number
 $2x$ = 2nd number

$$\begin{aligned}\text{Therefore } 5x - 2x &= 24 \\ 3x &= 24 \\ \frac{3x}{3} &= \frac{24}{3} \\ x &= 8\end{aligned}$$

$$\begin{aligned}\text{SO } 5x &= 5(8) = 40 \\ 2x &= 2(8) = 16 \\ \text{Difference} &= 24 \quad \text{Ans.}\end{aligned}$$

Application. The ratio of the measures of 3 angles of a triangle were 3:2:4. Find the measures of the angles.

(Remember, the sum of the measures of the angles of a triangle is 180 degrees).



$$\begin{aligned}\text{Therefore } 3x + 4x + 2x &= 180 \\ 9x &= 180 \\ \frac{9x}{9} &= \frac{180}{9}\end{aligned}$$

$$\begin{aligned}x = 20, \quad 4x &= 4(20) = 80^\circ \\ 3x &= 3(20) = 60^\circ \\ 2x &= 2(20) = 40^\circ \\ \text{Total} &= 180^\circ\end{aligned}$$

Drill and Practice:
Rising, p. 303-304
Dressler, p. 290-291

3. CONCEPT: PROPORTIONS

If, of 621 employees, 582 are hourly employees, the ratio

$$\frac{\text{No. of hourly employees}}{\text{No. of total employees}} \text{ is } \frac{582}{621}$$

This ratio can be reduced $\frac{582}{621} = \frac{582 \div 3}{621 \div 3} = \frac{194}{207}$

$\frac{581}{621}$ and $\frac{194}{207}$ are equal or equivalent ratios.

Therefore we can write

$$\frac{581}{621} = \frac{194}{207} \quad \text{This equation is called a proportion.}$$

Sometimes a proportion is written as

$$581:621 = 194:207 \quad \text{and is read}$$

"581 is to 621 as 194 is to 207"

The terms of the proportion have special names.

(first) $\frac{581}{621} = \frac{194}{207}$ (third) or $581:621 = 194:207$
 (second) $\frac{581}{621} = \frac{194}{207}$ (fourth) first:second = third:fourth

The first and fourth terms are called the extremes.
 Example: 581 and 207

The second and third terms are called the means.
 Example: 621 and 194

Notice that in a proportion:

$$\frac{2}{3} = \frac{4}{6} \quad \text{that } 2 \cdot 6 = 3 \cdot 4$$

$$12 = 12$$

In general if $\frac{a}{b} = \frac{c}{d}$ or $a:b = c:d$

$$\text{then } a \cdot d = b \cdot c.$$

The product of the extremes equals the product of the means.

This rule can be used to solve equations when the value of one of the terms in a proportion is unknown.

Example: $\frac{x}{4} = \frac{7}{8}$

$$x \cdot 8 = 4 \cdot 7$$

$$8x = 28$$

$$\frac{8x}{8} = \frac{28}{8}$$

$$x = \frac{14}{4} = 3 \frac{2}{4} = 3 \frac{1}{2}$$

Example: $5:x = 15:20$ 167

$$15x = 5 \cdot 20$$

$$15x = 100$$

$$\frac{15x}{15} = \frac{100}{15}$$

$$x = \frac{20}{3} = 6 \frac{2}{3}$$

Application 1. The ratio of tin to copper in an alloy is 5:1. In a sample which contains 115 grams of tin, how much copper would be found?

$$\text{Ratio of } \frac{\text{grams of tin}}{\text{grams of copper}} = \frac{5}{1}$$

Let x = no. grams of copper in sample

$$\text{Therefore } \frac{115 \text{ grams tin}}{x \text{ grams copper}} = \frac{5}{1}$$

$$\frac{115}{x} = \frac{5}{1}$$

$$5x = 115$$

$$\frac{5x}{5} = \frac{115}{5}$$

$$x = 23 \text{ grams of tin.}$$

Application 2. From a blueprint 1 centimeter represented 2 feet in a floor plan. How many feet does 3 centimeters in the print represent?

Let x = No. of feet represented by 3 centimeters.

$$\frac{1 \text{ cm}}{2 \text{ feet}} = \frac{3 \text{ cm}}{x \text{ (feet)}}$$

Notice that each ratio may have mixed dimensions as the exact dimensions appear in each ratio.

$$\text{Therefore } 1 x = 2 \cdot 3$$

$$x = 6 \text{ feet.}$$

Drill and Practice:
Rising, p. 306, 307

4. CONCEPT: PERCENT AND THE SOLUTION OF PROBLEMS INVOLVING PERCENT

Percent is a special ratio in which the second term is 100.

$$3\% = \frac{3}{100}$$

$$17\% = \frac{17}{100}$$

$$6.4\% = \frac{6.4}{100} \text{ or } \frac{64}{1000}$$

$$325\% = \frac{325}{100}$$

Using proportions of the form

$$\frac{r}{100} = \frac{P}{W}$$

where $\begin{matrix} P = \text{Part} \\ W = \text{Whole} \end{matrix}$

$$r\% = \frac{r}{100}$$

Many problems involving percent may be solved

Examples: Find 35% of 140.

$$\frac{r}{100} = \frac{P}{W}$$

$$\begin{aligned} r &= 35 \\ W &= 140 \\ \text{Find } P &(\text{Part}) \end{aligned}$$

$$\frac{35}{100} = \frac{P}{140}$$

$$\begin{aligned} 100P &= 35(140) \\ 100P &= 4900 \end{aligned}$$

$$\frac{100P}{100} = \frac{4900}{100}$$

$$P = 49$$

64 is 8% of what number?

$$\frac{r}{100} = \frac{P}{W}$$

$$\begin{aligned} r &= 8 \\ P &= 64 \\ \text{Find } W \end{aligned}$$

$$\frac{8}{100} = \frac{64}{W}$$

$$\frac{8W}{8} = \frac{6400}{8}$$

$$W = 800$$

Application 1.

Orders for overseas sales of machine tools were down 17% during May in comparison with April. If the overseas sales total for April was \$736,200, find the overseas sales total for May:

Let x = drop in overseas sales from April to May.

$$\frac{17}{100} = \frac{x}{736,200}$$

$$100x = 17(736,200)$$

$$100x = 12515400$$

$$\frac{100x}{100} = 12515400$$

$$x = \$125,154 \text{ Drop in Sales}$$

$$\begin{aligned} \text{Overseas sales total} &= 736,200 - 125,154 \\ \text{for May} &= \underline{\$611,046} \end{aligned}$$

Application 2.

The number of employees in the assembly department grew from 321 in March to 429 in September. Find the percent increase in employment.

$$\text{Actual increase} = 429 - 321 = 108 \text{ employees.}$$

Let x = percent increase.

$$\frac{x}{100} = \frac{108}{321}$$

Note:
Use original number of
employees as denominator

$$321x = 10800$$

$$x = \frac{10800}{321} = 33.64486 \text{ Readout}$$

Ans. 33.6% increase in employment.

Drill and Practice:

Rising, p. 310-312

Dressler, p. 383-384

Sullivan, p. 123 (1-10), 21, 22

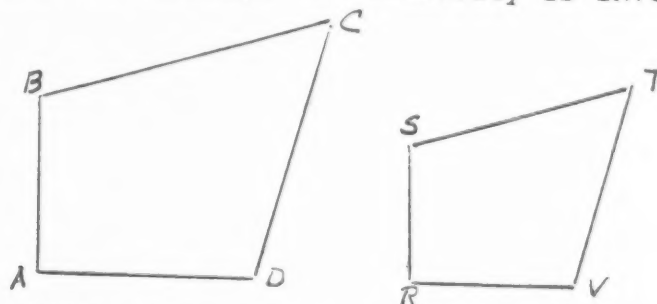
p. 124 (24, 26, 27-32, 34, 35)

**A FURTHER
DEVELOPMENT OF
GEOMETRY**

1. CONCEPT: SIMILAR POLYGONS; DEFINITION AND PROPERTIES

Any time that a reduction or enlargement of a drawing on a copier occurs, the concept of similarity is involved.

Consider:



If polygons ABCD and RSTV have the same shape, they are called similar polygons.

Definition: Two polygons are similar if their corresponding sides are proportional and their corresponding angles are congruent. The symbol $\sim$ means "is similar to". ABCD $\sim$ RSTV.

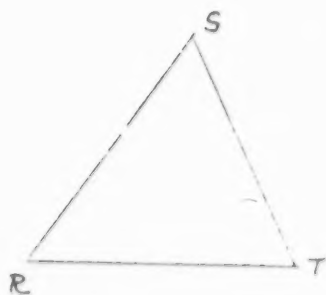
Therefore

$$\begin{array}{l} \angle A \cong \angle R \\ \angle B \cong \angle S \\ \angle C \cong \angle T \\ \angle D \cong \angle V \end{array} \quad \text{and} \quad \frac{AB}{SR} = \frac{BC}{ST} = \frac{CD}{TV} = \frac{AD}{RV}$$

Example: If $m\angle D = 92^\circ$, then $m\angle V = 92^\circ$
and If $m\angle C = 44^\circ$, then $m\angle T = 44^\circ$

Example: If $\frac{AB}{SR} = \frac{5}{4}$, then $\frac{BC}{ST} = \frac{5}{4}$ and $\frac{CD}{TV} = \frac{5}{4}$

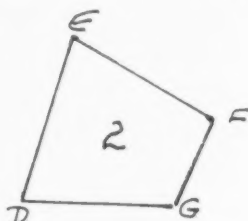
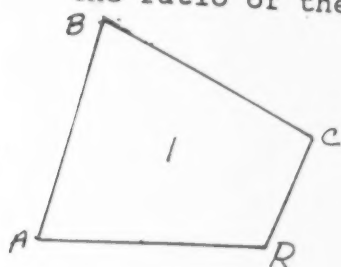
Example: If $\triangle RST \sim \triangle DEF$ and $\frac{RS}{DE} = \frac{3}{2}$



Find EF if $ST = 11$ cm.
Solution

$$\begin{array}{l} \frac{RS}{DE} = \frac{ST}{EF} \\ \frac{3}{2} = \frac{11}{x} \end{array} \quad \left| \quad \begin{array}{l} 3x = 22 \\ x = 7 \frac{1}{3} \text{ cm.} \end{array} \right.$$

In similar polygons, the ratio of their perimeters equals the ratio of the lengths of any two corresponding sides.



$$\frac{\text{Perimeter } ABCR}{\text{Perimeter } DEFG} = \frac{4}{3}$$

$$\text{or } \frac{P_1}{P_2} = \frac{S_1}{S_2}$$

Where P_1 = perimeter of polygon 1
 P_2 = perimeter of polygon 2
 S_1 = side of polygon 1
 S_2 = corresponding side of polygon 2

Example: If the ratio of the corresponding sides of two similar polygons is 7:10, find the perimeter of the second polygon if the perimeter of the first is 35 cm.

Let x = perimeter of second polygon

$$\frac{P_1}{P_2} = \frac{S_1}{S_2}$$

$$\frac{35}{x} = \frac{7}{10}$$

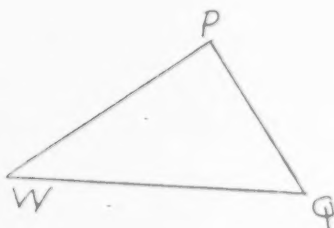
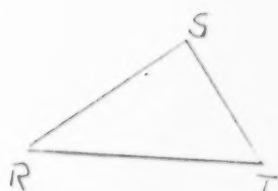
$$7x = 350$$

$$x = \frac{350}{7}$$

$$x = 50 \text{ cm.}$$

Two triangles are similar, if two angles in one triangle are congruent to two angles of the other triangle. (A A)

Example:

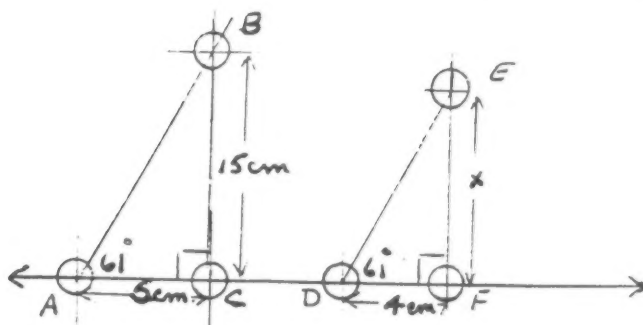


If $m\angle R = 31^\circ$ and $m\angle W = 31^\circ$
 and $m\angle T = 54^\circ$ and $m\angle Q = 54^\circ$
 Then $\triangle RST \sim \triangle WPQ$

Application 1:

Holes are to be drilled in a piece of metal where indicated; find the length of x .

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Solution: Since $m\angle BAC = m\angle EDF = 61^\circ$ ($\angle BAC = \angle EDF$) and $m\angle BCA = m\angle EFD = 90^\circ$ ($\angle BCA = \angle EFD$) then $\triangle ABC \sim \triangle DEF$. (A.A.) Therefore, the corresponding sides are proportional.

$$\frac{AC}{DF} = \frac{BC}{EF}$$

$$\frac{5}{15} = \frac{4}{x}$$

$$5x = 60$$

$$x = 12 \text{ cm.} = EF.$$

Application 2. The length of a weld along the edges of a metal plate in the form of a pentagon is 64 cm. Its shortest side is 12 cm. If the perimeter of a similar pentagon is 32, find the length of its shortest side.

Solution:

$$\frac{P_1}{P_2} = \frac{S_1}{S_2}$$

$$\frac{64}{32} = \frac{x}{12}$$

$$32x = 64(12)$$

$$32x = 768$$

$$x = 24 \text{ cm.}$$

Let x = length of shortest side of first pentagon

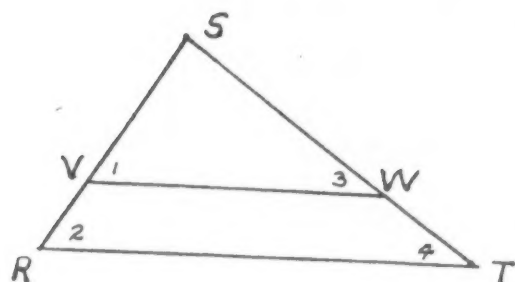
Drill and Practice:

Rising, p. 316-320

Dressler, p. 353-356

Smith, p. 165 (41), 215-216

2. CONCEPT: Theorem - If a line is parallel to one side of a triangle and intersects the other two sides a triangle is formed similar to the original triangle.



If $\overline{VW}$ is parallel to $\overline{RT}$
 then $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$
 and $\triangle VSW \sim \triangle RST$ (A.A.)

Further

$$\frac{SV}{SR} = \frac{SW}{ST} = \frac{VW}{RT}$$

(Corresponding sides of similar polygons are proportional.)

Example: If $\overline{VW} \parallel \overline{RT}$ and $VS = 8$, $RS = 15$ and $SW = 10$ find ST .

$$\frac{RS}{VS} = \frac{ST}{SW}$$

$$\text{Let } ST = x$$

$$\frac{15}{8} = \frac{x}{10}$$

$$8x = 150$$

$$x = \frac{150}{8} = 18.75$$

Example: If $\overline{VW} \parallel \overline{RT}$ and $VW = 11.2$, $RS = 8.6$ and $SV = 5.4$, find RT

$$\frac{SV}{RS} = \frac{VW}{RT}$$

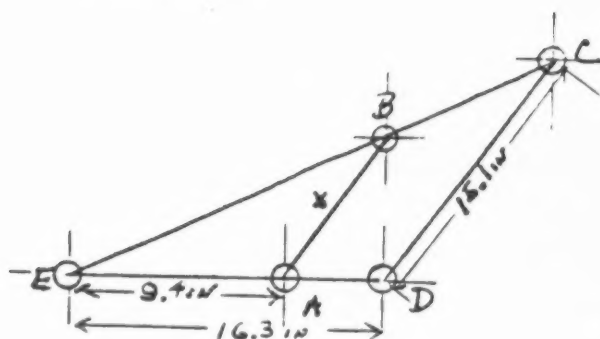
$$\text{Let } x = RT$$

$$\frac{5.4}{8.6} = \frac{11.2}{x}$$

$$5.4x = 11.2 (8.6)$$

$$5.4x = 96.32$$

$$x = \frac{96.32}{5.4} = 17.84$$

Application 1.

Given: $\overline{AB} \parallel \overline{CD}$
Find BA

Solution: $BA = x$

$$\frac{x}{15.1} = \frac{9.4}{16.3}$$

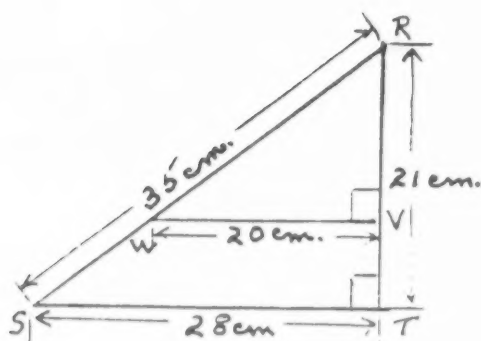
$$16.3x = (15.1)(9.4)$$

$$16.3x = 141.94$$

$$x = \frac{141.94}{16.3}$$

$$= 8.7079755$$

$$BA = x = 8.7 \text{ in.}$$

Application 2.

A weld is to be programmed along the perimeter of $\triangle RST$. If $WV = 20$ in. find the perimeter of $\triangle RWV$.

Solution: Since both $\angle RVW$ and $\angle RTS = 90^\circ$, $\overline{WV}$ and $\overline{ST}$ are both perpendicular to $\overline{RT}$ and are therefore parallel. This makes $\triangle RWV$ and $\triangle RST$ similar.

Let x = perimeter of $\triangle RWV$.

$$\frac{P(RWV)}{P(RST)} = \frac{WV}{ST}$$

$$P(RST) = 35 + 28 + 21 = 84 \text{ cm.}$$

$$\frac{x}{84} = \frac{20}{28}$$

$$28x = 1680,$$

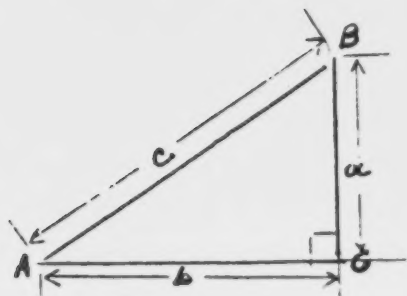
$$x = \underline{60 \text{ cm.}}$$

Drill and Practice:

Rising, p. 317 (5-11)
p. 318 (6-14, 20-23)
p. 319 (27-30)

Dressler, p. 356 (22-26)

Smith, p. 219-221 (4, 5, 6)

3. CONCEPT: THE PYTHAGOREAN THEOREM

In right triangle ABC, where $m\angle C = 90^\circ$, c = length of the hypotenuse a and b are the lengths of the legs. It can be shown the square of the length of the hypotenuse equals the sum of the squares of the lengths of the legs.

$$\text{or } \boxed{c^2 = a^2 + b^2}$$

Example: Given right triangle ABC, with $m\angle C = 90^\circ$

If $a = 3$, $b = 4$, find c .

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 3^2 + 4^2 \\ c^2 &= 9 + 16 \\ c^2 &= 25 \\ c &= 5 \end{aligned}$$

Example: Given right triangle ABC with $m\angle C = 90^\circ$

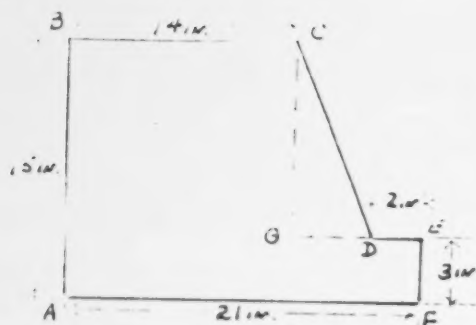
If $a = 5$, $c = 13$, find b .

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 13^2 &= 5^2 + b^2 \\ 169 &= 25 + b^2 \\ 169 - 25 &= b^2 \\ 144 &= b^2 \\ 12 &= b \text{ or } b = 12 \end{aligned}$$

Combinations of numbers such as 3, 4, 5 and 5, 12, 13 are called Pythagorean triples and are always the sides of a right triangle. The longest side is the hypotenuse. 3 - 4 - 5 triangles for centuries have been used by builders to produce right angles. If the square of the length of one side of a triangle equals the sum of the squares of the lengths of the other two sides the triangle is a right triangle.

Application 1.

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The design of a panel in a console is indicated in the drawing at the left.

Find the length CD.

Solution: Draw $\overline{CG} \perp \overline{GF}$ to complete right triangle CGD
 Now, $CG = BA - EF = 15 - 3 = 12$ in.
 $GD = AF - (BC + DE) = 21 - (14 + 2) = 5$ in.

$$\begin{aligned} (CD)^2 &= (CG)^2 + (GD)^2 && \text{Pythagorean Theorem.} \\ (CD)^2 &= 12^2 + 5^2 \\ (CD)^2 &= 144 + 25 \\ (CD)^2 &= 169 \\ CD &= \underline{13 \text{ in.}} \end{aligned}$$

Application 2.

The formula for calculation of impedance (in ohms) is an application of $c^2 = a^2 + b^2$.

Where $Z^2 = R^2 + X_L^2$ and $Z = \text{impedance (ohms)}$
 Find X_L if $R = 28,000$ ohms or 28 k. $R = \text{resistance (ohms)}$

And $Z = 35,000$ ohms or 35 k. and $X_L = \text{inductive reactance in ohms}$

Substituting

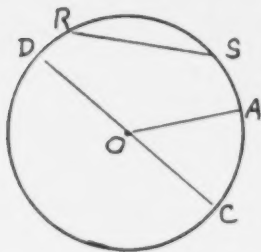
$$\begin{array}{l|l} (35)^2 = (28)^2 + (X_L)^2 & 441 = (X_L)^2 \\ 1225 = 784 + (X_L)^2 & X_L = 21 \text{ k} \\ 1225 - 784 = (X_L)^2 & \end{array}$$

Drill and Practice:

Rising, p. 328, 329
 Dressler, p. 660-663
 Smith, p. 221 (15-19)
 Sullivan, p. 145-147

4. CONCEPT: THE VOCABULARY OF THE CIRCLE

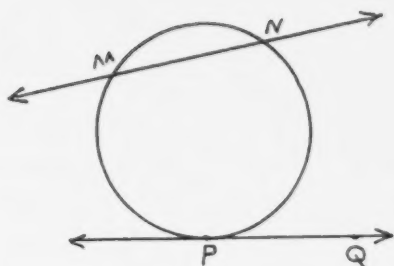
A circle (symbol $\odot$) is the set of all points in a plane at a given distance from a point O in that plane. O is called the center; the circle is named $\odot O$.



A radius is a segment drawn from the center to a point on the circle. $OA = \text{radius}$. The plural form of radius is radii.

A chord is a line segment whose end points are on the circle. RS is a chord.

A diameter is a chord which contains the center of the circle. DC is a diameter.



A secant is a line or segment which passes through two points on the circle. $\overleftrightarrow{MN}$ and $\overline{MN}$ are secants.

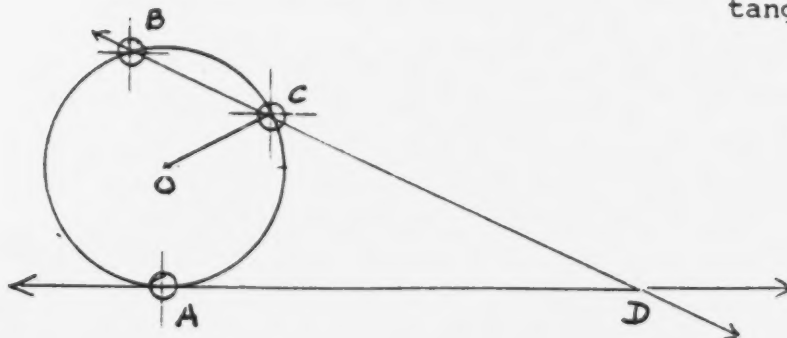
A tangent is a line or segment which has exactly one point in common with the circle. $\overleftrightarrow{PQ}$ is a tangent. P is a point of tangency.

Radii of the same circle are congruent. The length of a diameter of a circle is two times the length of its radius.
 $d = 2r$

Example: If the diameter = 6 meters, the radius equals 3 meters.

Two circles are congruent if they have congruent radii.

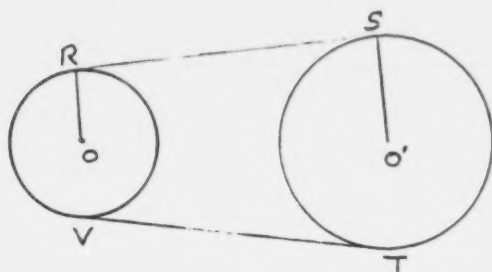
Application 1.



In this drawing $\overleftrightarrow{BC}$ and $\overline{BC}$ are secants and $\overline{AD}$ is a tangent. $\overline{OC}$ is a radius.

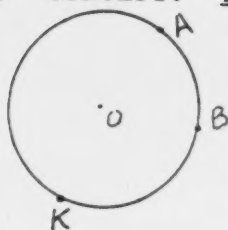
Application 2.

In a set of pulleys $\overline{RS}$
and $\overline{VT}$ are tangents.
 $\overline{OR}$ and $\overline{OS}$ are radii.



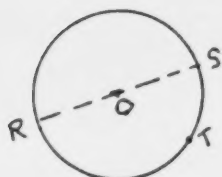
Drill and Practice:
 Rising, p. 346-348
 Dressler, p. 615-616
 Smith, p. 226

5. CONCEPT: THE ARCS OF A CIRCLE AND CENTRAL ANGLES

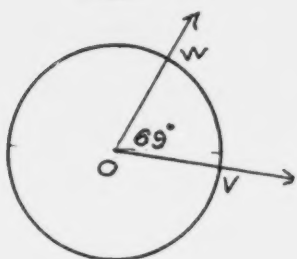


Given: Circle O; A and B are points on the circle.

Arcs are the set of points on a circle between and including two points on the circle.



There are two arcs on this circle with A and B for end points. The shorter arc AB ($\widehat{AB}$) is called the minor arc. The longer arc is called the major arc AKB ($\widehat{AKB}$). To avoid confusion an arc named by its end points is a minor arc. If RS is a diameter then $\widehat{RTS}$ is a semicircle.

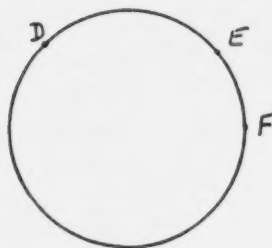


A central angle is an angle whose vertex is at the center of the circle.

$\angle WOV$ is a central angle.

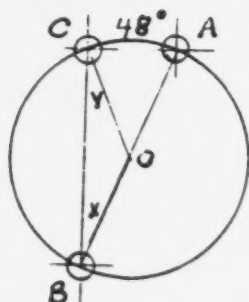
The measure of an arc equals the measure of its central angle, and conversely. Therefore if $m\angle WOV = 69^\circ$, then $m\widehat{WV} = 69^\circ$. The measure of a semicircle = 180° .

Arcs of the same circle which have only one common point can be combined.



$$m\widehat{DE} + m\widehat{EF} = m\widehat{DF}$$

Arcs with the same measure are congruent.

Application 1.

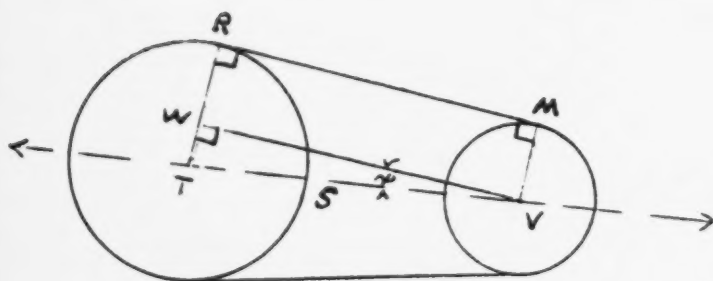
Holes are to be programmed to be drilled at A, B, C, all points on circle O. Find the measure of $\angle x$.

Note: $\therefore$ = "Therefore".

Solution:

$$\begin{aligned} m\widehat{CA} &= 48^\circ \\ m\angle COA &= 48^\circ \\ \text{and } m\angle COA &= m\angle x + m\angle y \\ m\angle x &= m\angle y \\ \text{so } m\angle COA &= 2m\angle x \\ \therefore 48 &= 2m\angle x \\ \text{and } m\angle x &= \frac{48}{2} = 24^\circ \end{aligned}$$

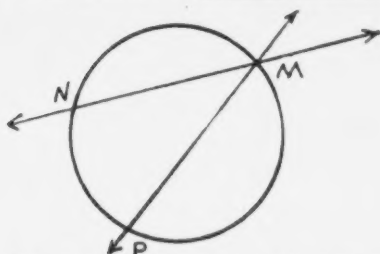
(Diagram)
(Central Angle)
(Ext. $\angle$ equals the sum of remote interior angles)
(Base angles of isosceles triangle)
(Substitution)
(Substitution)
(Division Prop. of = .)

Application 2.

In a pulley system
 $RS = 80^\circ$
Find $m\angle x$

Central angle $RTV = 80^\circ$
In right triangle WTV
 $\angle x = 90 - m\angle RTV$
 $\angle x = 90 - 80 = 10^\circ$

Drill and Practice:
Rising, p. 351-353
Dressler, p. 633-635

6. CONCEPT: THE INSCRIBED ANGLE OF A CIRCLE

An angle formed by two chords (or secants) which intersect on the circle is an inscribed angle.

Example: $\angle NMP$ is formed by two secants MN and MP which meet at point M on the circle.

$\angle NMP$ is said to intercept $\widehat{NP}$.

$\angle NMP$ is said to be inscribed in $\widehat{NP}$.

The measure of an inscribed angle is equal to one-half the measure of its intercepted arc.

Therefore $m\angle NMP = (1/2)m\widehat{NP}$. (See Concept 5, Application 1 for demonstration)

Examples: Given: Inscribed angles RST and WVM
Find $m\angle RST$, $m\angle WVM$.

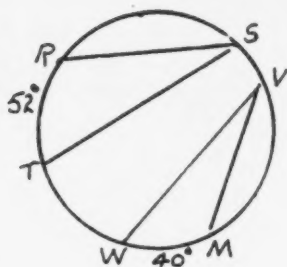
Solution:

$$m\angle RST = (1/2)m\widehat{RT}$$

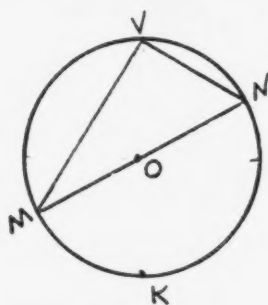
$$m\angle RST = (1/2)(52) = 26^\circ$$

$$m\angle WVM = (1/2)m\widehat{WM}$$

$$m\angle WVM = (1/2)(40) = 20^\circ$$



An angle inscribed in a semicircle is a right angle.



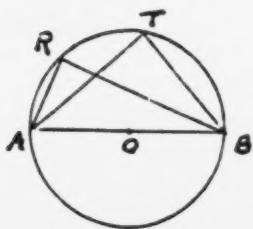
If $\overline{MN}$ is a diameter, $\widehat{MKN}$ is a semicircle with a measure of 180° .
The $m\angle MVN = (1/2)m\widehat{MKN}$.

$$\text{Therefore } m\angle MVN = (1/2)(180) = 90^\circ$$

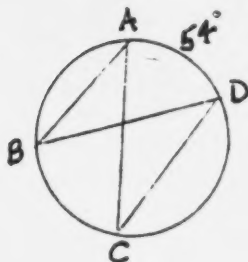
So $\angle MVN$ is a right angle.

Example:

If $\overline{AB}$ is a diameter
 $m\angle R = 90^\circ$ and $m\angle T = 90^\circ$



Example:

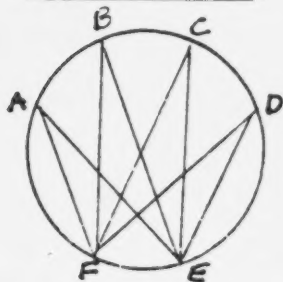


$$m\angle B = (1/2) \widehat{AD} = 1/2(54) = 27^\circ$$

$$m\angle C = (1/2) \widehat{AD} = 1/2(54) = 27^\circ$$

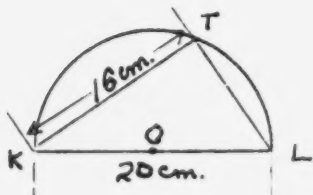
$\widehat{AD}$ is the intercepted arc for both $\angle B$ and $\angle C$.

Application 1.



In an ornamental iron grill fabricated by a welder, angles with vertices at A, B, C and D have the same measure, since they each equal one-half the measure of intercepted arc $\widehat{FE}$.

Application 2.



If $\overline{KL}$ is a diameter = 20 cm. and $\overline{KT}$ and $\overline{TL}$ are chords. Find TL if $KT = 16$ cm.

Solution: $\angle T$ is a RT. $\angle$. therefore

$$\triangle KTL \text{ is a right } \triangle \text{ and}$$

$$(KL)^2 = (KT)^2 + (TL)^2$$

$$(20)^2 = (16)^2 + (TL)^2$$

$$400 = 256 + (TL)^2$$

$$400 - 256 = (TL)^2$$

$$(TL)^2 = 144$$

$$TL = 12 \text{ cm.}$$

Drill and Practice:

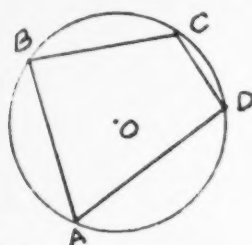
Rising, p. 356-357

Oral (1-9)

Written (1-14)

Dressler, p. 641-642

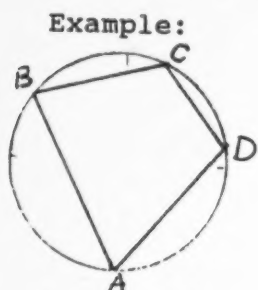
7. CONCEPT: THE INSCRIBED POLYGON AND THE CIRCUMSCRIBED CIRCLE



If each vertex of a polygon lie on a circle, we say that the polygon is inscribed within the circle and the circle is circumscribed about the polygon.

Example: ABCD is inscribed within circle O, and circle O is circumscribed about polygon ABCD.

If a quadrilateral is inscribed within a circle, the opposite angles are supplementary.



Application 1.

Suppose $m\widehat{ADC} = 200^\circ$.

Then the $m\widehat{ABC} = 360 - 200 = 160^\circ$

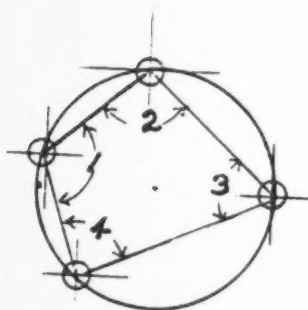
So $m\angle B = (1/2)(\widehat{ADC}) = (1/2)(200) = 100^\circ$

and $m\angle D = (1/2)\widehat{ABC} = (1/2)(160) = 80^\circ$

Therefore $m\angle B + m\angle D = 180^\circ$

In like manner we can show

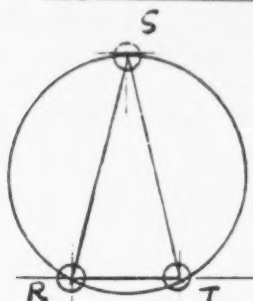
$m\angle A + m\angle C = 180^\circ$



If, in the accompanying drawing, $m\angle 1 = 140^\circ$. Find the $m\angle 3$.

Solution: $m\angle 3 = 180 - m\angle 1$
 $\angle 3 = 180 - 140 = 40^\circ$

Must we know the measure of $\angle 2$ in order to find $m\angle 4$? Answer, yes.

Application 2.

Triangle RST is inscribed in circle O. $RS = ST$, $m\angle S = 22^\circ$

Find $m\angle R$. Find $m\widehat{ST}$.

Solution: Since $RS = ST$, then $m\angle R = m\angle T$.
Since they are base angles in an isosceles $\triangle$.

Now, $m\angle S + m\angle R + m\angle T = 180$

$$\text{or } 22 + 2m\angle R = 180$$

$$2m\angle R = 180 - 22$$

$$2m\angle R = 158$$

$$m\angle R = 79^\circ$$

Now, $\angle R$ is an inscribed angle.

$$\text{so } m\angle R = \frac{1}{2} m\widehat{ST}$$

$$79 = \frac{1}{2} m\widehat{ST}$$

$$\text{and } 2(79) = 2 \left(\frac{1}{2} \right) m\widehat{ST}$$

$$158^\circ = m\widehat{ST}$$

$$\text{or } m\widehat{ST} = 158^\circ$$

Drill and Practice:

Rising, p. 359 (17, 18, 19)

8. CONCEPT: THE CIRCUMFERENCE OF A CIRCLE

The ratio $\frac{C}{d}$ is an irrational number where C is the circumference (distance around) of a circle and d is the diameter of the circle. The ratio $\frac{C}{d}$ is represented by the Greek letter pi or π . ($\pi = 3.14159....$) The ratio $\frac{C}{d} = \pi$ allows us to write a formula $C = \pi d$ by multiplying both numbers by d.

The circumference of a circle equals the product of π and the diameter of the circle.

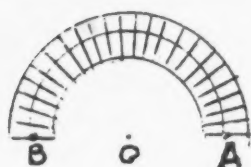
Example: Find the circumference of the circle whose radius is 7 cm.

$$\begin{aligned} C &= \pi d & d &= 2r \\ C &= 3.14(14) & d &= 2(7) = 14 \text{ cm.} \\ C &= 43.96 \text{ cm.} & \pi &= 3.14 \end{aligned}$$

Another version of this formula is

$$\begin{aligned} C &= 2\pi r & \text{since } d &= 2r. \\ \text{so } C &= 2(3.14) 7 \\ C &= 43.96 \text{ cm.} \end{aligned}$$

Application 1.



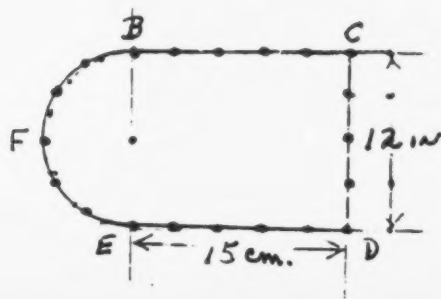
A conveyer is in the shape of a semicircle.

If $BA = 20$ ft.
find the length of the conveyer.

$$\begin{aligned} C &= \pi d \\ C &= 3.14(20) \\ C &= 62.8 \text{ ft.} \\ & \text{(circumference of} \\ & \text{complete circle)} \end{aligned}$$

$$\begin{aligned} \text{Length of conveyer (semicircle)} \\ &= \frac{1}{2}(62.8) = \underline{31.4} \text{ feet} \end{aligned}$$

Application 2.



Spot welds are to be programmed along the border of this piece of metal. EBCD is a rectangle. EFB is a semicircle. If 20 spot welds are to be programmed, how far apart will they be?

Total perimeter = length semicircle + BC + CD + ED.

$$\begin{aligned}\text{Length } \widehat{BFE} &= \frac{1}{2} [\pi \cdot d] \\ &= \frac{1}{2} [3.14(12)] \\ &= 18.84 \text{ in.}\end{aligned}$$

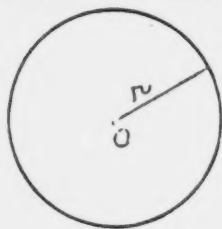
$$\begin{aligned}\text{Total perimeter} &= 18.84 + 15 + 12 + 15 \\ &= 60.84 \text{ in.}\end{aligned}$$

$$\begin{aligned}\text{Distance between welds} &= 60.84 \div 20 = 3.042 \\ &\text{or } 3.0 \text{ inches}\end{aligned}$$

Drill and Practice:

Rising, p. 369 (Oral 1, 5), 370 (1-4)

Dressler, p. 620-622

9. CONCEPT: THE AREA OF A CIRCLE

The area of a circle equals the product of π and the square of the radius.

$$A = \pi r^2$$

A = Area

$\pi = 3.1416$

r = Radius.

Examples:

Find the area of a circle with radius = 10.2 cm.

$$A = \pi r^2$$

$$A = 3.14(10.2)^2$$

$$A = 3.14(104.04)$$

$$A = 326.6856 \text{ or } 326.69 \text{ square centimeters.}$$

Find the radius of a circle whose area is 147.6 square feet.

$$A = \pi r^2$$

$$147.6 = 3.14 \cdot r^2$$

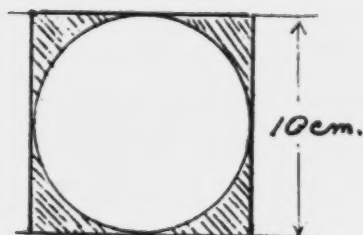
$$\frac{147.6}{3.14} = r^2$$

$$47.01 = r^2$$

$$\sqrt{47.01} = r$$

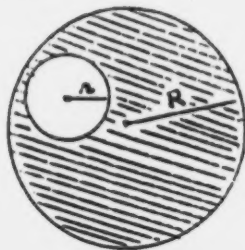
$$r = 6.9 \text{ feet.}$$

Application 1.



A heat resistant compound is to be applied to the shaded area shown at the left. ABCD is a square. The circle is inscribed within the square. Find the area of the shaded part.

$$\begin{aligned} \text{Shaded Area} &= \text{Area Square} - \text{Area Circle} \\ &= 10^2 - \pi(5)^2 \\ &= 100 - 3.14(25) \\ &= 100 - 78.5 \\ &= 21.5 \text{ square centimeters.} \end{aligned}$$

Application 2.

Find the area of the shaded part if

$R = 10$ cm and $r = 4$ cm,

$$\begin{aligned}
 \text{Shaded Area} &= \pi R^2 - \pi r^2 \\
 &= \pi (R^2 - r^2) \\
 &= 3.14(10^2 - 4^2) \\
 &= 3.14(100 - 16) \\
 &= 3.14(84) \\
 &= 263.76 \text{ square} \\
 &\quad \text{centimeters}
 \end{aligned}$$

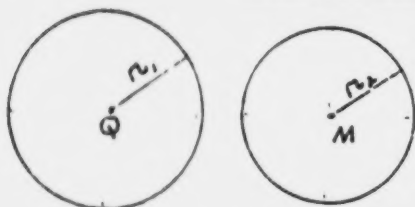
Note: A polygon is circumscribed about a circle and the circle is inscribed within the polygon if the sides of the polygon are tangent to the circle.

Drill and Practice:

Rising, p. 369-370 (2, 4, 7,), 370-371 (1-14)

Dressler, p. 626-628 (1-27)

10. CONCEPT: THE RATIO OF THE CIRCUMFERENCES OF TWO CIRCLES AND THE RATIO OF THEIR RADII



Given: Circle Q with
radius = r_1 ,
circle M with
radius = r_2 .

The circumference (C_1) of circle Q = $2\pi r_1$
The circumference (C_2) of circle M = $2\pi r_2$

Therefore
$$\frac{C_1}{C_2} = \frac{2\pi r_1}{2\pi r_2}$$

So that
$$\frac{C_1}{C_2} = \frac{r_1}{r_2}$$

The circumferences of two circles have the same ratio as their radii. (or diameters).

Example:

If the ratio of the radii of two circles is 5:4, what is the ratio of their circumferences? Answer. 5:4.

Example:

If the radius of one circle is 8 in. and the radius of another circle is 6 in., write the ratio of their radii.

$$\frac{r_1}{r_2} = \frac{8}{6} = \frac{4}{3}$$

Example:

The radius of one circle is 10 cm., and the radius of another circle is 15 cm. Find the circumference of the first circle if the circumference of the second circle is 100 cm.

$$\frac{C_1}{C_2} = \frac{r_1}{r_2} \quad \frac{C_1}{100} = \frac{10}{15}$$

$$15C_1 = 10000$$

$$C_1 = \frac{10000}{15}$$

$$= 667 \text{ cm.}$$

Application 1.

A circle was reduced in size by a copier. The diameter of the original was 10 cm. The diameter of the small circle was 6 cm. Find the ratio of two diameters.

$$\frac{10}{6} \quad \text{or} \quad \frac{5}{3}$$

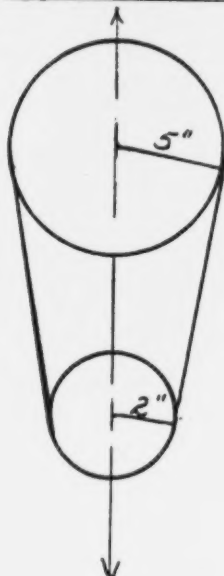
Note: $\frac{d_1}{d_2} = \frac{2r_1}{2r_2} = \frac{r_1}{r_2}$

Find the circumference of the original circle if the circumference of the reduced circle is 20 cm.

$$\frac{C_1}{C_2} = \frac{d_1}{d_2} \qquad \begin{array}{l} d_1 = \text{diameter (1)} \\ d_2 = \text{diameter (2)} \end{array}$$

$$\frac{C_1}{20} = \frac{5}{3}$$

$$\begin{aligned} 3C_1 &= 5(20) \\ 3C_1 &= 100 \\ C_1 &= 33.3 \text{ cm.} \end{aligned}$$

Application 2.

In a pulley system, the radius of the large wheel is 5 in., and the radius of the smaller wheel is 2 in.

If the circumference of the larger wheel is 31.4 in. Find the circumference of the smaller wheel.

$$\frac{C_1}{C_2} = \frac{r_1}{r_2}$$

$$\frac{31.4}{C_2} = \frac{5}{2}$$

$$5 C_2 = 62.8$$

$$C_2 = \frac{62.8}{5}$$

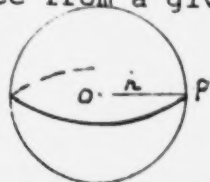
$$C_2 = 12.56 \text{ inc.}$$

Drill and Practice:

Rising, p. 370 (10-12, 15)
Dressler, p. 621-622 (18-21)

11. CONCEPT: THE SPHERE, CYLINDER AND CONE; VOCABULARY AND VOLUME

A sphere is the set of points, all of which are at a given distance from a given point called the center.



$\overline{OP} = r$ = the radius of the sphere.
Every point on the surface of the sphere is at the distance r from the center O .

A hemisphere is-in practical terms - one half a sphere.

A right circular cylinder has two congruent circles for bases and an altitude which is a segment perpendicular to the planes of the bases at their centers.



h = altitude of cylinder
 r = radius of bases

A right circular cone has one base and a point called the vertex. The altitude is drawn from the vertex perpendicular to the plane of the base at the center of the circle.



h = altitude of the cone
 r = radius of base

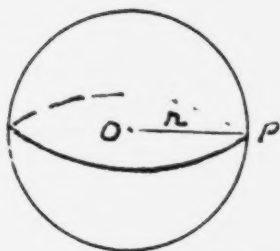
The volume of a sphere is given by the formula:

$$V = \frac{4}{3} \pi r^3$$

where V = volume
(cubic units)

$$\pi = 3.1416$$

r = radius of
the sphere
(linear units)



Examples: Find the volume of a sphere with a radius of 6 cm.

$$V = \frac{4}{3} \pi r^3 = \frac{4}{3} (3.1416) 6^3 = \frac{4}{3} (3.1416) \cdot 216$$

$$V = 904.7808 = 905 \text{ cubic centimeters.}$$

The volume of a right circular cylinder is given by the formula

$$V = \pi r^2 h$$



where V = volume of the cylinder
(cubic units)

$$\pi = 3.1416$$

r = radius of circular base
(linear units)

h = altitude of cylinder
(linear units)

Example: Find the volume of a right circular cylinder with radius equal to 7 in. and an altitude equal to 10 in.

$$\begin{aligned} V = \pi r^2 h &= 3.1416(7)^2 \cdot 10 = 3.1416(49) \cdot 10 = 1539.384 \\ &= 1539 \text{ cubic inches.} \end{aligned}$$

The volume of a right circular cone is given by the formula:

$$V = \frac{1}{3} \pi r^2 h$$



where

V = volume of cone (cubic units)
 $= 3.1416$

r = radius of base (linear units)

h = altitude (linear units)

Example: Find the volume of a cone with radius equal to 9.4 cm. and altitude equal 11.2 cm.

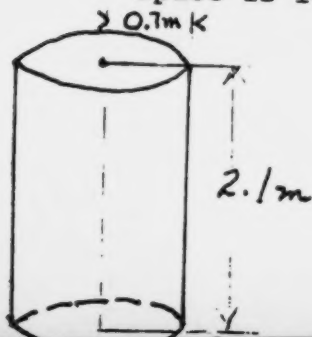
$$V = \frac{1}{3} \pi r^2 h = \frac{1}{3} (3.1416)(9.4)^2 (11.2)$$

$$= 1036.3426$$

$$= 1036.3 \text{ cubic centimeters}$$

Application 1.

In an engineering process 7000 spherical pellets are to be deposited in a large cylinder which has a radius of 1.7 meters and a height of 2.1 meters. If each pellet has an average radius of .04 meters how much empty space is left in the cylinder?



Volume of empty space = volume of
cylinder minus
volume of 7000 pellets

$$\begin{aligned} \text{Volume of cylinder} \\ = \pi r^2 h &= 3.1416(1.7)^2(2.1) = 3.23 \text{ cu. m.} \end{aligned}$$

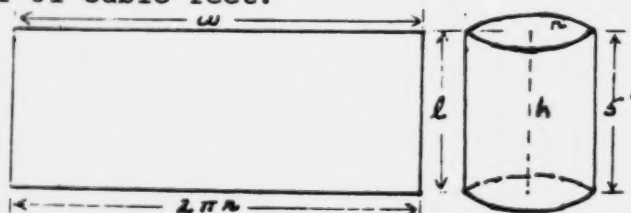
$$\begin{aligned} \text{Volume of one pellet} \\ = \frac{4}{3} \pi r^3 = \frac{4}{3} (3.1416) (.04)^3 = 0.000268 \text{ cu. m.} \end{aligned}$$

$$\begin{aligned} \text{Volume of 7000 pellets} \\ = 7000 (.000268) = 1.88 \text{ cu. m.} \end{aligned}$$

Therefore the volume of empty space = $3.23 - 1.88 = 1.35$ cu. m.

Application 2.

A piece of sheet metal 5 feet in length is to be fabricated into an open cylinder which is to have a volume of 31 cubic feet.



Calculate the width (w) which needs to be cut, by

(1) Finding the radius (r) of the cylinder

$$\begin{aligned} V &= \pi r^2 h \\ 31 &= 3.1416 r^2 (5) \\ 31 &= 15.708 r^2 \\ r^2 &= \frac{31}{15.708} \end{aligned}$$

$$r^2 = 1.97$$

$$r = \sqrt{1.97} = 1.4 \text{ feet}$$

Then (2) Finding the circumference of the base which equals the width

$$\begin{aligned} C &= 2\pi r \\ C &= 2(3.1416)(1.4) \\ C &= 8.8 \text{ feet} = \text{width.} \end{aligned}$$

Drill and Practice:

Rising, p. 374-375, 377-379

Dressler, p. 647-648

Smith, p. 51 (13, 22, 23)

Sullivan, p. 116-117 (13, 15)

STATISTICS

1. CONCEPT: STATISTICS - DEFINITION AND VOCABULARY

Statistics is the mathematics of numerical data. It is also the data itself.

Example: For one week, production of auto radio chassis in a fabrication department was as follows:

Monday	521
Tuesday	727
Wednesday	499
Thursday	520
Friday	478

This information is numerical data.

Descriptive statistics is a method of collecting, organizing and analysing data.

A frequency table is used to tally data.

Example: In an inservice educational program a test of analytical skills is given to 80 employees. The results were as follows:

Frequency Table

<u>Scores</u>	<u>Tally</u>	<u>Frequency</u>
0 - 20	11	2
21 - 40	<del> </del> 111	8
41 - 60	<del> </del> <del> </del> <del> </del> <del> </del> <del> </del> <del> </del> 1	31
61 - 80	<del> </del> <del> </del> <del> </del> <del> </del> 11	22
81 - 100	<del> </del> <del> </del> <del> </del> 11	17

Notice that from this table other information can be drawn. For example, most persons scored above 40. The exact number is $31+22+17 = 70$.

A cumulative frequency table shown here helps in further analysis.

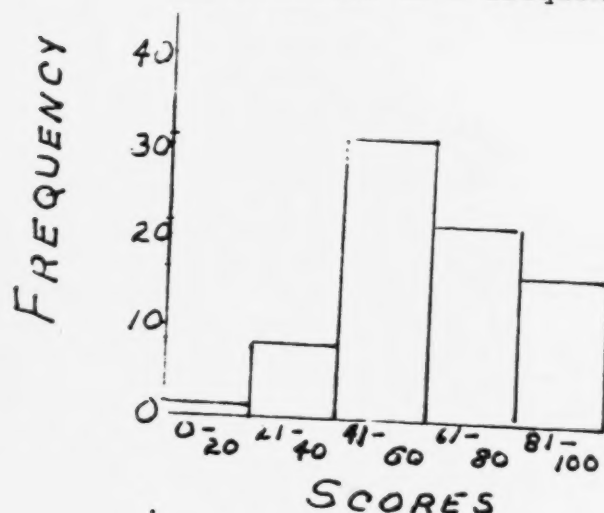
<u>Scores</u>	<u>Frequency</u>	<u>Cumulative Frequency</u>
0 - 20	2	2
21 - 40	8	10
41 - 60	31	41
61 - 80	22	63
81 - 100	17	80

From this table, we can determine the number of persons scoring 40 or below 10.

The number of persons scoring 60 or below is 41.

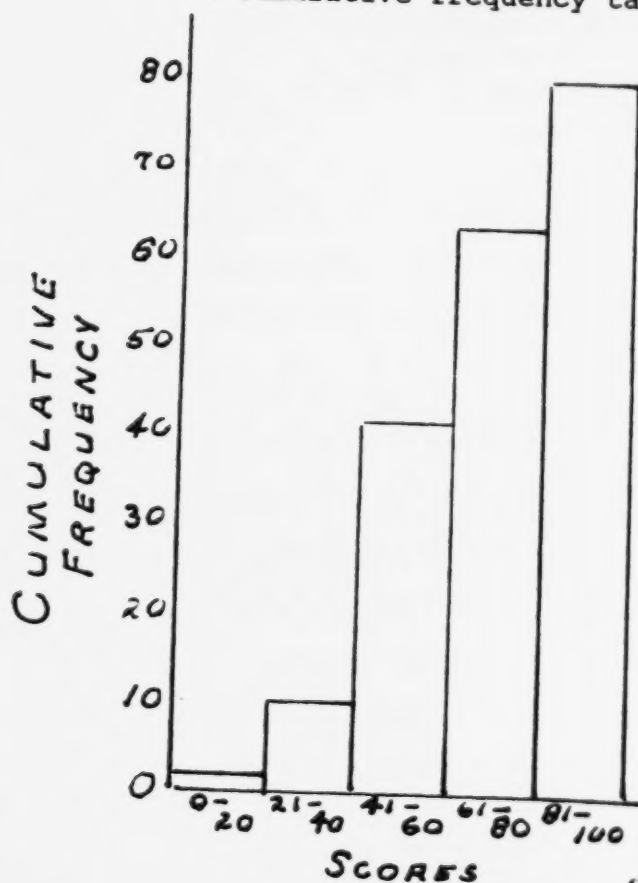
The interval in the above table is 20. There are 5 intervals of equal size. Subtract the starting point of any interval from the starting point of the next higher interval. 195
 Ex: $61-40 = 20$, $41-20 = 20$ (the exception is the first interval when it starts with 0.)

A histogram is a bar graph which illustrates the information in a frequency table.



HISTOGRAM

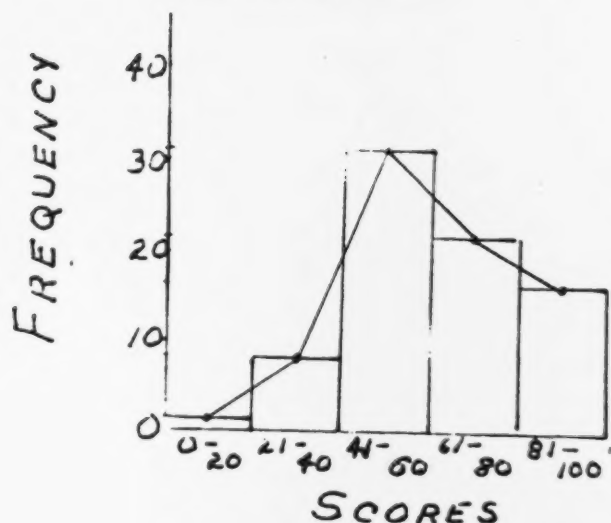
A cumulative histogram illustrates the information in a cumulative frequency table.



CUMULATIVE
FREQUENCY
HISTOGRAM

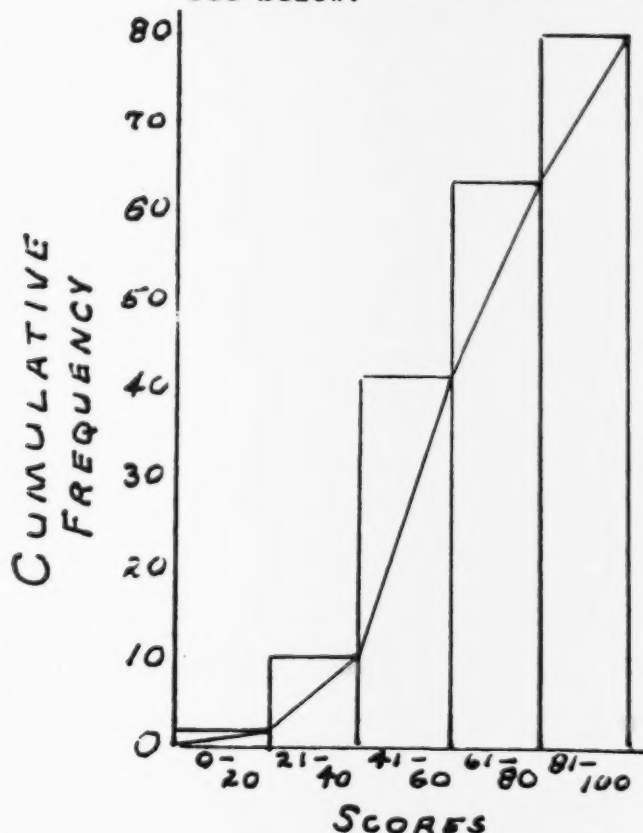
A frequency polygon is a line graph. Drawn by connecting the midpoints of the tops of consecutive bars at a frequency polygon. See below.

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FREQUENCY
POLYGON

A cumulative frequency polygon is a line graph drawn by connecting the upper right hand corner of each bar. See below.



CUMULATIVE
FREQUENCY
POLYGON

The cumulative frequency polygon tells us, (at any point on the polygon) the numbers of persons placing at or below a certain score.

Drill and Practice:

Rising, p. 439-441, 444-445

Dressler, p. 496-498, 505-508

2. CONCEPT: THE MEAN, THE MEDIAN, AND THE MODE

There are some special numbers called the measures of central tendency which described the data under analysis. These numbers are the mean, the median, and the mode.

The mean is the arithmetic average.

$$\text{Mean} = \frac{\text{Sum of the numbers}}{\text{The number of numbers}}$$

Example: Find the mean of 86, 54, 72, 80.

$$\text{Mean} = \frac{86+54+72+80}{4} = \frac{292}{4} = 73$$

Example: Find the mean.

-8, +2, 0, -4, +7, +10

$$\text{Mean} = \frac{-8+2+0-4+7+10}{6} = \frac{+7}{6} = +1\frac{1}{6}$$

The median is the middle number after the numbers have been ranked in order of size.

Example: Find the median

4, 13, 17, 8, 7, 20, 2

Arrange in order: 2, 4, 7, 8, 13, 17, 20

The median is the middle number: 8

If the number of numbers is even, take the average of the two middle numbers.

Example: Find the median.

32, 48, 56, 74, 83, 21

Arrange in order: 21, 32, 48, 56, 74, 83

$$\text{The median} = \frac{48+56}{2} = \frac{104}{2} = 52$$

The mode is the number which occurs most often.

Example: Find the mode.

71, 80, 54, 80, 54, 72, 54

Mode = 54 (occurs three times)

Example: Find the mode.

74, 63, 74, 83, 91, 63

Mode = 74 and 63 (two modes)

Application:

In a testing laboratory, it was determined that liter samples of antifreeze decreased the freezing point of the radiator liquids by the following amounts (degrees Celsius).

3.1°, 4.2°, 3.3°, 3.0°, 3.4°, 3.1°
3.3°, 3.5°, 3.1°, 3.2°, 3.7°, 3.6°

Find the mean, the median, and the mode.

The mean = $\frac{3.1+4.2+3.3+3.0+3.4+3.1+3.3+3.5+3.1+3.2+3.7+3.6}{12}$

= $\frac{40.5}{12} = 3.375 = 3.4^\circ$ Celsius.

The median:

Arrange in order: 3.0, 3.1, 3.1, 3.1, 3.2, 3.3,
3.3, 3.4, 3.5, 3.6, 3.7, 4.2

Median: Average of 3.3, 3.3, = $\frac{3.3+3.3}{2} = 3.3^\circ$ Celsius

The mode:

The number which occurs most often is 3.1° C.

Drill and Practice:

Rising, p. 448

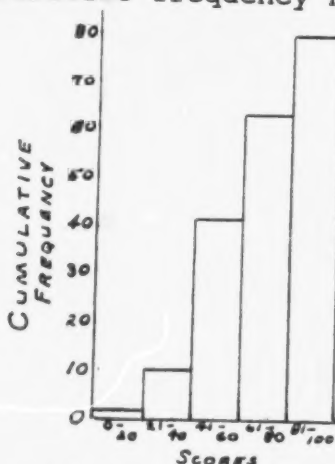
Dressler, p. 513-516, 522-523

3. CONCEPT: THE QUARTILE AND THE PERCENTILE

Consider once again the grades on the analytical skills test administered to 80 employees (see Statistics, Concept 1).

<u>Scores</u>	<u>Frequency</u>	<u>Cumulative Frequency</u>
0 - 20	2	2
21 - 40	8	10
41 - 60	31	41
61 - 80	22	63
81 - 100	17	80

Let's plot the cumulative frequency histogram.



80 employees (or 100% of the employees) scored 100 or lower.

60 employees (or 75% of the employees) scored 78 or lower.

40 employees (or 50% of the employees) scored 59 or lower.

20 employees (or 25% of the employees) scored 47 or lower.

Now we see that since 25% of the employees received a score of 47 or less, 47 represents the lower quartile (or first quartile).

Since 50% of the employees received a score of 59 or less, then 59 represents the middle quartile (or second quartile). (59 is also called the median.)

Since 75% of the employees received a score of 78 or less, then 78 represents the upper quartile (or third quartile).

A percentile is a score which represents what percent of the total population scored at or below that number.

Application:

Sarah received a score of 84 from the cumulative frequency polygon. It can be seen that 84 is at the 80th percentile level. This means that Sarah has a score equal to or better than 80% of her colleagues.

Drill and Practice:

Rising, p. 452-455

Dressler, p. 529-530

4. CONCEPT: RANDOM SAMPLES AND SIMULATION

To control quality in a production plant it is often difficult to test every item produced. To make certain that quality is maintained, random sampling is often employed; that is, some of the objects are chosen for testing.

To facilitate this choice, random number tables are employed.

A random number table could be generated by the spinning of a wheel with numbers 1 - 100 and recording the results on a table.

For example: 25 numbers

Table A

17	8	84	74	44
83	3	27	17	53
44	17	32	32	72
61	99	38	66	68
05	76	90	87	54

Random number tables with almost unlimited range can be generated by computer programs. See Table B, page 201A

Example:

Random number tables can be used to simulate (imitate) certain experiments. Using Table A simulate the experiment of determining the probability of obtaining tails in a coin-flipping exercise can be simulated. Let an even number represent heads and an odd number represent tails. Considering 25 flips, the result is 11 tails and 14 heads.

This means that perhaps the random sample is not large enough. Now try the same simulation with Table B, page 201 A

Use the first four squares, of 25 numbers each, in the first column of squares on the left.

Outcome: 49 tails, 51 head - much closer to the expected result.

Table of Random Numbers

84 60 95 82 32	03 99 11 04 61	66 08 32 46 53	88 61 81 91 61	93 71 61 68 94
29 73 34 77 62	38 55 59 55 54	08 35 56 08 60	71 29 92 38 53	32 88 65 97 80
59 07 60 79 76	17 54 67 37 04	55 12 12 92 81	27 95 45 89 09	92 05 24 62 15
76 15 59 37 79	32 64 35 28 61	00 91 19 89 36	80 86 30 05 14	95 81 90 68 31
29 54 96 96 16	69 57 26 87 77	14 06 04 06 19	33 56 46 07 80	39 51 03 59 05
61 95 87 71 00	24 12 26 65 91	14 84 54 66 72	90 89 97 57 54	27 69 90 64 94
17 24 77 09 43	61 19 63 02 31	41 83 95 53 82	78 03 87 02 67	92 96 24 17 71
10 06 16 88 29	30 53 22 17 04	39 68 52 33 09	55 98 66 64 85	10 27 41 22 02
35 20 83 33 74	03 78 89 75 99	74 41 65 31 66	87 53 90 88 23	75 86 72 07 17
35 66 35 29 78	48 22 86 33 79	53 15 26 74 33	16 81 86 03 11	85 78 34 76 19
01 91 82 83 14	60 36 59 46 53	42 61 42 92 97	98 95 37 32 31	35 87 53 39 49
70 07 11 47 36	83 79 94 24 02	34 99 44 13 74	09 95 81 80 65	56 62 33 44 42
11 13 30 75 86	32 96 00 74 05	99 38 54 16 00	15 91 70 62 53	36 40 98 32 32
78 13 86 65 59	19 32 25 38 45	66 49 76 86 46	19 64 09 94 13	57 62 05 26 06
27 48 24 54 76	11 22 09 47 47	48 50 92 39 29	85 24 43 51 59	87 39 93 74 08
88 65 12 35 96	31 75 15 72 60	15 47 04 83 55	03 15 21 91 21	68 98 00 53 39
74 84 39 34 13	88 49 29 93 82	20 09 49 89 77	22 10 97 85 08	14 45 40 45 04
28 59 72 04 05	30 93 44 77 44	73 78 80 65 33	94 20 52 03 80	07 48 18 38 28
74 02 28 46 17	22 88 84 88 93	60 53 04 51 28	82 03 71 02 68	27 49 99 87 48
65 74 11 40 14	78 21 21 69 93	44 37 21 54 86	87 48 13 72 20	35 90 29 13 86
74 91 06 43 45	41 84 98 45 47	34 67 75 83 00	19 32 58 15 49	46 85 05 23 26
61 31 83 18 55	46 35 23 30 49	45 30 50 75 21	14 41 37 09 51	69 24 89 34 60
76 50 33 45 13	11 08 79 62 94	59 74 76 72 77	39 64 37 75 44	14 01 33 17 92
11 65 49 98 91	52 70 10 83 37	16 52 06 96 76	02 18 16 81 61	56 30 38 71 15
92 85 25 58 66	57 27 53 68 98	68 65 22 73 76	88 44 80 35 84	81 30 44 85 85

Table B

Application:

An engineering staff has decided to test 9 pocket radios out of a production of 90 radios.

- Procedure:
- (1) Assign numbers 0-89 to each of the radios produced.
 - (2) Choose a method of choosing the radios to be tested.
 - (3) From Table B, page 201-A read the first nine numbers listed in the first column at the left:
Result: 84, 29, 59, 76, 29, 61, 17, 10
 - (4) These numbers represent a random selection of radios to be tested.

Note: If a different method were used, a different random set would result; for example, choose the numbers in the upper right hand corner of each square of 25 numbers moving from left to right.

Result: 84, 03, 66, 88, 93, 61, 24, 14, 90

The radios identified by these numbers would be the items tested.

Drill and Practice:

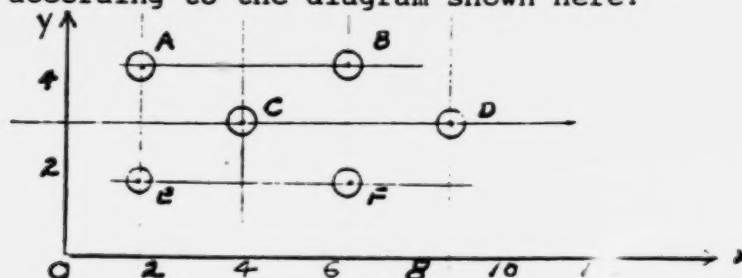
Rising, p. 457-458

Dressler, p. 493

**THE
MATHEMATICS
OF LINEAR
EQUATIONS**

1. CONCEPT: PROGRAMMING IN THE XY PLANE

A machinist wishes to program the drilling of holes in a metal part according to the diagram shown here.



An XY set of coordinates has been superimposed on the drawing.

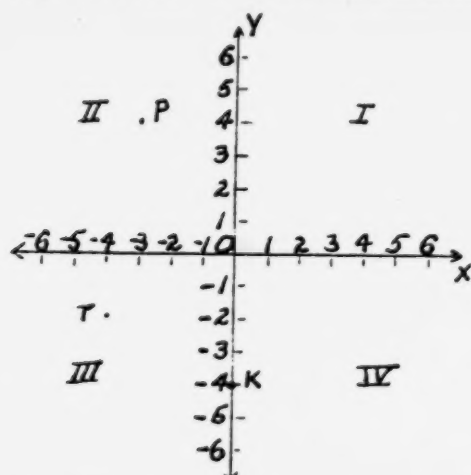
The line $\overrightarrow{OX}$ is called the X - axis.

The line $\overrightarrow{OY}$ is called the Y - axis.

O is called the origin.

Consider the center of hole C. From the diagram we read that C is located by C (4.0, 3.0). This was obtained by reading the intersection of $\overrightarrow{OX}$ and a line through C perpendicular to $\overrightarrow{OX}$, (4.0). Then we read the intersection of $\overrightarrow{OY}$ and a line through C perpendicular to $\overrightarrow{OY}$, (3.0). Thus, C (4.0, 3.0) indicates that C is located by moving 4.0 units to the right of the origin on the X - axis ($\overrightarrow{OX}$) and 3.0 units up from the origin of the Y axis ($\overrightarrow{OY}$).

The XY set of coordinates can be extended to include the positive and negative real numbers. This is called a rectangular coordinate system.



Thus, the X - axis and the Y - axis include all real numbers from positive infinity to negative infinity. (0.0) at O is called the origin.

Thus, point P can be located by P (-3, 4)

K is at K (0, -4)

T is at T (-4, -2)

In general P(X,Y) represents a point P with coordinates X and Y. The X - coordinate is called the abscissa. The Y coordinate is called the ordinate.

The Roman Numerals I, II, III, IV represent 4 quadrants counting in the counter clockwise direction.

$P(-3, 4)$ indicates an ordered pair and point P is the graph of $(-3, 4)$.

Example: Locate Q $(-5, 2)$.

Solution: Place a point Q at the intersection of the $x = -5$ vertical line, and $y = 2$ horizontal line. This is called plotting a point.

Application: Given the instructions that one-inch holes are to be drilled at these centers, plot the points.

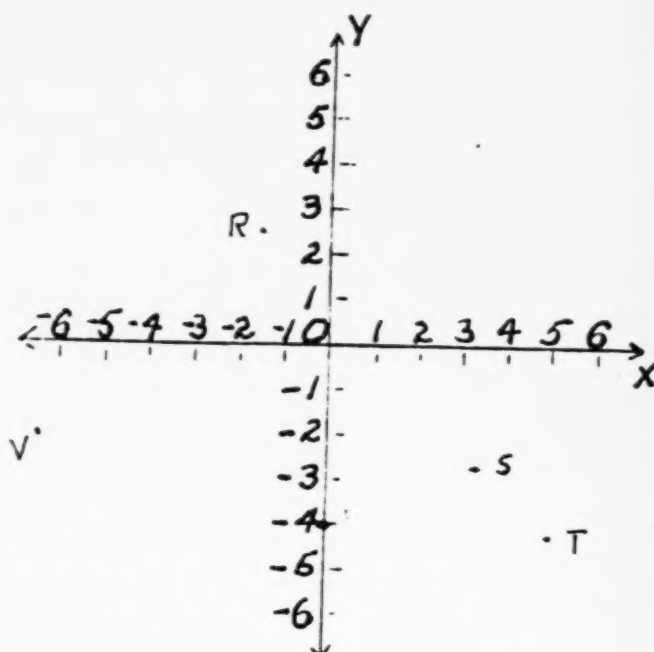
$R(-1.5, 2.5)$

$S(3.4, -2.7)$

$T(5.0, -4.2)$

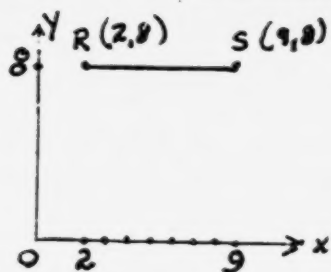
$V(-6.4, -2.1)$

Solution:



Drill and Practice:
 Rising, p. 474-475
 Dressler, p. 533
 Sullivan, p. 334-335

2. CONCEPT: THE DISTANCE BETWEEN TWO POINTS ON THE SAME HORIZONTAL OR VERTICAL LINE



$\overline{RS}$ is a line segment whose end points are $R(2, 8)$ and $S(9, 8)$.

Find the length of RS :

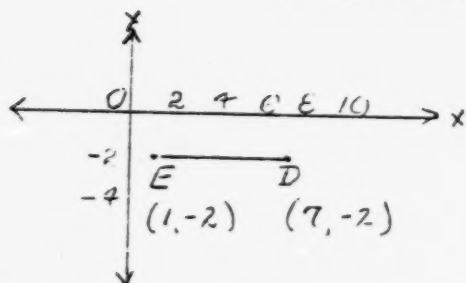
Method 1. Count units. 7

Method 2.

$$RS = |9 - 2| = |7| = 7$$

The distance between two points on a horizontal line is the absolute value of the difference of their x-coordinates.

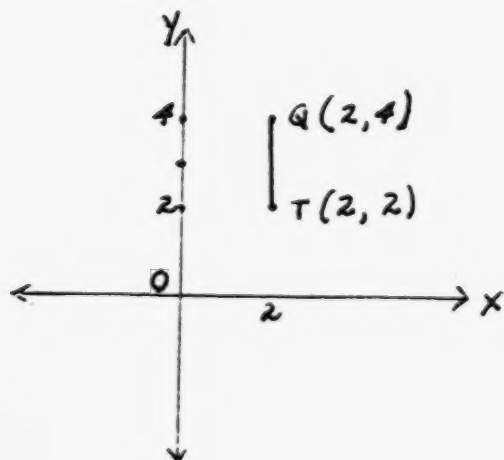
Example: Find the length of $\overline{DE}$ if $D(7, -2)$ and $E(1, -2)$.



1) Counting units = 6

$$2) DE = |7 - 1| = |6| = 6$$

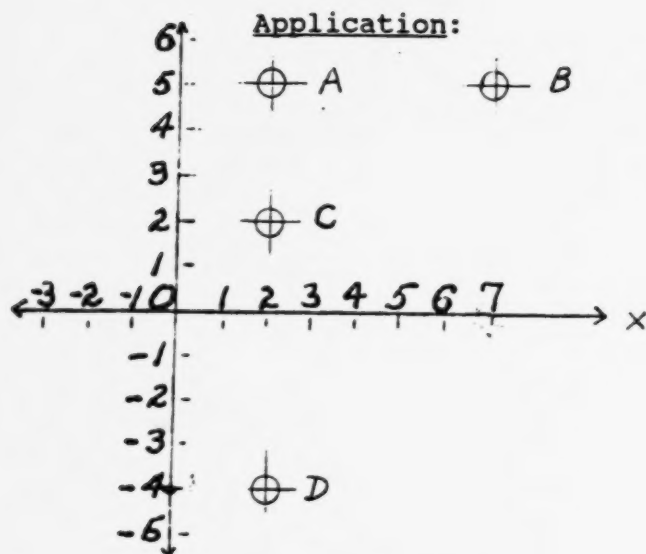
Likewise, the distance between two points on a vertical line is the absolute value of the difference of their y-coordinates.



Find QT .

$QT = 2$ (Count)

$$QT = |4 - 2| = 2 = 2$$



Four holes are to be drilled with centers at A, B, C, D. Find the distances AB, AC, CD.

Solution: merely count units.

Count: AB = 5

or $AB = |7-2| = |5| = 5$

Count: AC = 3

or $AC = |5-2| = |3| = 3$

Count: CD = 6

or $CD = |2-(-4)| = |2+4| = |6| = 6$

Drill and Practice:

Rising, p. 474 (14-19, 20-34), 475

Dressler, p. 41 (1, 2, 7, 8, 21-28)

3. CONCEPT: THE GRAPH OF A LINEAR EQUATION

The graph of a linear equation containing x and y is all of those points whose coordinates make the equation a true statement.

Example: If $3x + 4y = 12$

Then $(2, 1.5)$ is a point on the graph of $3x + 4y = 12$ since $3(2) + 4(1.5) = 12$ is true. But $(3, -5)$ is a point which is not on the graph of $3x + 4y = 12$ since $3(3) + 4(-5) = 12$ is false.

Three-Point Method of plotting the graph of a linear equation in x and y .

Graph: $2x + y = 7$

Choose any three values for x and calculate the corresponding values for y . This will produce three sets of ordered pairs whose graph will locate the graph of $2x + y = 7$.

$$\begin{aligned}\text{Let } x &= 2 \\ 2(2) + y &= 7 \\ 4 + y &= 7 \\ y &= 7 - 4 \\ y &= 3 \\ (2, 3)\end{aligned}$$

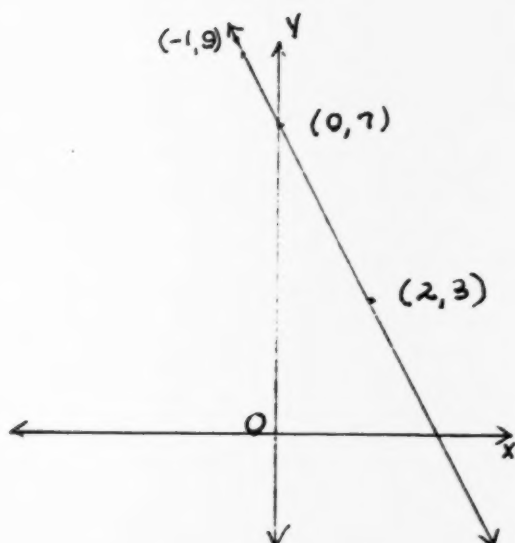
$$\begin{aligned}\text{Let } x &= 0 \\ 2(0) + y &= 7 \\ 0 + y &= 7 \\ y &= 7 \\ (0, 7)\end{aligned}$$

$$\begin{aligned}\text{Let } x &= -1 \\ 2(-1) + y &= 7 \\ -2 + y &= 7 \\ y &= 7 + 2 \\ y &= 9 \\ (-1, 9)\end{aligned}$$

These results can be tabulated.

x	y	(x, y)
2	3	$(2, 3)$
0	7	$(0, 7)$
-1	9	$(-1, 9)$

Graph the points in the table.



The line drawn through $(-1, 9)$, $(0, 7)$, and $(2, 3)$ includes all points whose coordinates satisfy the equation $2x + y = 7$.

Points like K have coordinates which do not satisfy the equation.

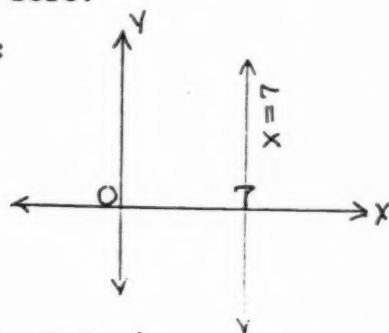
Note: More than three points can be used. Two are sufficient to determine a line. The use of three helps to reduce the possibility of arithmetic or algebraic error.

General form of a linear equation:

$$Ax + By = C$$

Where A, B and C are real; both A and B may not equal zero.

Graph:

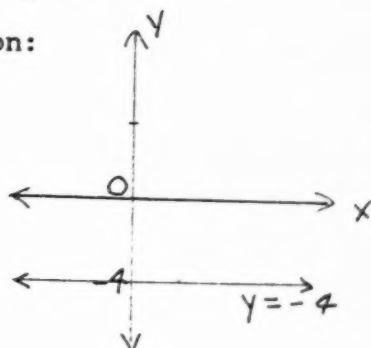


This is a linear where
 $B = 0$; $x + 0y = 7$

The graph is a vertical
line, along which $x = 7$.

Graph: $y = -4$

Solution:



The Intercept Method of plotting the graph of a linear equation in x and y.

Consider the equation $3x + 4y = 12$.

If we choose $x = 0$, then $3(0) + 4y = 12$

$$4y = 12$$

$$y = 3$$

so $(0, 3)$ represents the point at which the graph crosses the y-axis. This point is called the y-intercept.

If we chose $y = 0$

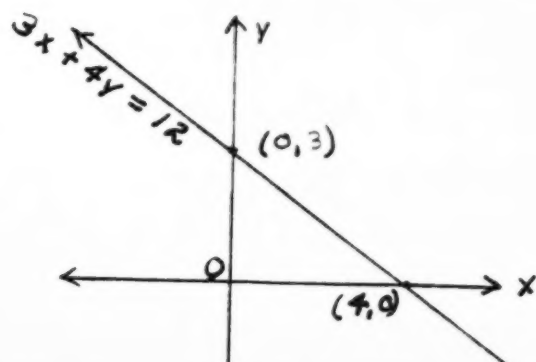
$$\text{then } 3x + 4(0) = 12$$

$$3x = 12$$

$$x = 4$$

so $(4, 0)$ represents the
x intercept.

Drawing a line through the
x intercept produces the
graph of $3x + 4y = 12$



Example: Graph $6x - 5y = 30$. Using the intercept method.

Let $x = 0$

$$6(0) = 30$$

$$0 - 5y = 30$$

$$-5y = 30$$

$$y = -6$$

y-intercept = $(0, -6)$

Let $y = 0$

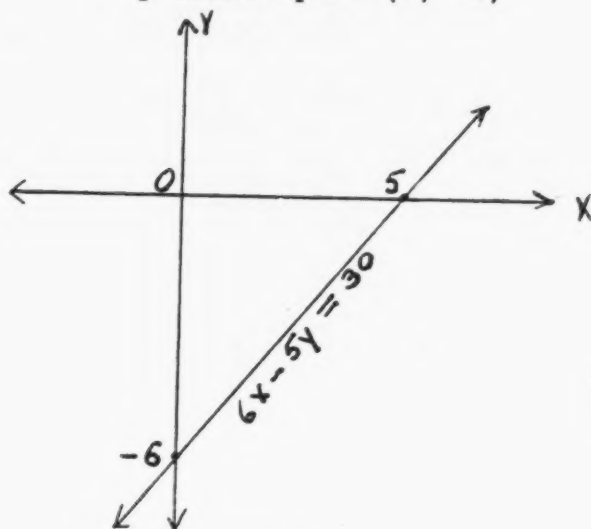
$$6x - 5(0) = 30$$

$$6x - 0 = 30$$

$$6x = 30$$

$$x = 5$$

x-intercept = $(5, 0)$



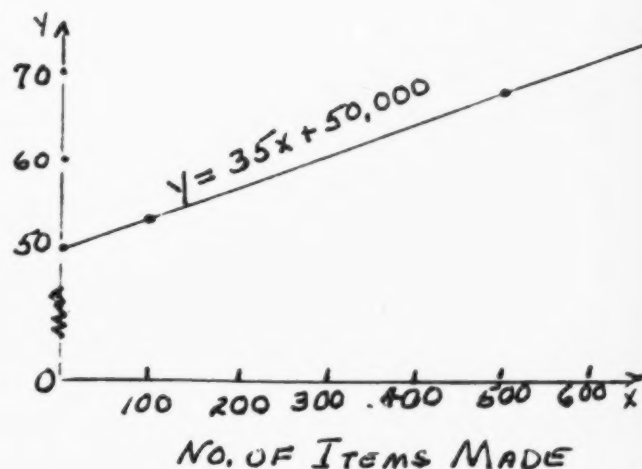
Application 1.

$y = 35x + 50000$ is a linear equation which represents the relationship between the number of items manufactured (x) at \$35 per item and the total production cost (y). The overhead expense is 50,000.

Graph this equation.

x	y
0	50,000
100	53,500
500	67,500

TOTAL PRODUCTION COST (THOUSANDS)



Application 2.

A truck cost \$15,000. The company which owns the truck considers it to be depreciating at the rate of \$2500 per year for 5 years.

Therefore if y = the value of the truck

x = the age of the truck

$y = 15000 - 2500x$ is a linear equation which relates the age of the truck to its value.

Plot the graph of this equation.

Let $x = 0$

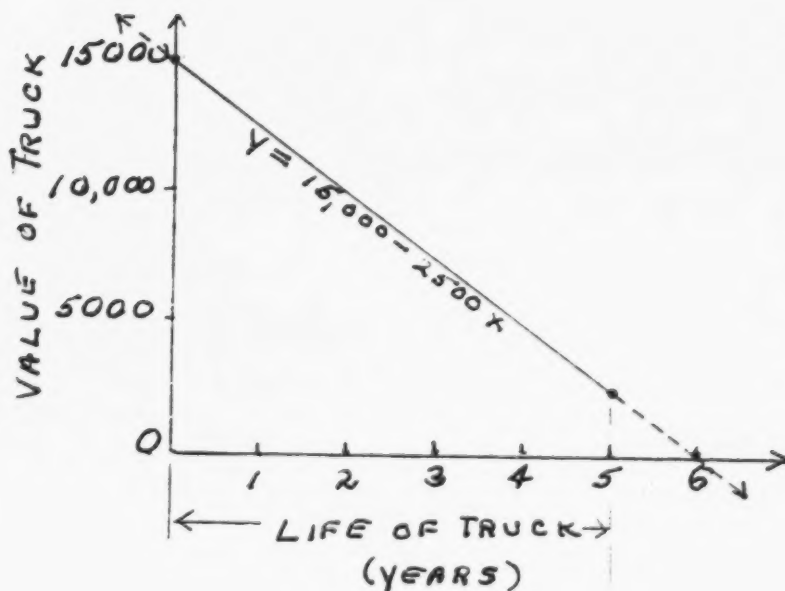
y int. = 15,000

Let $y = 0$

$0 = 15,000 - 2500x$

$2500x = 15,000$

x int. = 6

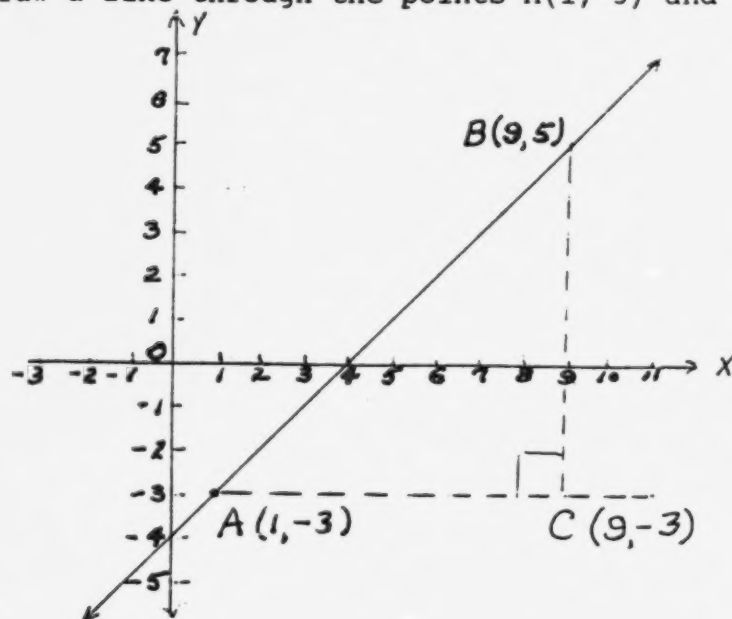
Drill and Practice:

Rising, p. 478

Dressler, p. 541-542

4. CONCEPT: THE SLOPE OF A LINE

Draw a line through the points $A(1, -3)$ and $B(9, 5)$.



An indication of the steepness of this line is called the slope.

Complete the right triangle ABC by drawing the vertical line BC until it intersects the horizontal line AC at C. C, then, has the coordinates $(9, -3)$.

The slope (m) is defined as Rise where the
Run

"Rise" is the difference $y_1 - y_2$ and the "Run" is the difference $x_1 - x_2$ when $A(x_1, y_1)$ and $B(x_2, y_2)$.

The slope:
 $m =$

$$\frac{y_1 - y_2}{x_1 - x_2}$$

The slope of
AB =

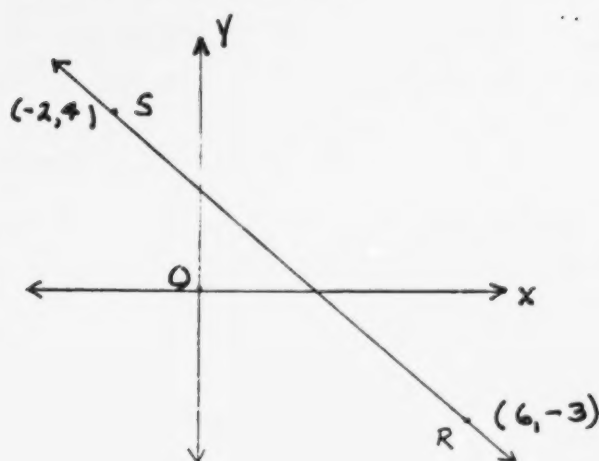
$$\frac{y_1 - y_2}{x_1 - x_2}$$

$$\text{OR } m(AB) = \frac{5 - (-3)}{9 - 1} = \frac{5 + 3}{9 - 1} = \frac{8}{8} = 1$$

Example: Find the slope of $\overleftrightarrow{RS}$ if $R(6, -3)$ and $S(-2, 4)$.

$$m(\overleftrightarrow{RS}) = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-3 - 4}{6 - (-2)} = \frac{-7}{6+2} = \frac{-7}{8}$$

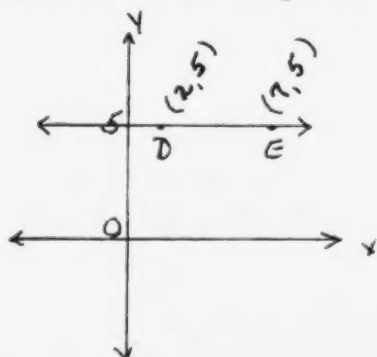
Plotting this line.



Note that $\overleftrightarrow{RS}$ travels "down to the right". $\overleftrightarrow{RS}$ has a negative slope, $\frac{-7}{8}$.

- Rule: 1. When the graph of a line points down to the right the slope is negative.
 2. When the graph of a line points up to the right the slope is positive.

Determine the slope of a horizontal line.

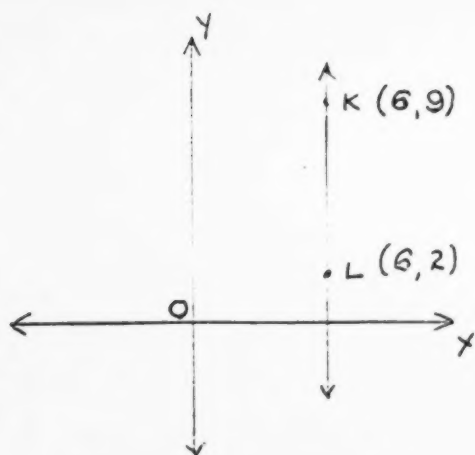


A line $\overleftrightarrow{DE}$ is horizontal and therefore parallel to the x-axis.

$$\text{Slope} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{5-5}{2-7} = \frac{0}{-5} = 0$$

Slope = 0 or "zero slope".

Rule: The slope of a horizontal line is zero.



KL is parallel to the y-axis.

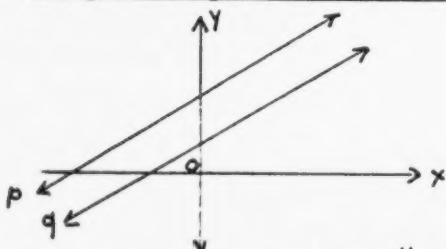
213

$$\text{Slope} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{9-2}{6-6} = \frac{7}{0}$$

$\frac{7}{0}$ is undefined.

Rule: The slope of a vertical line is labelled "no slope".

Slopes of parallel lines



If line p is parallel to line q, then p and q have the same slope, and conversely.

Example: If $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and if the slope of $\overleftrightarrow{AB}$ is $\frac{5}{2}$, then the slope of $\overleftrightarrow{CD}$ is $\frac{5}{2}$.

Application 1.

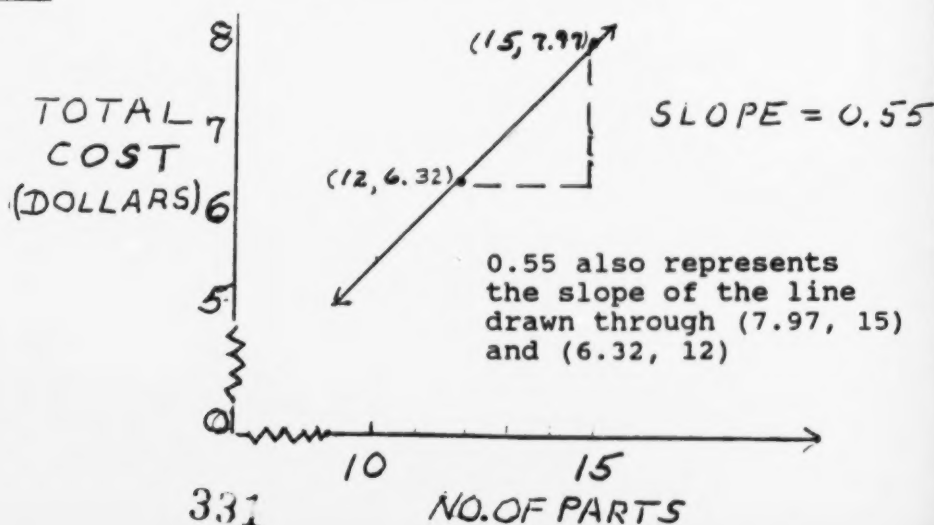
It costs \$6.32 to produce 12 machine parts. It costs \$7.97 to produce 15 machine parts. Calculate the variable cost per part.

Solution:

The Cost/Part = $\frac{\text{Difference in Costs}}{\text{Difference in No. of Parts}}$

No. Parts	Cost
15	\$7.97
12	6.32

$$= \frac{7.97-6.32}{15-12} = \frac{1.65}{3} = \$0.55/\text{Parts}$$



Application 2.

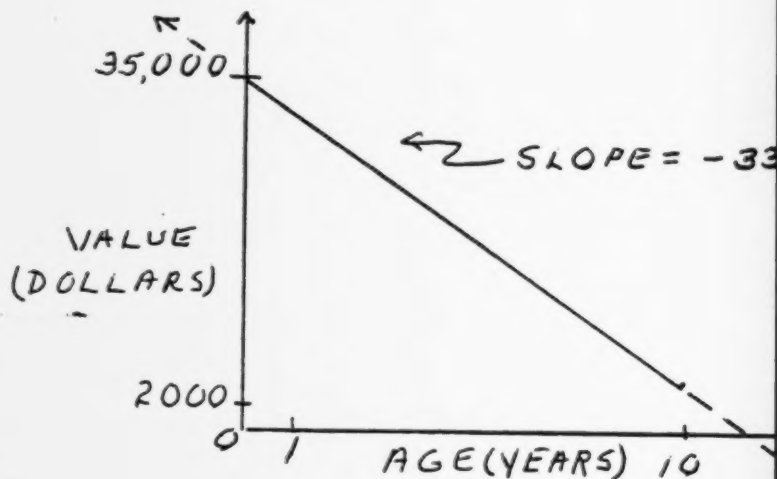
A corporation buys an earth mover for \$35,000. At the end of ten years, the earth mover has a scrap value of \$2000. Find the annual rate of depreciation.

Age	Value
0	35,000
10	2,000

$$\begin{array}{l} \text{Annual Rate} = \frac{\text{Loss in Value}}{\text{No. of Years}} = \frac{35,000 - 2,000}{10} \\ \text{of} \\ \text{Depreciation} \end{array}$$

$$= \frac{33,000}{10} = -\$3,300/\text{year}.$$

-3300 is also the slope of the line drawn between (0, 35,000) and (10, 2,000)

Drill and Practice:

Rising, p. 482-485

Dressler, p. 548-550

5. CONCEPT: THE SLOPE - INTERCEPT FORM OF A LINEAR EQUATION

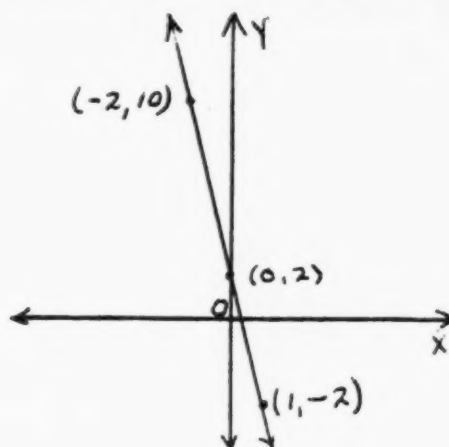
A linear equation has the form $4x + y = 2$. This is considered standard form of a linear equation ($Ax + By = C$).

If we rewrite the equation, solving for y in terms of x , we get

$$y = -4x + 2$$

Plotting this equation using the three-point method produces

X	Y
1	-2
0	2
-2	10



Note:

$$m = \text{slope} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{10 - 2}{-2 - 0} = \frac{8}{-2} = -4$$

a graph of a linear equations in which the y -intercept is 2 and the slope is -4.

If we examine the equation $y = -4x + 2$, we see that the slope of -4 appears as the coefficient of x and the y intercept of 2 appears as the constant term.

Using the m as the value of the slope and b as the value of the y -intercept, this form of the equation becomes

$$y = mx + b \quad \text{which is}$$

the slope-intercept form of a linear equation.

Example: Write a linear equation with a slope of 7 and a y -intercept of 3.

Solution: $y = mx + b$ where $m = 7$, and $b = 3$
The equation is $y = 7x + 3$.

Example: Given the equation $3x + 4y = 16$
Find the slope and the y-intercept.

Solution: $3x + 4y = 16$. Solve this equation
for y in terms of x.

$$4y = -3x + 16$$

$$y = \frac{-3x}{4} + 4$$

The equation is now in slope-intercept form:

$$y = mx + b.$$

$$m = \text{slope} = \frac{-3}{4} \text{ and } b = \text{y-intercept} = 4$$

Example: Write the equation (in slope-intercept form)
of the line passing through R (6, 4) and
S(2, -5).

Solution: $y = mx + b$. We need to obtain the
slope (m) and the y-intercept (b)

$$\text{slope} = m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{4 - (-5)}{6 - 2} = \frac{4+5}{4} = \frac{9}{5}$$

$$\text{Therefore } y = \frac{9}{5}x + b$$

Since RS passes through R(6, 4) substitute
 $x = 6$ and $y = 4$ in the equation and calculate b.

$$y = \frac{9}{5}x + b$$

$$4 = \frac{9}{5}(6) + b$$

$$4 = \frac{54}{5} + b$$

$$4 = 10\frac{4}{5} + b$$

$$4 - 10\frac{4}{5} = b$$

$$b = -6\frac{4}{5}$$

$$y = \frac{9}{5}x - 6\frac{4}{5} \text{ is the equation in slope-intercept form.}$$

Example: Write $y = \frac{9}{4}x - 6\frac{4}{5}$ in standard form
 $Ax + By = C$

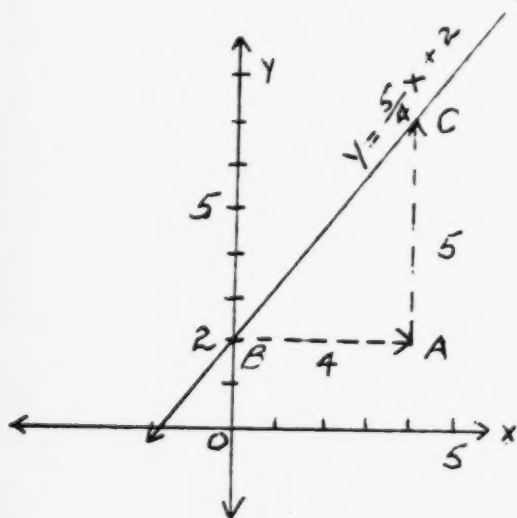
$$y = \frac{9}{5}x - 6\frac{4}{5}$$

$$5y = 9x - 34 \quad \text{multiply both members by 5}$$

$$-9x + 5y = 34 \quad \text{standard form.}$$

Example: Graph $y = \frac{5}{4}x + 2$

$$m = \frac{5}{4} = \text{slope} = \frac{\text{rise}}{\text{run}}, \quad b = \text{y-intercept} = 2$$



- 1) Plot the y-intercept (0, 2)
call this point B
- 2) Slope = $\frac{\text{a rise of } 5}{\text{a run of } 4}$
- 3) Count off from (0, 2)
4 units to the right (Point A)
- 4) From A count 5 units up.
to C.
- 5) Draw a line through B and C
- 6) Label the line

$$y = \frac{5}{4}x + 2$$

Application. Energy costs for a department varied according to this chart.

No. Kits Produced(x)	Energy Cost(y)
1462	\$ 976
2522	1294

Write a linear equation which describes the relationship between energy cost (y) and no. of kits produced (x).

Solution:

The equation will take the form $y = mx + b$.

$$\text{The slope} = m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{976 - 1294}{1462 - 2522} = \frac{-318}{-1060} = 0.294$$

$$\text{Therefore } y = 0.294x + b$$

Substitute (1462, 976) for (x, y)

$$976 = 0.294 (1462) + b$$

$$976 = 429.8 + b$$

$$b = 976 - 429.8$$

$$b = 546.2$$

Therefore

Energy cost $y = 0.294x + 546.2$ Where x is the number of kits produced.

The slope 0.294 or \$.29 represents the variable cost per kit and the y-intercept 546.2 represents \$546.20 the fixed cost (overhead).

Drill and Practice:

Rising, p. 487-488

Dressler, p. 552-553, 554-555,
556-557

6. CONCEPT: THE GRAPH OF A LINEAR INEQUALITY IN TWO VARIABLES

Let's consider for a moment, the graph of an inequality in one variable

$$\begin{array}{r} x - 4 \geq 10 \\ +4 \quad +4 \end{array}$$

$x \geq 14$ is the solution and this is the graph if we are considering only the real number line.

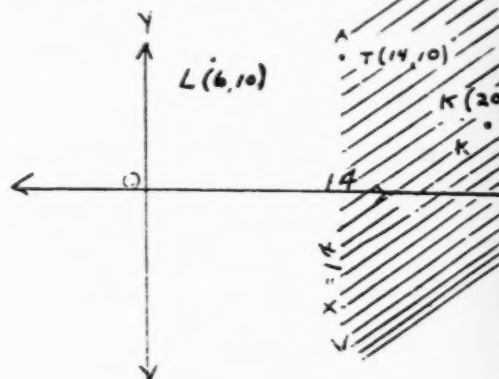


Let's expand our concept of graph to include the entire XY plane.

Now plot $x \geq 14$

The line $x = 14$ separates the plane into 3 areas.

- 1) The points to the left of line $x = 14$
- 2) The points on $x = 14$
- 3) The points to the right of line $x = 14$



All points such as K fit the conditions of $x = 14$ since $x = 20$.

All points such as T fit the conditions of $x = 14$ since $x = 14$.

All points such as L do not fit the conditions of $x = 14$ since $x = 6$.

Therefore the graph of the solution set of this inequality is the shaded area plus the solid line $x = 14$.

Let's try the graph of $x + 4y > 8$

Solution: The inequality for x in terms of y

$$x + 4y > 8$$

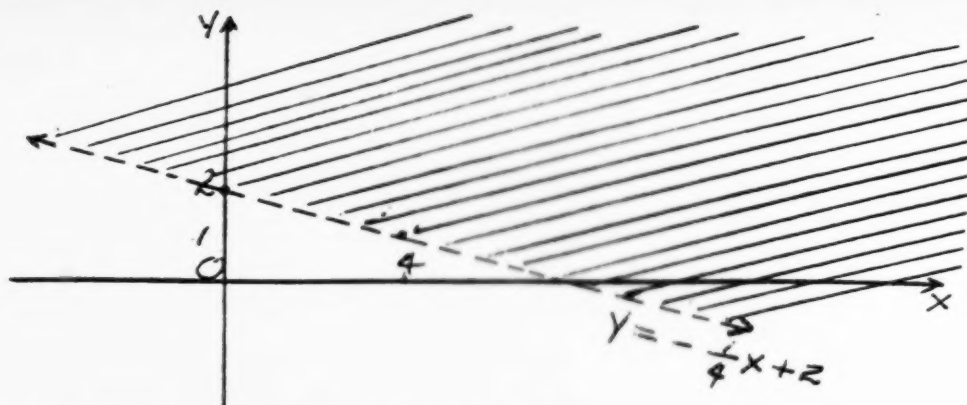
$$4y > -x + 8$$

$$y > -1/4x + 2$$

Now consider the equality $y = -\frac{1}{4}x + 2$. This line when

plotted divides the XY plane into three parts. Since the symbol $>$ is used, make the graph a broken line, not a solid line.

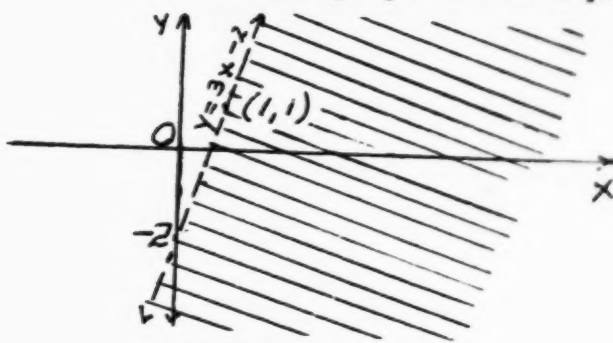
Since the inequality in this form $y > -1/4x + 2$ indicates that the value of y must always be greater than the value of y in the equation we shade above the line.



Example: Graph $y < 3x - 2$

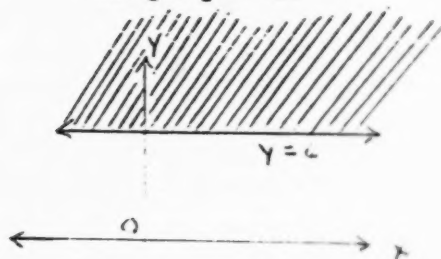
This inequality is already solved for y in terms of x . Plot the graph of the equation $y = 3x - 2$.

Draw graph of equation as a broken line. Since $<$ is used shade below the line.



Example: Graph $y \geq 6$
Graph $y = 6$.

Use a solid line since $y \geq 6$. Shade above the line because $>$ symbol.



Application 1.

Automobile headlights are processed in a production department allowing 30 minutes per headlight.

Truck headlights are processed in the same department and need 45 minutes per headlight.

If 8250 production minutes are available, write an inequality which relates the number of auto headlights (x) and the number of truck headlights which could be processed (y).

Solution: Let x = no. of auto headlights
 y = no. of truck headlilghts

$$30x + 45y \leq 8250$$

The total number of minutes available for auto headlights + the total number of minutes available for truck headlights must be less than or equal to 8250.

Application 2. Graph the inequality
 $30x + 45y \leq 8250$

Let $x = 0$

Then y -intercept = 184 (0,184)

Let $y = 0$

Then x -intercept = 275 (275,0)

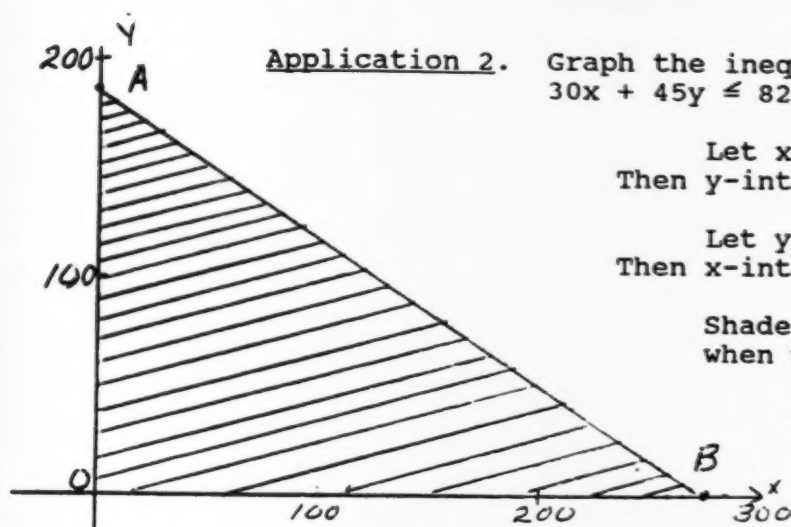
Shade below the line because when we solve for y

$$45y \leq -30x + 8250$$

$$y \leq \frac{-30x}{45} + \frac{8250}{45}$$

$$y \leq -\frac{2}{3}x + 184$$

The symbol $<$ is used.



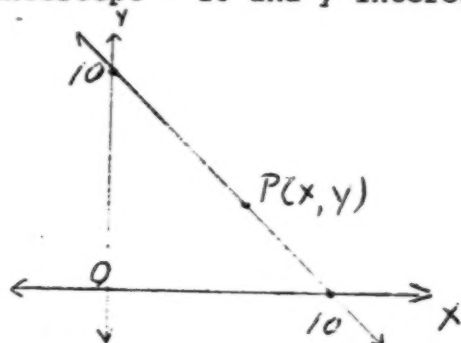
Shade only in first quadrant because production (x and y) must be positive. For same reason do not continue AB beyond A and B.

Drill and Practice:
 Rising, p. 491-492
 Dressler, p. 562-563

7. CONCEPT: THE GRAPHIC SOLUTION OF A SYSTEM OF LINEAR EQUATIONS

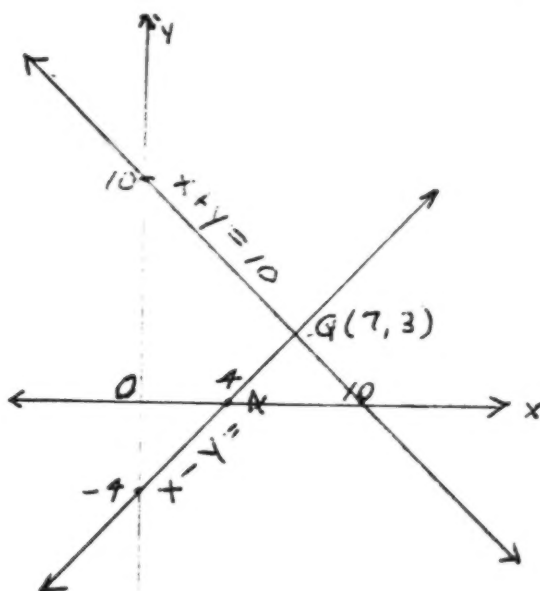
If we consider the equation $x + y = 10$, there are an infinite number of ordered pairs (x, y) which satisfy this equation: $(1, 9)$, $(2, 8)$, $(-1, 11)$, $(5 \frac{1}{2}, 4 \frac{1}{2})$, $(18, -8)$, $(1.25, 8.75)$, and others.

If we graph this equation using the intercept method, we get x -intercept = 10 and y -intercept = 10.



The coordinates (x, y) every point P on this line satisfy the equation.

If, however, we consider $x + y = 10$ together with $x - y = 4$, we now have a system of equations. Graphing them both we have an intersection of the two lines at point $Q(7, 3)$



Therefore the solution of this system of equations is said to be $x = 7$
 $y = 3$.

The accuracy of a graphic solution depends upon the type of graph paper and the care exercised by the person making the graph.

Results however can usually be checked.

$$\begin{aligned} x + y &= 10 \\ 7 + 3 &= 10 \\ 10 &= 10 \end{aligned}$$

$$\begin{aligned} x - y &= 4 \\ 7 - 3 &= 4 \\ 4 &= 4 \end{aligned}$$

Be sure to check both equations.

3:0

Example: Solve for x and y .

223

$$3x - 4y = 12$$

$$2x + 2y = 15$$

Plot both graphs.

Use intercept method.

$$x = 6$$

$$y = 1 \frac{1}{2}$$

Check:

$$3(6) - 4(1 \frac{1}{2}) = 12 \quad 2(6) + 2(1 \frac{1}{2}) = 15$$

$$18 - 6 = 12$$

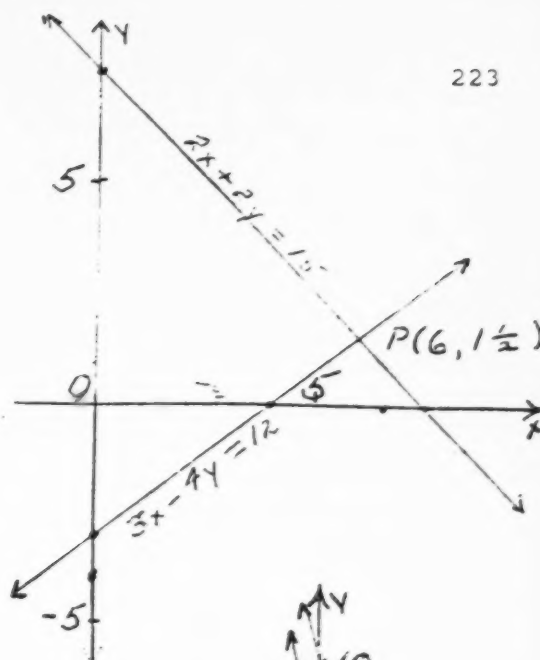
$$12 + 3 = 15$$

$$12 = 12$$

$$15 = 15$$

ck.

ck.

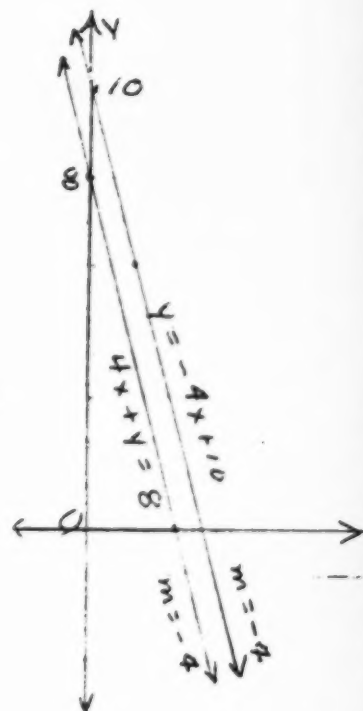


Example: Graph the solution

$$4x + y = 8$$

$$y = -4x + 10$$

Since $4x + y = 8$ can be written as $y = -4x + 8$ it has the same slope as $y = -4x + 10$. Therefore these two lines are parallel and have no common solution.

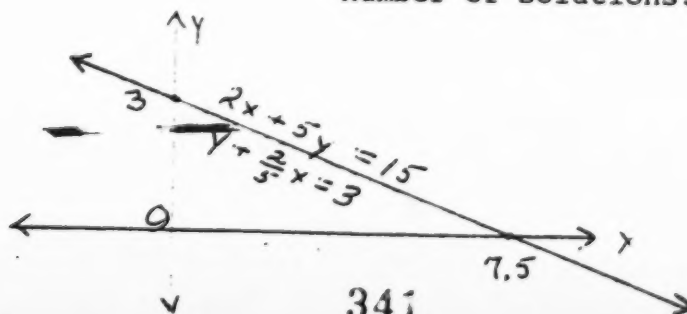


Example: Graph the solution of

$$2x + 5y = 15$$

$$\text{and} \quad y + \frac{2}{5}x = 3$$

Writing these in $y = mx + b$ form we see that $2x + 5y = 15$ becomes $y = -\frac{2}{5}x + 3$ and that $y + \frac{2}{5}x = 3$ becomes $y = -\frac{2}{5}x + 3$. Therefore both equations have the same slope and the same intercept, and consequently the same graph. There are an infinite number of solutions.



Application.

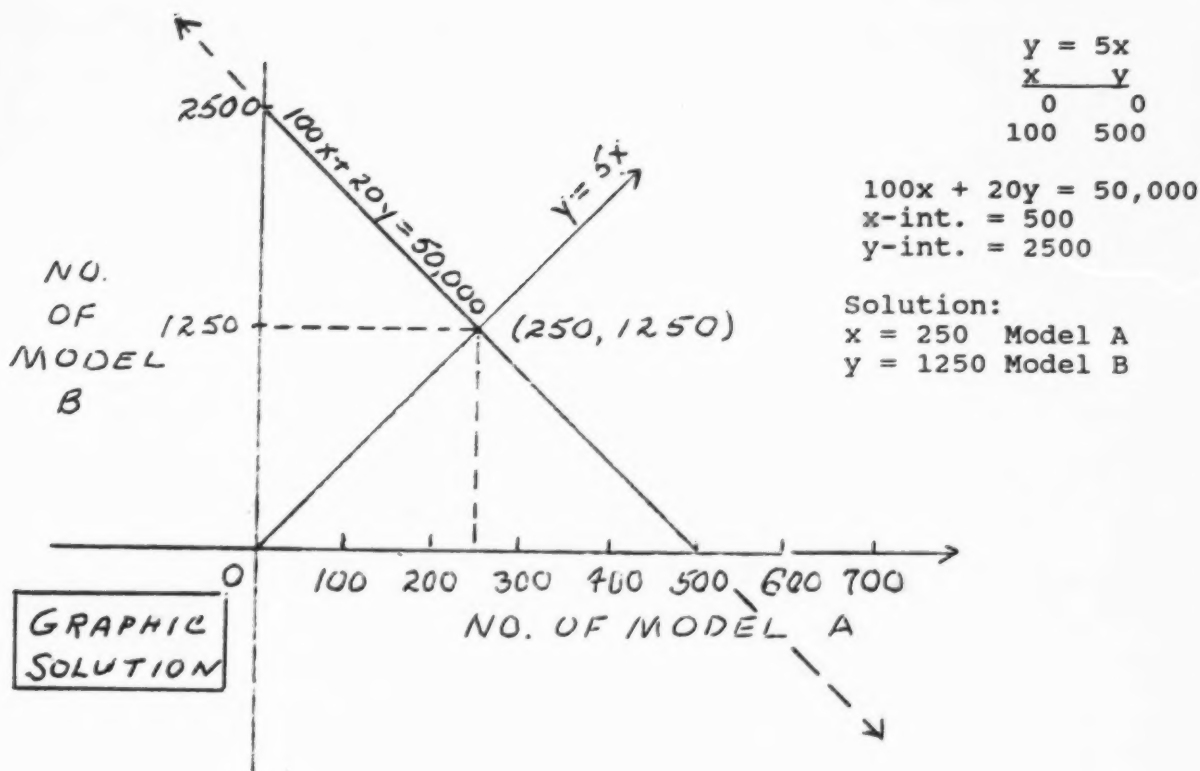
A small manufacturer desires to make a net profit of \$50,000 on the production of two different portable telephones. Model A provides a net profit of \$100 per unit and Model B provides a profit of \$20 per unit.

If the demand is for 5 times as many Model B phones as Model A phones, calculate the numbers of each model which can be produced.

Solution: Let x = no. of Model A phones
then y = no. of Model B phones

$$y = 5x$$

$$\text{and } 100x + 20y = 50,000$$

Drill and Practice:

Rising, p. 506, 507

Dressler, p. 567, 568

8. CONCEPT: SOLVING A SYSTEM OF TWO LINEAR EQUATIONS USING THE SUBSTITUTION METHOD

Consider the equations:

$$\begin{aligned}x + 3y &= 13 \\ 3x - 5y &= -17\end{aligned}$$

Note that the coefficient of x in the first equation is 1. Therefore a method called substitution is often very useful.

Step 1. In the equation $x + 3y = 13$ solve for x in terms of y .
 $x + 3y = 13$
 $x = -3y + 13$

Step 2. Substitute $-3y + 13$ for x in the other equation.
 $3x - 5y = 17$
 $3(-3y + 13) - 5y = -17$
 $-9y + 39 - 5y = -17$
 $-14y = -17 - 39$
 $-14y = -56$
 $y = 4$

Step 3. Now substitute $y = 4$ into either equation.
 $x + 3y = 13$
 $x + 3(4) = 13$
 $x + 12 = 13$
 $x = 13 - 12$
 $x = 1$

Solution Set = $(1, 4)$ or $x = 1$
 $y = 4$

Step 4. Check both equations by substituting the solution set.

$x + 3y = 13$	$3x - 5y = 17$
$1 + 3(4) = 13$	$3(1) - 5(4) = 17$
$1 + 12 = 13$	$3 - 20 = 17$
$13 = 13$ Check	$-17 = -17$ Check

Examples: Solve for x

$$9x - y = 8$$

$$4.5x = y/2 + 5 \quad \left(\begin{array}{l} \text{Multiply} \\ \text{by 2.} \end{array} \right)$$

$$9x - y = 8$$

$$9x = y + 10$$

Solve $9x - y = 8$ for y .

$$-y = 8 - 9x$$

$$y = 9x - 8$$

$$\begin{array}{ll}
 \text{Substitute} & 9x = (9x-8) + 10 \\
 \text{in other} & 9x = 9x-8 + 10 \\
 \text{equation.} & 9x-9x = 2 \\
 & 0 = 2
 \end{array}$$

This is a contradiction.

The system has no solution. The graphs of the two equations are parallel lines.

Example: (1) $6x + y = 10$ Solve (1) for y $y = 10-6x$
 (2) $12x+2y = 20$

Substitute $y = 10-6x$ in (2)

$$12x + 2(10-6x) = 20$$

$$12x + 20 - 12x = 20$$

$$20 = 20 \quad \text{This is an identity.}$$

The system has an infinite number of solutions. The graphs of the two equations are the same line.

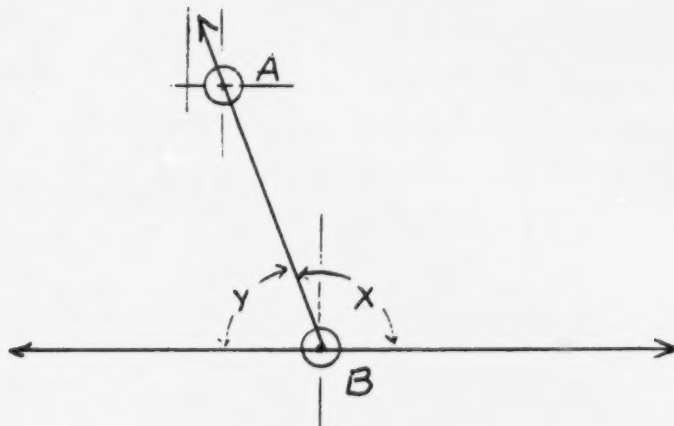
Application 1.

The design of a machine part demands that two holes are to be drilled, one at A, the other at B.

The measure of $\angle x$ is 42° more than the measure of $\angle y$. CB is a straight line.

Find the measures of $\angle x$ and $\angle y$.

Note: If two numbers are unknown, two equations must be written.



$$\text{Let } x = m\angle x$$

$$y = m\angle y$$

Write two equations

$$x = y + 42$$

$$x + y = 180 \quad (\angle x \text{ and } \angle y \text{ are supplementary})$$

Substitution method

$$\text{Since } x = y + 42$$

$$\text{then } (y+42) + y = 180$$

$$y+42+y = 180$$

$$2y+42 = 180$$

$$2y = 180 - 42$$

$$2y = 138$$

$$y = 69^\circ$$

$$\text{and } x = y + 42 = 69+42 = 111^\circ$$

Check: $x = y + 42$ $x + y = 180$
 $111 = 69 + 42$ $111 + 69 = 180$
 $111 = 111$ $180 = 180$
 check

This shows that,
 if your
 equations are
 correct, the
 solutions
 check.

Check
 the

Problem: Is m/x 42 more than m/y ? Yes
 (check the "sense" of your solution)

Application 2.

An engine part sells at the manufacturer's plant for \$47. The revenue equation is $y = 47x$ where x is the number of parts manufactured and y is the total revenue in dollars.

Since the unit cost of manufacture is \$13 and the overhead is \$6800, the cost equation is

$$y = 13x + \$6800$$

At what point will the revenue equal the cost.

$$y = 47x$$

$$y = 13x + 6800$$

Substituting

$$47x = 13x + 6800$$

$$47x - 13x = 6800$$

$$34x = 6800$$

When $x = 200$ units, the revenue will equal the cost.

Drill and Practice:

Rising, p. 506-507

Dressler, p. 574-575

9. CONCEPT: SOLVING A SYSTEM OF TWO LINEAR EQUATIONS USING THE ADDITION-SUBTRACTION METHOD

Consider the system of equations

$$6x - 5y = 23$$

$$2x = 3 - 3y.$$

Notice that none of the coefficients of x and y are equal to 1. Although the substitution method may be used, an alternate method called the addition-subtraction method is often useful:

- 1) Align the coefficients on same side of the equal

$$6x - 5y = 23 \quad \text{sign.}$$

$$2x + 3y = 3 \quad \text{Multiply this equation by 3.}$$

- 2)

$$6x - 5y = 23$$

Since $6x$ and $6x$ are both positive, use rules for subtraction.

Remember

$$6x + 9y = 9$$

change sign

$$-14y = 14$$

$$y = 1$$

- 3) Now substitute $y = 1$ in one of the original equations.

$$6x - 5(-1) = 23$$

$$6x + 5 = 23$$

$$6x = 23 - 5$$

$$6x = 18$$

$$x = 3$$

$$x = 3$$

$$y = -1$$

Check:

$$6x - 5y = 23$$

$$6(3) - 5(-1) = 23$$

$$18 + 5 = 23$$

$$23 = 23$$

Check

$$2x + 3y = 3$$

$$2(3) + 3(-1) = 3$$

$$6 - 3 = 3$$

$$3 = 3$$

Check

Now consider

$$\begin{array}{lcl} 2x - 3y = -5 & \left(\begin{array}{l} \text{Mult. by 7} \\ \text{Mult. by 3} \end{array} \right) & 14x - 21y = -35 \\ 5x + 7y = 2 & & 15x + 21y = 6 \end{array}$$

Notice that the y terms have coefficients which are equal but opposite in sign. Use the rules for addition. Do not change signs of subtrahend.

$$14x - 21y = -35$$

$$15x + 21y = 6$$

$$29x = -29$$

$$x = -1$$

Substitute $x = -1$ into one of the equations and calculate y .

$$\begin{array}{rcl}
 2(-1) - 3y & = & -5 \\
 -2 - 3y & = & -5 \\
 -3y & = & -5 + 2 \\
 -3y & = & -3 \\
 y & = & 1
 \end{array}
 \qquad
 \begin{array}{rcl}
 x & = & -1 \\
 y & = & 1
 \end{array}$$

Remember: Check.

Remember: If the results are a contradiction such as $7 = 5$, there are no solutions. Graphs are parallel.

If the results are an identity such as $6 = 6$, there are an infinite number of solutions. The graphs are identical.

Application 1.

The parts department of an auto dealership sold 175 sets of windshield wipers. Some were sold at \$5.95 others were sold at \$6.45. The total receipts were \$1073.75. How many sets were sold at each price?

Let x = no. of sets sold at \$5.95 each
 y = no. of sets sold at \$6.45 each

$$\begin{array}{rcl}
 x + y & = & 175 \\
 5.95x + 6.45y & = & 1073.75
 \end{array}
 \qquad
 \text{Mult. by } 5.95$$

$$\begin{array}{rcl}
 5.95x + 5.95y & = & 1041.25 \\
 5.95x + 6.45y & = & 1073.75
 \end{array}$$

Since the coefficients of y are equal and positive use the rules for subtraction.

$$\begin{array}{rcl}
 5.95x + 5.95y & = & 1041.25 \\
 5.95x + 6.45y & = & 1073.75 \\
 \hline
 -.50y & = & -32.50 \\
 y & = & 65
 \end{array}$$

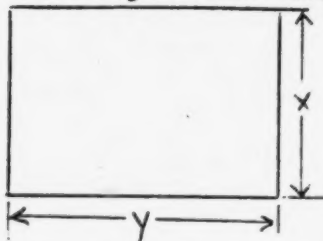
Substituting

$$\begin{array}{rcl}
 x + y & = & 175 \\
 x + 65 & = & 175 \\
 x & = & 175 - 65 \\
 x & = & 110
 \end{array}
 \qquad
 \begin{array}{rcl}
 x & = & 110 \text{ sets at } \$5.95 \\
 y & = & 65 \text{ sets at } \$6.45
 \end{array}$$

Remember: Check

Application 2.

The perimeter of a rectangular piece of sheet metal is 14 feet. If the length of the sheet metal is one foot longer than the width, find the dimensions.



Let x = width
 y = length

$$2x + 2y = 14 \text{ (Perimeter Formula)}$$

$$y = x + 1$$

$$2x + 2y = 14$$

$$-x + y = 1 \text{ (Multi. by 2)}$$

$$2x + 2y = 14$$

$$\underline{-2x + 2y = 2}$$

$$4y = 16 \text{ (Rule: For Addition)}$$

$$y = 4$$

$$y = x + 1$$

$$4 = x + 1$$

$$4 - 1 = x \quad x = 3 \text{ ft.}$$

$$3 = x \quad y = 4 \text{ ft.}$$

Drill and Practice:

Rising, p. 510

Dressler, p. 571-572

10. CONCEPT: GRAPHING SYSTEMS OF LINEAR INEQUALITIES

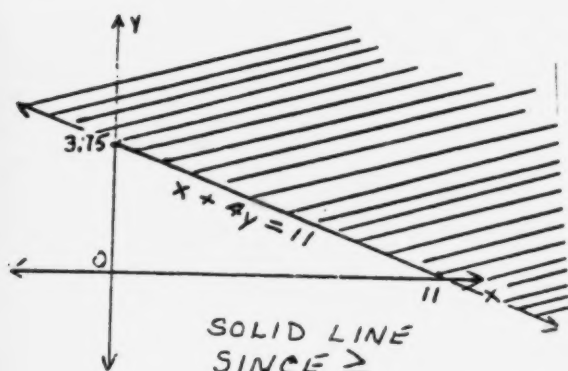
Solve the system graphically:

$$\begin{aligned}x + 4y &\geq 11 \\ 2x - 5y &> 9\end{aligned}$$

Using a suitable method (intercept method) graph the equations $x + 4y = 11$ and $2x - 5y = 9$. These lines are the boundaries of the regions which include points which do or do not satisfy the inequality. Sometimes as in the case of $2x + 5y > 9$, the graph of the equation itself satisfies the inequality.

Graph: $x + 4y = 11$

y-intercept = 3.75
x-intercept = 11

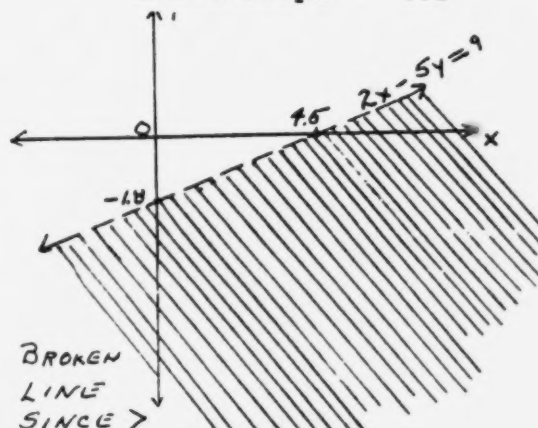


Now solve inequality for y:

$$\begin{aligned}x + 4y &\geq 11 \\ 4y &\geq -x + 11 \\ y &\geq \frac{1}{4}x + 3.75\end{aligned}$$

Since $\geq$, shade above line.Graph: $2x - 5y = 9$.

y-intercept = -1.8
x-intercept = 4.5

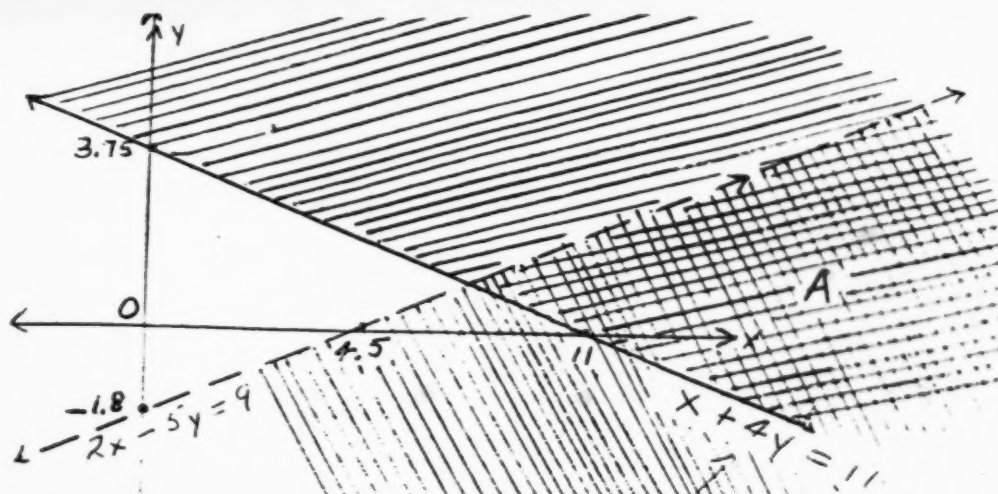


Now solve inequality for y:

$$\begin{aligned}2x - 5y &> 9 \\ -5y &> -2x + 9 \\ y &< \frac{2}{5}x - 1.8\end{aligned}$$

Notice that the direction of the inequality sign changes with division by (-5) . Therefore, since $<$, shade below the line.

Now place both graphs on same set of axes.



"A" represents the area in which all points occur which satisfy both equations. This includes the points on $x + 4y = 11$ which bound "A".

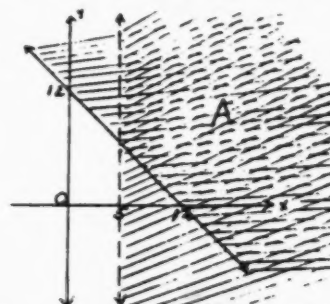
Examples:

Graph Solution

$$x > 5$$

$$x + y \geq 12$$

Solution: A.



Application.

The Allied Co. manufactures automobile mufflers. Models A and B. Each model must pass through two processes. There is a limit to the total number of minutes available for each process and profits should be at a maximum.

After reading the table below calculate graphically the number of mufflers of each model which should be manufactured in order to gain maximum profit.

	Model A	Model B	Total
	\$8/ Muffler	\$10/ Muffler	Time Available
Unit Profit			
Process 1	10 min./muffler	8 min./muffler	1600 min.
Process 2	9 min./muffler	12 min./muffler	2100 min.

Let x = no. of mufflers (Model A)

y = no. of mufflers (Model B)

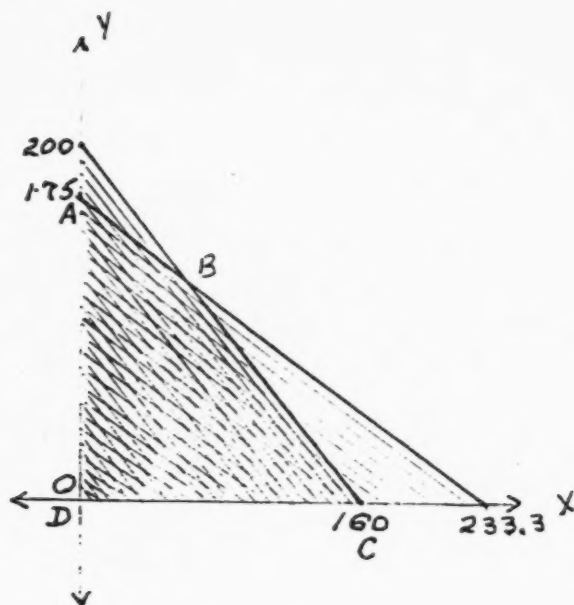
$$(\text{Process 1}) \quad 10x + 8y \leq 1600$$

$$(\text{Process 2}) \quad 9x + 12y \leq 2100$$

We must also consider that $x \geq 0$ and $y \geq 0$ since negative production is not meaningful.

Total profit is determined by

$$8x + 10y \quad (\text{see unit profit in table})$$



Graph: $10x + 8y \leq 1600$
 $9x + 12y \leq 2100$
 $x \geq 0$
 $y \geq 0$

Find the coordinates of B by solving

$$\begin{array}{rcl} 10x + 8y = 1600 & (\text{Divide by } 2) & 5x + 4y = 800 \\ 9x + 12y = 2100 & (\text{Divide by } 3) & 3x + 4y = 700 \\ \hline & & 2x = 100 \\ & & x = 50 \end{array}$$

Solve for y

$$\begin{array}{rcl} 10(50) + 8y = 1600 & & \\ 500 + 8y = 1600 & & \\ 8y = 1600 - 500 & & \\ 8y = 1100 & & \\ y = 137.5 \text{ or } 138 & & \end{array}$$

Coordinates of B
 $(50, 138)$

To find the maximum (or minimum) of the expression, substitute the coordinates of the vertices of ABCD, into $8x + 10y$.

A(0, 175)	B(50, 138)	C(160, 0)	D(0, 0)
$8(0) + 10(175)$	$8(50) + 10(138)$	$8(160) + 10(0)$	$12(0) + 10(0)$
\$1750	$400 + 1380$	\$1280	\$0
	\$1780		

Therefore maximum profits are earned when 50 Model A mufflers are manufactured and 138 Model B mufflers are manufactured.

Drill and Practice:

Rising, p. 517, 520-521

Dressler, p. 582

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Expanding from the CORE curricula in math this program completes the trip through blueprint reading skills.

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The sets of decision making skills needed to solve a workplace related problem in a work team are presented.

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